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The Equivalence Principle Meets the Uncertainty Principle

by

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ABSTRACT. — We argue that the very existence of the weak Equivalence Principle can only be understood in terms of the Uncertainty Principle.

RÉSUMÉ. — Nous présentons des arguments montrant que l'existence même du Principe d'Équivalence Faible ne peut être comprise qu'en relation avec le Principe d'Incertitude.

Louis De Broglie was, as was Albert Einstein, a master iconoclast —a constant challenger of universally held myths.

It is in this grand, but difficult, tradition of being willing to step on everyone's toes, that Jean-Pierre Vigier was trained as De Broglie's assistant, and in which he has fully matured. Not only in his professional life as a physicist, but in his private life as well, has he set his own direction and followed his own conscience.

In the same spirit I would like to tell you why I think the search for a classical unified field theory failed. I think it also explains why I expect the search for a quantized version of gravity to also fail. It is primarily because I believe the search is heading in the totally wrong direction. The problem is not the mathematical complexity of the theory, but rather that the physics is wrong. I believe that there are still many clues being offered by the equivalence principle, and insights into this principle, that we have not made use of. And they lead in a totally different direction. (I recognize

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that electrodynamics has been combined with the weak interaction. Nonetheless I believe that there *is* a unification with gravity to be had, only that it will proceed along very different lines than have so far been investigated.)

I believe that the very existence of the weak equivalence principle can only be understood in terms of the uncertainty principle, and I will try here to explain this. It is easiest to describe these ideas in terms of a simple extension of special relativity, which takes the mass more seriously as a dynamical variable. (The details of this theory were worked out in Ref. [*1*].)

Specifically, the theory says that in fact we live not in a 4-dimensional spacetime continuum, but rather in a 5-dimensional one, and the fifth dimension is not a hidden, folded-up, mathematical fiction, but is as real as the other four. It is the proper time, which should be treated as an independent degree of freedom. Why? Because like any true degree of freedom, we can not only arbitrarily set it, but we can arbitrarily determine its rate of change.

One can always (in principle) erect a spherical mass sheet around a region and this will affect the gravitational potential inside, and hence the rate at which the proper time of a particle runs relative to coordinate time there, without exerting any forces in the region or otherwise disrupting the local physics. (Thus it is only the second derivative, the « acceleration » of the proper time, that is affected by the laws of motion.)

The dynamical variable conjugate to the proper time is the mass, and together they enter the Hamiltonian of the problem in the same way as x and p. This in turn determines the equations of motion, which yield not only x as a function of the time, t, but also the proper time, τ, as a function of t (so that this needs no longer to be postulated as part of the kinematics). Also, just as p can change as a function of time, so too can m change as a function of time. Thus the theory is a classical theory of changing mass, and represents a new and simple way to treat decaying particles, even classically.

But for our purposes, we merely write as an illustration, the Hamiltonian for a free, stable particle

$$\mathrm{H} = \sqrt{(pc)^2 + (mc^2)^2}\,.$$

Now in ordinary physics, m here is just a parameter, and there is no explanation for the symmetry between m and p.

However the independent variables here are x and τ, to be determined as $f(t)$ by solving the equations of motion. Their conjugate variables are p and m. For a free particle the change of p and m are given by

$$\dot{p} = -\frac{\partial \mathrm{H}}{\partial x} = 0, \qquad c^2\dot{m} = -\frac{\partial \mathrm{H}}{\partial \tau} = 0\,,$$

which merely says that momentum and mass are conserved here, as is expected. But one sees that just as one could have H depend on x, producing a force which changes p, one could equally as well make H depend on τ, which would produce a « force » that would change the mass of the particle.

