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Quasi-compactness and uniform ergodicity of Markov operators

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RÉSUMÉ. — Il est démontré que, pour des opérateurs Markoviens, la quasi-compacité est équivalente à la convergence ergodique uniforme vers une projection de rang fini. Pour des probabilités de transition ergodiques, la quasi compacité est équivalente à la convergence ergodique forte, même pour un espace d'états de type non-dénombrable.

SUMMARY. — It is shown that, for Markov operators, uniform ergodicity with finite dimensional fixed points space is equivalent to quasi-compactness. For ergodic transition probabilities, strong convergence of the averages is shown equivalent to quasi-compactness, even when the σ-algebra is not countably generated.

The study of a quasi-compact linear operator T on a Banach space was given by Yosida and Kakutani [*16*], in order to obtain some of the limit theorems of Doeblin [*3*]. Horowitz [*8*] and Brunel [*1*] studied quasi-compact conservative and ergodic contractions in $L_1(m)$ (see also Lin [*9*]). Brunel and Revuz [*2*] studied the quasi-compactness of Harris recurrent transition probabilities on a countably generated σ-algebra.

The result of Yosida and Kakutani is that for T quasi-compact with

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$\| T \| \leq 1, \frac{1}{N} \sum_{j=1}^{N} T^j$ converges uniformly to a finite dimensional projection (see [5] for the treatment of the case where only $T^n/n \to 0$ weakly is assumed). In this work we show that for Markov operators (on L_∞ or on C(K)), uniform ergodic convergence to a finite dimensional projection implies quasi-compactness. The result is used to remove the countability assumption from Brunel and Revuz' result [2].

An important tool is the following « uniform ergodic theorem » [10]. *If* $\| T \| \leq 1$, *then* $\frac{1}{N} \sum_{j=1}^{N} T^j$ *converges uniformly if and only if* $I - T$ *has a closed range*. The limit is a projection E on the fixed points of T.

We deal with C(X) after treating the L_1 case (« abstract » Markov operators), for which we need the following lemma.

LEMMA 1. — *Let* T *be a conservative and ergodic positive contraction of* $L_1(X, \Sigma, m)$, *with* m *σ-finite . If* $|\lambda| = 1$ *and* λ *is an eigenvalue of* T^*, *then* $\{ f \in L_\infty : T^*f = \lambda f \}$ *is finite dimensional.*

Proof. — If $T^*f = \lambda f$, then $T^* | f | \geq | T^*f | = | f |$, and $T^* | f | = | f |$ since T is conservative, hence $| f | =$ constant by ergodicity, and we may assume $| f | \equiv 1$.

Now identify L_∞ with a space C(K), K compact Hausdorff. T^* corresponds to a positive contraction S on C(K). If $T^*f_1 = \lambda f_1$ and $T^*f_2 = \lambda f_2$, with $| f_1 | = | f_2 | = 1$, then $S\hat{f}_1 = \lambda \hat{f}_1$, $S\hat{f}_2 = \lambda \hat{f}_2$ (with $\hat{f}_i$ the images of f_i in C(K)). By a well-known result on unimodular eigenfunctions (see Schaefer [15]), $S(\hat{f}_1 \hat{f}_2^{-1}) = \hat{f}_1 \hat{f}_2^{-1}$, so $T(f_1 f_2^{-1}) = f_1 f_2^{-1}$. Hence $f_1 f_2^{-1}$ is constant, and $f_1 \doteq \alpha f_2$ with $|\alpha| = 1$, and the lemma holds.

Remark. — For λ a root of unity, one can avoid the representation of L_∞ and use the result of Foguel and Weiss [7] on the finiteness of the invariant sets of T^{*n}. The proof is *not* shorter.

