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Cosmological event horizons, entropy and quantum particles

by

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ABSTRACT. — We review various aspects of the thermal properties of event horizons in connection with the nonlocal nature of quantum states.

RÉSUMÉ. — Nous présentons divers aspects des propriétés thermiques des horizons des événements en liaison avec la nature non locale des états quantiques.

1. DE SITTER HORIZONS

Nothing better illustrates the connection between quantum nonlocality and the global structure of spacetime than the remarkable thermal properties of event horizons discovered by Bekenstein [*1*] and Hawking [*2*], and subsequently elaborated by many authors. The essence of the connection is that a pure quantum state defined on the whole (i. e. maximally extended) spacetime manifold appears as a thermal state to an observer restricted to the region of spacetime « outside » an event horizon. Particle states are defined in terms of modes which straddle the event horizon, and the thermal nature of the system outside the horizon can be attributed to the observer's obligation to relinquish information about the quantum state beyond the horizon. It is hence a direct consequence of the nonlocal nature of the quantum state.

Bekenstein and Hawking supported these ideas by introducing the concept of black hole entropy, defined as

$$S_{bh} = 2\pi A \tag{1.1}$$

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where A is the event horizon area of the hole, and I have used units with $\hbar = c = 8\pi G = k = 1$. Then a « generalized second law of thermodynamics » may be stated: the *total* entropy, consisting of S_{bh} plus the entropy of any matter fields, S_m, is non-decreasing. Thus

$$\dot{S}_{bh} + \dot{S}_m \geqq 0 . \tag{1.2}$$

The generalized second law has been verified in a variety of scenarios involving the exchange of energy and entropy between a black hole and its environment [*3*]. As a result, the interpretation of the event horizon area of a black hole as a measure of the hole's entropy is now generally accepted.

Black holes are, however, only one type of global structure that contain event horizons. Many cosmological models also have event horizons. The thermodynamic status of cosmological horizons remains to be clarified. Is the horizon area still a measure of entropy? Will the generalized second law continue to hold for reasonable assumptions about the matter content of the spacetime?

One cosmological case has received a lot of attention, namely, de Sitter space. Gibbons and Hawking [*4*] have asserted that the generalized second law extends to de Sitter horizons, and detailed investigation [*5*] confirms this.

On the other hand, the thermodynamic status of the event horizon in de Sitter space differs in a rather deep way from the black hole case. First, the thermal character of the horizon is rather subtle. An inertial particle detector in a de Sitter-invariant vacuum state responds as if it is immersed in a bath of thermal radiation of temperature

$$T_h = \frac{H}{2\pi} \tag{1.3}$$

where H is the (constant) Hubble parameter for de Sitter space [*5*]. However, unlike in the black hole case, the stress-energy-momentum tensor of this quantum state does *not* correspond to that of thermal radiation [*6*]. Indeed, any thermal radiation present in de Sitter space is rapidly redshifted away by the expansion. There is no asymptotically flat spacetime region where the thermal radiance of the horizon may be compared to that of an ordinary hot body. A related fact is that different inertial observers see differently located event horizons.

Secondly, in the black hole case it is possible to quantify the entropy of the hole in terms of the loss of information concerning the matter that imploded to form the hole in the first place [*1*]. This argument depends upon the existence of a well-defined black hole mass-energy, and the use of the first law of thermodynamics (conservation of mass-energy). The horizon structure of de Sitter space is quite different. The observer is located « inside » rather than « outside » the horizon. The absence of an asymptotic flatness precludes a meaningful definition of the mass-energy of de Sitter space, and hence precludes a use of the first law.

In some sense, the amount of information hidden in the infinite volume of space behind the de Sitter horizon is infinite. This reflects the fact that there are an infinite number of non-overlapping regions of de Sitter space each containing a (different) horizon. Alternatively, however, one can try to quantify the horizon entropy around a given location by « growing » de Sitter space from a matter-dominated cosmology.

Take, for example, a radiation filled $k = 0$ Friedmann model with non-zero cosmological constant Λ. The Friedmann equation is readily solved for the scale factor

$$a(t) = (B/\Lambda)^{\frac{1}{4}} \sinh^{\frac{1}{2}} [2(\Lambda/3)^{\frac{1}{2}}t] \tag{1.4}$$

which has the form $a \sim t^{\frac{1}{2}}$ near $t = 0$ and $a \sim \exp [(\Lambda/3)^{\frac{1}{2}}t]$ as $t \to \infty$. The constant B is related to the energy density ρ of the radiation by $B = \rho a^4$.