Yet even at this simple level, there is further information to be gained, for the other equations are

$$\frac{dx}{dt} = v = \frac{\partial \mathrm{H}}{d\tau}, \qquad \frac{d\tau}{\partial t} = \frac{1}{c^2}\frac{\partial \mathrm{H}}{\partial m}.$$

These equations determine

$$v = \frac{c^2 p}{\sqrt{(pc)^2 + (mc^2)^2}}, \qquad \text{or} \qquad p = \frac{mv}{\sqrt{1 - (v^2/c^2)}},$$

and also

$$\frac{d\tau}{dt} \equiv \dot{\tau} = \frac{mc^2}{\sqrt{(pc)^2 + (mc^2)^2}} = \sqrt{1 - (v^2/c^2)}\,.$$

So this latter equation need not be postulated separately, as a property of the metric, but follows directly from the symmetry of the theory. In a sense, when you were taught relativity, you were taught only half of the subject. (Also, in this theory, if the particle decays, this will affect the rate of passage of proper time.)

It is not necessary here to consider a more complicated system. We merely have to point out that in this dynamics, besides the usual uncertainty principles

$$\Delta x \Delta p \sim \hbar\,,$$
$$\Delta \mathrm{E} \Delta t \sim \hbar\,,$$

there is a new one

$$c^2 \Delta m \Delta \tau \sim \hbar\,.$$

This last equation says that when one tries to measure the mass, or the proper time, directly, there is a limit to the combined accuracy of the two, which is determined by ordinary quantum considerations.

For example, let us assume we want to measure the mass, M, of a heavy particle by gravitationally scattering off of it a light particle of known mass, m, and velocity v (see the figure). The time of maximum interaction, T, is given by

$$\mathrm{T} \sim b/v\,.$$

The transverse momentum picked up by the light particle will be

$$\mathrm{P}_y \sim \mathrm{F}_y \mathrm{T} \sim \frac{\mathrm{GM}m}{b^2}\cdot\frac{b}{v} \sim \frac{\mathrm{GM}m}{bv}\,.$$

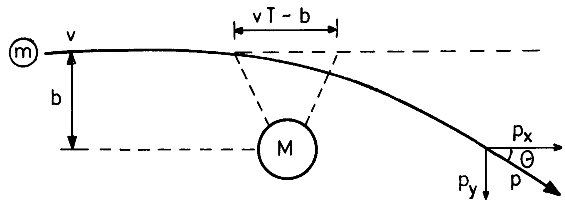

FIG. 1. — The Uncertainty Principle for Mass and Proper Time. If one measures the mass M of a heavy particle by scattering a light mass off of it gravitationally, then the angle θ determines M. But the light particle exerts an unknown gravitational potential on M, to the extent that b is unknown, so that $c^2 \Delta m \Delta \tau \simeq \hbar$, where τ is the proper time read by a clock located on M.

and the deflection will be approximately (for small θ)

$$\theta \sim \frac{p_y}{p} \sim \frac{\mathrm{G}\mathrm{M}m}{bv} \Big/ mv \sim \frac{\mathrm{GM}}{bv^2},$$

which yields a measure of M. But also

$$\delta \mathrm{M} \sim \frac{bv}{\mathrm{G}m} \delta p_y .$$

This gives both the mass and the accuracy to which it is known.

However if there were a clock sitting on particle M, then even if we knew its time exactly before we performed the experiment, we would no longer know it accurately after the experiment has been performed. This is because while the light particle passed by, it exerted a gravitational potential on M, which will affect the clock reading. If the distance b is known to within δb, then

$$\delta \tau \sim \frac{\delta \varphi}{c^2} \mathrm{T} \sim \left(\frac{\mathrm{G}m}{c^2 b^2} \delta b \right) \left(\frac{b}{v} \right) \sim \frac{\mathrm{G}m \delta b}{c^2 bv},$$

and one has

$$c^2 \delta \mathrm{M} \delta \tau \sim \left(\frac{c^2 bv}{\mathrm{G}m} \delta p_y \right) \left(\frac{\mathrm{G}m}{bv} \delta b \right) \sim \delta p_y \cdot \delta b \sim \hbar .$$

So the extent to which one can accurately measure the mass of the particle limits the knowledge one has of its proper time, and *vice versa*. One sees that this only depends on the usual uncertainty principle between p_y and y, operating on the light particle, and so is unavoidable. (For further examples of the uncertainty principle, see the second paper of ref. [*1*].) There are any number of examples of this type, and they all show that as long as there is an uncertainty relation between E and t, and p and x, then there must be one between m and τ.