THEOREM 2. — *Let* T *be a positive contraction of* $L_1(m)$. (*i*) T *is quasi-compact if and only if* $N^{-1} \sum_{j=1}^{N} T^j$ *converges uniformly to a finite dimensional projection.* (ii) *In this case,* $I - \lambda T$ *has closed range for every* $|\lambda| = 1$, *and* $\sigma(T) \cap \{ |\lambda| = 1 \}$ *consists of finitely many points, which are all roots of unity, and are eigenvalues with finite dimensional eigenspaces.*

Proof. — Assume $\left\| \frac{1}{N}\sum_{j=1}^{N} T^j - E \right\| \to 0$, with $EL_1(m)$ finite dimensional.
Let C be the conservative part of T. Since $L_1(C)$ is T-invariant [6], the uniform ergodic theorem implies that there is an $f \in L_1(C)$, $f > 0$ a. e. on C, satisfying $Tf = f$. By taking an equivalent finite measure m' such that $dm'/dm = f$ on C, we may and do assume $T1_C = 1_C$ ($T'f' = (dm/dm')$ $T(f'dm'/dm)$ is the necessary transformation). We shall assume also $m(C) = 1$. Clearly now $E1_C = 1_C$. The restriction of T to $L_1(C)$ is a contraction of $L_\infty(C)$, and since $\{ f : Tf = f \}$ is finite dimensional, there are finitely many disjoint sets $A_1, \ldots, A_K$ with union C such that $TL_1(A_j) \subset L_1(A_j)$ and T is ergodic on A_i [6].

Let $|\lambda| = 1$, $1 \neq \lambda \in \sigma(T)$. We show that $\lambda I - T$ has closed range. Let $(\lambda I - T)f_n \to f$ in L_1 norm. Define $g_{nk} = |f_n - f_k| - |T(f_n - f_k)|$. Then we have,

$$\| g_{nk} \| = \int \left| \, |f_n - f_k| - |T(f_n - f_k)| \, \right| dm$$
$$\leq \int |\lambda(f_n - f_k) - T(f_n - f_k)| \, dm \xrightarrow[n,k\to\infty]{} 0.$$

Since by definition $g_{nk} + T|f_n - f_k| - |f_n - f_k| \geq 0$ a. e.,

$$\| (I - T)|f_n - f_k| \| \leq \| (I - T)|f_n - f_k| - g_{nk} \| + \| g_{nk} \|$$
$$= \int [g_{nk} + T|f_n - f_k| - |f_n - f_k|] dm + \| g_{nk} \|$$
$$\leq \int g_{nk} dm + \| g_{nk} \| \leq 2 \| g_{nk} \| \xrightarrow[n,k\to\infty]{} 0.$$

By the uniform ergodic theorem, $I - T$ is invertible on $(I - T)L_1$. Since $|f_n - f_k| - E|f_n - f_k| \in (I - T)L_1$, and $(I - T)E = 0$, we have

$$\| (I - T)\{ |f_n - f_k| - E|f_n - f_k| \} \| \xrightarrow[n,k\to\infty]{} 0$$

and therefore

$$\| \, |f_n - f_k| - E|f_n - f_k| \, \| \xrightarrow[h,k\to\infty]{} 0.$$

Denote $a_{nki} = \int_{A_i} |f_n - f_k| \, dm$. But $E|f_n - f_k| = \sum_{i=1}^{K} a'_{nki} 1_{A_i}$. Hence

$$\int_D |f_n - f_k| \, dm + \sum_{i=1}^{K} \int_{A_i} \left| \, |1_{A_i} f_n - 1_{A_i} f_k| - a_{nki} \right| dm \to 0\,.$$

Applying lemma VI.2 of [2] K times, we have that $\{ f_n \}$ contains a Cauchy subsequence. If $f_{n_i} \to g$, $(\lambda I - T)g = \lim (\lambda I - T)f_{n_i} = f$ and $(\lambda I - T)L_1$ is closed (We extend the argument of [*2*]). Applying the uniform ergodic theorem to λT, we have that λ is an eigenvalue of T.

We now have that if $\lambda \in \sigma(T)$ with $|\lambda| = 1$, λ is an eigenvalue of T, hence of T^*. λ is isolated in $\sigma(T)$, since $\lambda I - T$ is invertible on its range. Hence $\sigma(T) \cap \{ \lambda : |\lambda| = 1 \}$ consists of finitely many eigenvalues $\lambda_1, \ldots, \lambda_r$.