The proper radius of the event horizon is given (for $k = 0$ models) by

$$R_h = a(t) \int_t^\infty \frac{dt'}{a(t')}. \tag{1.5}$$

For the scale factor (1.4)

$$R_h \to 3.71\, a(t)(B\Lambda)^{-\frac{1}{4}} \sim t^{\frac{1}{2}}, \qquad t \to 0 \tag{1.6}$$

$$\to (3/\Lambda)^{\frac{1}{2}}, \qquad t \to \infty . \tag{1.7}$$

Thus initially $R_h \to 0$ and the horizon area (hence entropy) is negligible. There will, however, be matter (radiation) within the horizon volume at all times as $t \to 0$. As the universe expands, some of the radiation flows across the horizon and is lost to a hypothetical observer within the horizon volume. There is a back reaction on the gravitational dynamics that causes the horizon area to change. This back reaction is given *exactly* by the solution (1.4). Eventually, all the radiation is essentially lost, and the horizon settles down to the de Sitter value (1.7).

Now one could regard the growth of the de Sitter horizon area from zero to $12\pi^{-1}$ as « paid for » by the information conveyed across the horizon by the particles of matter. To test this hypothesis assume that the universe is filled with identical particles of mass m. Each particle carries $\ln 2 \simeq 1$ bit of information. The total number of particles within the horizon volume as $t \to 0$ is $4\pi\rho R_h^3/3m$. If one now makes the restriction that the wavelength of the particle must be less than the horizon size R_h to qualify for the designation « within the horizon volume », then the maximum information content is obtained by letting $m \approx R_h^{-1}$. The total information content is then approximately $\rho R_h^4 = B(R_h/a)^4 \approx \Lambda^{-1}$, using (1.6). But the final de Sitter horizon area is also approximately Λ^{-1}, so one may conclude that the de Sitter entropy is indeed given quantitatively (to within an order of magnitude) by the information conveyed across the horizon by the particles during the transition from a matter-dominated Friedmann model to de Sitter space.

If, instead of N particles of mass m, one considers thermal radiation, then the entropy density of the radiation $\approx \rho^{\frac{3}{4}} = B^{\frac{3}{4}}/a^3$. The radiation entropy in a horizon volume R_h^3 is thus, as $t \to 0$, approximately $\Lambda^{-\frac{3}{4}}$, whereas the final horizon entropy is Λ^{-1}. The entropy therefore rises by a factor $\Lambda^{-\frac{1}{4}} \approx (R_h/\text{Planck length})^{\frac{1}{2}} \gg 1$. The generalized second law is thus well satisfied. The reason for the rise in entropy is due to the fact that the radiation temperature is $T_r \sim B^{\frac{1}{4}}/a$ whereas the instantaneous horizon temperature is $T_h \sim R_h^{-1} \approx (B\Lambda)^{\frac{1}{4}}/a$. Thus $T_r \approx T_h\Lambda^{-\frac{1}{4}} \gg T_h$. The system is therefore far from thermodynamic equilibrium and so the heat flow across the horizon generates a lot of entropy.

2. MORE GENERAL COSMOLOGICAL HORIZONS

In spite of the pleasing consistency of the thermodynamic quality of de Sitter horizons, questions still remain concerning more general cosmological horizons. Is the horizon area still a satisfactory measure of entropy? Can the horizon area shrink as a result of quantum processes, and if so will the generalized second law still apply?

The first step is to establish an analogue of the Hawking area theorem [*8*] for black holes. In the case of a Friedmann universe filled with a perfect fluid with pressure p and energy density ρ, the following may be shown:

THEOREM. — If $\rho + p \geqq 0$ and $a(t) \to \infty$ as $t \to \infty$ then the horizon area is non-decreasing with time.

A proof of the theorem (which applies for $k = 0$ and ± 1) will be given elsewhere [*9*].

If the universe recontracts to a final singularity at $t = t_s$, at which $a = 0$, the horizon radius integral (1.5) must be truncated at the singularity. One finds [*9*]

$$R_h = a(t) \int_t^{t_s} \frac{dt'}{a(t')} \to 0, \qquad t \to t_s \tag{2.1}$$

so that, in the approach to the singularity, the horizon area shrinks.

This shrinkage can be attributed to the rapid contraction of the universe near the singularity. Note that the condition $\rho + p \geqq 0$ necessary for the validity of the area theorem is also the condition required by the singularity theorem [*9*]. If $\rho + p < 0$, it is possible for the universe to « bounce », avoiding the singularity. Under these circumstances the horizon could shrink during the period of rapid contraction prior to the bounce.