It seems to me that this uncertainty principle between m and τ leads to an important insight into the nature of the weak equivalence principle, because this principle states that in an external gravitational field the motion of a particle is independent of its mass, m.

Thus, in an external gravitational field one may know the mass very poorly, and yet still accurately determine the position and velocity of the particle. In other words, one can have (nonrelativistically)

$$\Delta x \sim 0\,,$$
$$\Delta v \sim 0\,,$$
$$\Delta m \sim \infty\,,$$
$$\Delta p \sim v\Delta m \sim \infty\,.$$

So without violating the uncertainty principle, one can know the trajectory of a particle very accurately, provided one does not know its mass, because in this case, one can know the velocity without knowing the momentum. But this situation, so totally different from the usual quantum case where one cannot know trajectories, is possible only because of the equivalence principle. For example, Kepler's law states that for a particle in a circular orbit in a field with a $1/r$ potential, $r^3/T^2 = GM/4\pi^2$, independently of the mass of the orbiting particle m. Thus even if m is very poorly known (indeed, because it is poorly known), one can measure the radius r accurately, and determine the period T accurately and therefore calculate the velocity v $(= 2\pi r/T)$ and the proper time τ $(= \sqrt{1 - v^2/c^2 t})$.

Therefore one very important index of the equivalence principle is that it allows one to work in the limit

$$\Delta\tau \sim 0, \qquad \Delta m \sim \infty\,.$$

Of course one can have intermediate cases where m is known to some extent. But because the mass drops out of the equation of motion in an external gravity field, it gives rise to this very special property, that one can determine trajectories.

This is to be strongly contrasted with the case in ordinary quantum theory, where we usually know the mass fairly accurately. But in that situation we know almost nothing about the proper time, τ—a statement that needs to be explained.

In our discussion we have assumed that one is going to measure m. This measurement yields the uncertainty relation $c^2\Delta m\Delta\tau \simeq \hbar$. Even if one assumes that τ was accurately known before the experiment, the uncertainties introduced by the act of measurement will destroy this knowledge to the extent necessary. If one does not perform such an experiment but assumes, say, that one has an electron, then one must ask « Since when has this electron been around, in order to start the clock for τ running? ».

If it is a stable particle, then $\Delta m = 0$, and one has no idea when it was created, and $\Delta\tau = \infty$. If it is an unstable particle one usually writes $\Delta E\Delta t \simeq \hbar$, where ΔE is the width of the particle, and Δt is the lifetime. But in fact this « ΔE » is really a mass uncertainty. Similarly, there is no real Δt associated with the particle—one may know the laboratory time quite accurately. It is more properly a statement about the uncertainty of a clock located on the particle, that runs so long before the particle decays. So many of our usual lifetime-width uncertainty relations are more appropriately represented as mass-proper time relations.

The above relates to the case where the mass is taken as « given », and one does not set about to determine it experimentally. If instead, a particle decays, and one wants to know what product one has, one can measure its mass, and thereby reset the proper time, τ, as described, but only to the extent governed by the uncertainty principle. So τ can be naturally « set » by the formation of the particle, or later « reset » by a specific experiment, one measuring the mass, or τ directly.