If $\lambda \in \sigma(T)$ with $|\lambda| = 1$, there is an $f \in L_1$ with $Tf = \lambda f$, and $T|f| \geq |Tf| = |f|$, hence $T|f| = |f|$ a. e., and $\{ |f| \neq 0 \} \subset C$. To show $\{ f \in L_1 : Tf = \lambda f \}$ is finite dimensional we may and do assume T to be conservative. Since $T(1_{A_j} f) = 1_{A_j} Tf = \lambda 1_{A_j} f$, we may restrict ourselves to the invariant set A_j, and have T also ergodic. Let T_j be the restriction of T to $L_1(A_j)$. If λ *is* an eigenvalue of T_j it is of T_j^*. Apply now the lemma to obtain that $\{ f \in L_1(A_j) : Tf = \lambda f \}$ is finite dimensional, hence $X_\lambda = \{ f \in L_1 : Tf = \lambda f \}$ is finite dimensional.

Let $E_k = \lim \frac{1}{N} \sum_{j=1}^{N} \lambda_k^{-j} T^j$ (which exists uniformly). Then

$$L_1 = \left(\sum_{k=1}^{r} E_k \right) L_1 \oplus \bigcap_{k=1}^{r} (\lambda_k I - T) L_1$$

and putting $Q = \sum_{k=1}^{r} \lambda_k E_k$, we have

$$\| T^n - Q^n \| = \left\| T^n \left(I - \sum_{k=1}^{r} E_k \right) \right\| \to 0,$$

since on $\bigcap_{k=1}^{r} (\lambda_k I - T) L_1$ the restriction of T has no spectral points of unit modulus. Q is compact ($E_k L_1$ is finite dimensional) and T is quasi-compact.

Quasi-compactness implies the uniform ergodic theorem [*16*] [*10*].

The fact that the eigenvalues λ_k are roots of unity follows (looking at T_j^* as before), from the above mentioned lemma of [*15*], which shows that $\lambda_k^n \in \sigma(T)$ for every *n*. (One can use directly the result in [*5*], since T^* is also quasi-compact).

Remarks. — 1. When T is conservative, Horowitz' result [*8*] is that *strong* convergence of $\frac{1}{N} \sum_{j=1}^{N} T^{*j}$ to a finite dimensional projection implies quasi-compactness.

2. For non-positive contractions the result is false. The example in [*11*] is valid in l_1.

THEOREM 3. — *Let* P *be a positive contraction of* C(X), (X *compact Hausdorff*). *The following conditions are equivalent.*

(*i*) P *is quasi-compact.*

(*ii*) $N^{-1}\sum_{i=1}^{N} P^i$ *converges uniformly to a finite dimensional projection.*

(*iii*) (I – P)C(X) *is closed and* $\{ f : Pf = f \}$ *is finite dimensional.*

(*iv*) (I – P*)C(X)* *is closed and the space of invariant measures is finite dimensional.*

(*v*) (I – P)C(X) *is closed and the space of invariant measures is finite dimensional.*

Proof. — (*i*) ⇒ (*ii*) is due to Yosida and Kakutani [*16*] (see [*10*] for another proof). (*ii*) ⇒ (*iii*) is immediate, by the uniform ergodic theorem.

(*iii*) ⇒ (*iv*). By the uniform ergodic theorem $\frac{1}{N}\sum_{i=1}^{N} P^i$ converges uniformly to a projection E on $\{ f = Pf \}$. Hence dim E^*X^* = dim EX $< \infty$. Since $\frac{1}{N}\sum_{i=1}^{N} P^{*i} \to E^*$ uniformly, (*iv*) follows.

(*iv*) ⇒ (*v*). By the uniform ergodic theorem $\frac{1}{N}\sum_{i=1}^{N} P^{*i}$ converges uniformly, and so does $\frac{1}{N}\sum_{i=n}^{N} P^i$, implying (I – P)C(X) is closed.

(*v*) ⇒ (*ii*). Follows again from the uniform ergodic theorem.