The (so-called dominant) energy condition

$$\rho + p \geqq 0 \tag{2.2}$$

whilst physically reasonable, is by no means sacrosanct. Recently it has been questioned by Morris and Thorne [*10*]. In fact, it is known to fail under a number of circumstances. One of these is in the vicinity of a black hole in which the matter environment is in certain quantum vacuum states. It can then happen that $\rho < 0$, leading to a failure of (2.2). This produces the so-called Hawking effect, in which the black hole shrinks due to the inward propagation of negative energy across its horizon.

In spite of the fact that the area theorem is violated by the Hawking effect, the combined quantity $2\pi A + S_m$, where S_m is the entropy of the matter field, remains non-decreasing. This is because thermal radiation is produced by the black hole, which « pays » for the loss of the horizon area.

It is easy to establish an analogous scenario for cosmological event horizons. In this case there is no corresponding evaporation effect. However, if one relaxes the dominant energy condition, not by allowing ρ to become negative, but by allowing $p < -\rho$, then the horizon area will again shrink. One way to do this is to consider the effect of bulk viscosity. If the cosmological medium has equation of state $p = (\gamma - 1)\rho$ and bulk viscosity $\eta = \alpha\rho$ ($\alpha =$ constant > 0), then the effective pressure $p' = p - 3H\alpha\rho$. If $\gamma < 3H\alpha$ then $p' < -\rho$.

To investigate this, one can solve exactly the Friedmann equations for a $k = 0$ model with bulk viscosity:

$$3H^2 = \rho \tag{2.3}$$

and

$$\dot{H} = -\frac{1}{2}(\rho + p) = \frac{3}{2}H^2(3H\alpha - \gamma). \tag{2.4}$$

Then

$$\int_t^{t_f} \frac{dt'}{a(t')} = \int_a^{a_f} \frac{da'}{a'\dot{a}'} = \int_a^{a_f} \frac{da'}{a'^2 H} \tag{2.5}$$

where t_f and a_f are the final values of t and a. Integrating by parts and using (2.4)

$$R_h(t) = a(t)\int_t^{t_f} \frac{dt'}{a(t')} = \frac{1}{1 - 3\gamma/2}\left[\frac{1}{H} - \frac{9\alpha}{2}\left(1 - \frac{a}{a_f}\right)\right] \tag{2.6}$$

and

$$\dot{R}_h(t) = \frac{3/2}{1 - 3\gamma/2}\left[\gamma - 3H\alpha\left(1 - \frac{a}{a_f}\right)\right]. \tag{2.7}$$

Direct solution of (2.4) yields

$$\ln a + Ca^{3\gamma/2} = \gamma^2(t - t_0)/2\alpha \tag{2.8}$$

and

$$H = (Ca^{3\gamma/2} + 3\alpha/\gamma)^{-1} \tag{2.9}$$

where C and t_0 are constants.

The behaviour of the solutions depends crucially on the sign of

$$3H\alpha - \gamma = - Ca^{3\gamma/2}/(Ca^{3\gamma/2} + 3\alpha/\gamma). \tag{2.10}$$

There are four distinct classes. The case $C = 0$ corresponds to $3H\alpha = \gamma$, $H =$ constant. This is a curious de Sitter solution discovered by Barrow [*11*]. Unlike the standard vacuum de Sitter model, which exists in thermodynamic equilibrium, this is a viscous-driven model, rather akin to a stable « dissipative structure » familiar from far-from-equilibrium thermodynamics [*12*]. Entropy is generated throughout, and flows continuously away across the horizon to produce a steady state. The de Sitter horizon temperature and the temperature of the viscous medium which fills the space, are different.

For $C > 0$, $H > 0$ for all time, so $a_f = \infty$. An event horizon will exist if $\gamma < 2/3$, and one sees from (2.10) that $3H\alpha - \gamma > 0$. It follows from (2.7) that $\dot{R}_h > 0$ and the horizon area increases with time.

If $- 3\alpha/\gamma < A < 0$, then $3H\alpha - \gamma < 0$, so $\dot{H} < 0$, $H > 0$ and a increases with time. Examination of (2.10) shows that when $Ca^{3\gamma/2} = - 3\alpha/\gamma$ then $H = \infty$. Computation of the scalar curvature confirms that there is a spacetime singularity here. The universe expands monotonically, from $a \to 0$ at $t \to - \infty$ to $a = a_f = | 3\alpha/C\gamma |^{2/3\gamma}$ at the singularity.

Substituting (2.9) into (2.7) we find

$$\dot{R}_h = \left(\frac{3\gamma/2}{1 - 3\gamma/2}\right)\left(\frac{a}{a_f}\right)\left[\frac{1 - (a/a_f)^{3\gamma/2 - 1}}{1 - (a/a_f)^{3\gamma/2}}\right] < 0 \tag{2.11}$$

because $a/a_f < 1$ and $\gamma < \frac{2}{3}$. Thus the horizon area shrinks from $3\alpha/\gamma$ at $t = - \infty$ to 0 at $a = a_f$.