When the mass is taken as given, the usual case in quantum theory, and one applies an electric (or other non-gravitational) field to accelerate the particle, the usual restrictions on its orbit apply, and one cannot accurately know its trajectory, which is needed to calculate τ. As described above, one does not know τ because one does not know when to start counting. Over and above that, there is an extra uncertainty because of ones lack of knowledge of the trajectory. Even in the case when one ignores the initial lack of knowledge of τ, and tries to determine the passage of τ since the start of the experiment, one will be limited by the mass-proper time uncertainty relation, where now Δm will be $\Delta E/c^2$. One can consider this experiment to be a new measure of the mass, and τ, but after the system loses its coherence, $\Delta\tau$ will become infinite again.

So one can measure $\Delta\tau$ in this manner in an external non-gravitational field, even if the initial mass is known, and the experiment then yields a finite $\Delta\tau$. However there will be a minimum possible $\Delta\tau$ that one can obtain in such an experiment. One cannot make $\Delta\tau = 0$, because one cannot gain complete knowledge of the trajectory. For a free particle, the best one can do is determined by the fact that

$$\tau = \sqrt{1 - (v^2/c^2)}(t - t_0) .$$

Here, one does not know t_0, the creation time, at all, as we have discussed. However, if one wants to make a new determination of τ, one will be limited by the fact that (assuming for convenience a nonrelativistic particle) one can not know both E and t simultaneously, even though one would need to, in order to determine τ, since

$$\tau = \sqrt{1 - (2E/mc^2)}t .$$

If m is taken as known, then

$$\Delta\tau = \frac{t\Delta E}{mc^2\sqrt{1 - (2E/mc^2)}} + \sqrt{1 - (2E/mc^2)}\Delta t .$$

One can then take $\Delta E \simeq \hbar/\Delta t$, and minimize $\Delta\tau$ with respect to Δt. One finds approximately

$$\Delta\tau_{\min} \sim \sqrt{\frac{\hbar t}{mc^2}},$$

and one sees that $\Delta\tau$ grows in time. For a neutron, one would find that over its lifetime there is a cumulated uncertainty of $c\Delta\tau_{\min} \simeq 1$ cm, although in practice, it would likely be a good deal longer.

We are now in a position to see that the equivalence principle places classical electromagnetic and gravitational fields in a unique perspective. In an external gravitational field, one need not know the mass at all, and yet one can accurately measure the trajectory of particles, and one can then be in the situation where

$$\Delta m = \infty, \qquad \Delta\tau = 0.$$

On the other hand in an electric field, one often knows the mass exactly, and one can be in the situation where

$$\Delta m = 0, \qquad \Delta\tau = \infty.$$

Of course, in either case one may have partial knowledge of either variable, but the distinction between the extreme cases comes about because of the equivalence principle. The two cases shown above correspond to two opposite extremes of the uncertainty relation, much as in the case of wave vs. particle knowledge in ordinary quantum theory. In that case, which we are used to, one says that a quantum mechanical object is subject to quantum laws, which in the two extremes reduce to the classical characterisation as particle motion or wave motion. In general, neither is an adequate characterisation.

I would characterise the combined gravitational and electromagnetic field as a quantum field, which in the two extreme limits can be characterised as a gravitational field, subject to the equivalence principle, or else as an electromagnetic field. In general it is something more complicated than either. (I do not think the weak interaction changes this scheme.)

So I would expect the search for a unified field to produce this fully quantum mechanical entity, governed by the uncertainty principle, which is neither electrical nor gravitational in nature. It is only in the extreme limits that we can characterise it as a classical field of one nature or the other. Only when this generalized field is understood will it become possible to tackle the problem we refer to as « quantum gravity ». I suspect that if present attempts to quantize classical gravity were to succeed, it would be a serious setback for physics, since everyone would rush to accept the theory, which would probably be wrong, and there would be no way to test it.

In the light of these remarks, one can see that I believe that our present ideas as to the nature of gravitational and electromagnetic fields are still hopelessly naive, from the standpoint of a « final » theory. I would not for a moment question the mathematical sophistication of present theories, but I believe that there are still many physical insights that we have not made use of. Certainly the equivalence principle still has much to teach us.

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