Assume now conditions (*ii*) – (*v*). We can extend P to the space of complex continuous functions, so we may and do assume that this has been done.

Let $|\lambda| = 1$, $\lambda \in \sigma(P)$. We show that $I - \lambda P$ has closed range. Let $(I - \lambda P^*)\mu_n \to \mu$. Define $m_1 = \sum_{n=0}^{\infty} \alpha_n |\mu_n| + \alpha |\mu|$, with α_n, $\alpha > 0$ such that m_1 is a probability. Let $m = \sum_{n=0}^{\infty} 2^{-(n+1)} P^{*n} m_1$. Then $P^*m \ll m$ and we obtain an operator T on $L_1(m)$ by $Tu = d(P^*\nu)/dm$ when $u = d\nu/dm$. By (*iv*) $\frac{1}{N}\sum_{i=1}^{N} T^i$ converges to a finite dimensional projection. Since $(I - \lambda T)u_n \to u$ in $L_1(m)$, when $u_n = d\mu_n/dm$, then by theorem 2, $u = (I - \lambda T)v$ with $v \in L_1(m)$, and $\mu = (I - \lambda P^*)\nu$ with $d\nu/dm = v$. Hence $I - \lambda P^*$ has closed range, and $\frac{1}{N}\sum_{i=1}^{N} \lambda^i P^i$ converges, by the uniform ergodic theorem, with limit $E_{\bar\lambda}$, which projects on $\{ f : Pf = \bar\lambda f \}$.

We show that $E_{\bar\lambda}^*$ has finite dimensional range. If not, let $\{ \nu_n \}_{n=1}^{\infty}$ be a sequence of linearly independent (complex) finite measures with $P/\nu_n = \bar\lambda \nu_n$. Assume $\| \nu_n \| = 1$. Let $m_1 = \sum_{n=1}^{\infty} 2^{-n} | \nu_n |$ and $m = \sum_{n=0}^{\infty} 2^{-(n+1)} P^{*n} m_1$. Again, we have a positive contraction T on $L_1(m)$, with $Tu_n = \bar\lambda u_n$, where $u_n = d\nu_n/dm$. But by theorem 2, all eigenvalues of T have finite dimensional eigenspaces, which yields a contradiction. Since $\frac{1}{N}\sum_{n=1}^{N} \lambda^{-n} P^n$ converges uniformly for $\lambda \in \sigma(P)$ with $|\lambda| = 1$, λ is isolated in $\sigma(P)$, hence $\sigma(P) \cap \{ \lambda : |\lambda| = 1 \}$ is finite.

The end of the proof of theorem 2 can be used to show that P is quasi-compact.

Remark. — Horowitz' result [*8*] (see previous remarks) fails for general positive contractions of C(X), as observed in [*10*].

COROLLARY 4. — *Let* P *be a positive contraction of* C(X). *If* $\frac{1}{N}\sum_{j=1}^{N} P^j$ *converges uniformly to a finite dimensional projection, then, for every* $k \geq 1$, $\frac{1}{N}\sum_{j=1}^{N} P^{kj}$ *converges uniformly.*

Remarks. — Sawashima and Niiro [*14*] give an example where the corollary fails if the limit is infinite dimensional. Their example also shows that a positive contraction T on L_1 may satisfy the uniform ergodic theorem, while T is not quasi-compact on $(I - T)L_1$.

For comparison, we note the following result concerning the Markov operator $Pf(x) = \int f(y)P(x, dy)$ induced by a transition probability. We omit the additional requirement of a countably generated σ-algebra which appears in [*2*]. The Harris condition and the existence of an invariant measure need not be assumed *a priori*. For simplification we assume P to be ergodic, i. e., $Pf = f$ implies f is constant.

THEOREM 5. — *Let* Σ *be a* σ*-algebra of subsets of* X *and let* $P(x, A)$ *be a transition probability inducing an ergodic (Markov) operator on the space* $B(X, \Sigma)$ *of bounded measurable functions. Then the following conditions are equivalent.*

(*i*) P *is quasi-compact.*

(*ii*) $\frac{1}{N}\Sigma P^i$ *converges uniformly* (necessarily to a one dimensional projection).

(*iii*) $(I - P)B(X, \Sigma)$ *is closed.*

(*iv*) $(I - P^*)M(X, \Sigma)$ *is closed.* ($M(X, \Sigma) = \{$ finite signed measures $\}$).

(*v*) *Every* P^**-invariant functional is a measure.*

(*vi*) *The space of* P^* *invariant functionals is one-dimensional.*

(*vii*) $\frac{1}{N}\sum_{i=1}^{N} P^i$ *converges strongly.*

(*viii*) P *satisfies Doeblin's condition.*

Proof. — (*i*) $\Rightarrow$ (*ii*) is well-known [*16*] [*10*].

(*iii*) $\Leftrightarrow$ (*ii*). By the uniform ergodic theorem and then (*ii*) $\Rightarrow$ (*iv*).

(*iv*) $\Rightarrow$ (*ii*). Let T be the restriction of P^* to $M(X, \Sigma)$. By the uniform ergodic theorem $\frac{1}{N}\sum_{i=1}^{N} T^i$ converges uniformly, so does $\frac{1}{N}\Sigma T^{*i}$, which is $\frac{1}{N}\sum_{i=1}^{N} P^i$ when restricting T^* to $B(X, \Sigma)$. Hence (*ii*), (*iii*) and (*iv*) are equivalent.

(*ii*) $\Rightarrow$ (*v*). $\left\| \frac{1}{N}\sum_{i=1}^{N} P^i - E \right\| \to 0$, and Ef is invariant for any f, so it is constant by ergodicity.

Let m be any probability on Σ. Let

$$\mu(\mathrm{A}) = \mathrm{E}1_{\mathrm{A}} = \int \lim_{\mathrm{N}\to\infty} \frac{1}{\mathrm{N}} \sum_{i=1}^{\mathrm{N}} \mathrm{P}^i 1_{\mathrm{A}} dm = \lim_{n\to\infty} \frac{1}{\mathrm{N}} \sum_{i=1}^{\mathrm{N}} \mathrm{P}^{*i} m(\mathrm{A}).$$

Then μ is a probability, and $\langle \mu, f \rangle = \mathrm{E}f$, showing $\mathrm{P}^*\mu = \mu$.

If $\mathrm{P}^*\nu = \nu$ then

$$\nu(f) = \left\langle \frac{1}{\mathrm{N}} \sum_{i=1}^{\mathrm{N}} \mathrm{P}^{*i}\nu, f \right\rangle = \left\langle \nu, \frac{1}{\mathrm{N}} \sum_{i=1}^{\mathrm{N}} \mathrm{P}^i f \right\rangle \to \langle \nu, \mathrm{E}f \rangle = \mathrm{E}f.\nu(1).$$

Hence $\nu = \nu(1).\mu$ and is a measure.

(*v*) $\Rightarrow$ (*vi*). Let $\nu \neq \mu$ be two invariant probabilities. Let $m = \nu + \mu$. Then $\mathrm{P}^*m = m$, and T defined in $\mathrm{L}_1(m)$ by $\mathrm{T}(d\mu/dm) = d(\mathrm{P}^*\mu/dm)$ is a positive contraction of $\mathrm{L}_1(m)$, with $\mathrm{T}1 = 1$. Let $f \in \mathrm{B}(\mathrm{X}, \Sigma)$ with $\int f d\mu \neq \int f d\nu$. $\mathrm{T}^*f = \mathrm{P}f m$ a. e., and by the ergodic theorem $\lim \frac{1}{\mathrm{N}} \Sigma \mathrm{T}^{*i} f = \hat{f}$ exists a. e. and $\mathrm{T}^*\hat{f} = \hat{f}$ a. e. (m). But $\mathrm{T}^*(d\mu/dm) = d\mu/dm$, $\mathrm{T}^*(d\nu/dm) = d\nu/dm$, so that

$$\int \hat{f} d\mu = \int \lim \frac{1}{\mathrm{N}} \sum_{i=1}^{\mathrm{N}} \mathrm{T}^{*i} f d\mu = \int f d\mu \neq \int \hat{f} d\nu.$$

Let $g = \limsup \frac{1}{\mathrm{N}} \sum_{i=1}^{\mathrm{N}} \mathrm{P}^i f$. Then $\mathrm{P}g \geq \limsup \frac{1}{\mathrm{N}} \sum_{i=2}^{n+1} \mathrm{P}^i f = g$, and $h = \lim \mathrm{P}^k g$ is P invariant. Hence h is constant (P is ergodic). Since $h = \hat{f}$ a. e. (m), $\int \hat{f} d\mu = \int h d\mu = \int h d\nu = \int \hat{f} d\nu$, a contradiction. Since the positive and negative parts of an invariant functional are invariant, (*vi*) is proved.

(*vi*) $\Rightarrow$ (*vii*). Let ν be a functional on $\mathrm{B}(\mathrm{X}, \Sigma)$ such that $\nu(f) = 0$ for $f \in \overline{(\mathrm{I} - \mathrm{P})\mathrm{B}(\mathrm{X}, \Sigma)} \oplus \{\text{constants}\}$. Then $\mathrm{P}^*\nu = \nu$ and $\nu(1) = 0$. But condition (*vi*) implies $\nu = \nu(1)\mu$ where μ is an invariant positive functional. Hence $\nu = 0$. By the Hahn-Banach theorem

$$\overline{(\mathrm{I} - \mathrm{P})\mathrm{B}(\mathrm{X}, \Sigma)} \oplus \{\text{constants}\} = \mathrm{B}(\mathrm{X}, \Sigma),$$

and (*vii*) follows.

(*vii*) $\Rightarrow$ (*ii*). Let $(\mathrm{I} - \mathrm{P})f_n \to g$ in norm. By Doob [*4*, p. 209] there is a countably generated σ-algebra Σ_0 such that for $\mathrm{A} \in \Sigma_0 \mathrm{P}(x, \mathrm{A})$ is Σ_0 measurable, and f_n, g are Σ_0-measurable (Σ_0 is the admissible σ-algebra generated by the sets $\{r \leq f_n \leq s\}$, $\{r \leq g \leq s\}$, r, s rationals).

Let $P_0(x, A) = P(x, A)$ for $x \in X$, $A \in \Sigma_0$. For $f \in B(X, \Sigma_0)$, $P_0 f = Pf$, so P_0 satisfies (*vii*). (*vii*) $\Rightarrow$ (*v*), so by Brunel and Revuz [*2*] (who use the fact that Σ_0 is countably generated) $\frac{1}{N}\sum_{i=1}^{N} P_0^i$ converges uniformly and $I - P_0$ has closed range. Hence $g = (I - P_0)f$ with $f \in B(X, \Sigma_0)$, so $g = (I - P)f$. Therefore $I - P$ has closed range and (*ii*) holds by the uniform ergodic theorem.

(*ii*) $\Rightarrow$ (*i*). $B(X, \Sigma)$ is isometrically and order-isomorphic to $C(K)$ and theorem 3 applies, since the limit projection has one-dimensional range by ergodicity.

(*i*) $\Leftrightarrow$ (*viii*) is shown in [*13*].

Remarks. — 1. Condition (*v*) is equivalent to. (*v'*) There is no purely finitely additive P^*-invariant functional (since the countably additive part of an invariant functional is invariant).

2. Condition (*vii*) clearly implies the Harris condition:

$$m(A) = \lim \frac{1}{N}\sum_{n=1}^{N} P^n 1_A$$

is (the unique) invariant probability, and

$$m(A) > 0 \Rightarrow \sum_{n=1}^{\infty} P^n 1_A(x) = \infty .$$

Note that Moy [*12*] establishes the existence of a kernel under this condition without Σ being countably generated. It is not clear how to obtain a compact operator in $B(X, \Sigma)$, bounded by P^{n_0} (which is easy in $L_\infty(m)$) without the countable additivity, and this is why we use Σ_0 in proving (*vii*) $\Rightarrow$ (*ii*).

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