The nature of the singularity is very strange. Usually spacetime singularities are associated with catastrophic gravitational collapse, in which the curvature diverges as $a \to 0$. Here the curvature diverges as a result of catastrophic *expansion* of the universe, and occurs at a *finite* value of a. The divergence arises because the expansion rate becomes infinite. This type of singularity is the counterpart for cosmic repulsion (i. e. Λ-type terms) of the more familiar singularities associated with gravitational attraction. The occurence of such a singularity is determined by the sign of C, which in turn fixes the Hubble parameter H at a given value of a (or t). If H (i. e. the expansion rate) is great enough, the viscous term $3H\alpha$ will start to dominate, and runaway expansion will ensue. The accelerated expansion is assisted by γ being close to zero.

For the special case $\gamma = 0$, direct solution of (2.4) yields

$$a \propto \exp\left[-\frac{2}{3}(t_0 - t)^{\frac{1}{2}}/\alpha^{\frac{1}{2}} \right] \tag{2.12}$$

$$R_h = \frac{9\alpha}{2}\left\{ \exp\left[-\frac{2}{3}(t_0 - t)^{\frac{1}{2}}/\alpha^{\frac{1}{2}} \right] - 1 + \frac{2}{3}(t_0 - t)^{\frac{1}{2}}/\alpha^{\frac{1}{2}} \right\}, \quad t < t_0 \tag{2.13}$$

which clearly demonstrates how the horizon shrinks steadily from ∞ at $t \to -\infty$ to 0 at the singularity at $t = t_0$.

The regime $C < -3\alpha/\gamma$ corresponds to a contracting universe ($H < 0$), as may be seen from (2.9). In this case the $3H\alpha$ term in (2.4) acts in the opposite sense, enhancing the γ term, and accelerating the collapse. Inspection of (2.8) shows that $a \to \infty$ as $t \to -\infty$. The universe contracts at an accelerating rate, and becomes singular, not at $a = 0$, but at $a = a_f = |3\alpha/C\gamma|^{2/3\gamma}$. Once again the viscosity term causes a singularity at finite a. The horizon area shrinks throughout.

To investigate the generalized second law of thermodynamics I shall restrict the analysis to quasi-equilibrium, in which the model departs only slightly from ($k = 0$) de Sitter space and the viscous fluid is in thermal equilibrium with the horizon at a common temperature T_h.

First note that when the cosmological model departs from de Sitter space, the spectrum of radiation registered by an inertial particle detector is no longer thermal, and the concept of a well-defined temperature breaks down. However, if a particle detector is adiabatically switched on and off for a period of time during which the change in the value of $H \equiv \dot{a}/a$ is negligible, then it can be shown that the spectrum is thermal (to that approximation), with an « instantaneous » Hawking temperature $1/2\pi R_h$.

Small departure from de Sitter space requires $\gamma \approx 0$ and $9\alpha/2 \ll H^{-1}$ (small viscosity). It then follows from (2.6) and (2.7) that

$$\dot{S}_h \equiv 2\pi\dot{A} = 16\pi R_h \dot{R}_h \approx -72\pi^2\alpha(1 - 3\gamma/2)^{-2}. \tag{2.14}$$

This loss of horizon entropy will be offset by the gain in conventional entropy resulting from the bulk viscosity of the fluid generating heat. Following Weinberg [*13*], the latter is given by

$$\dot{S}_m = (4\pi R_h^3/3)(9H^2\alpha\rho/T) \tag{2.15}$$

for a horizon volume. Putting $T = 1/2\pi R_h$, and using (2.3) and (2.6), one finds

$$\dot{S}_m \approx 72\pi^2\alpha(1 - 3\gamma/2)^{-2} \tag{2.16}$$

which is exactly the negative of (2.14). Thus, the generalized second law is satisfied in this case. Note that if the temperature of the fluid were greater than that of the horizon, $T > T_h$, then (2.16) will not be large enough to offset (2.14). But in this case extra entropy will be generated as heat flows from the hot fluid across the cooler horizon, thereby rescuing the second law (see ref. [5]).

3. DISCUSSION AND SPECULATIONS

Ever since Hawking's discovery of black hole entropy, there has been a widespread assumption that *all* event horizons possess an associated entropy. In the general case, however, the issue is far from trivial. The existence of an area theorem for general Friedmann cosmological horizons subject to $\rho + p \geqq 0$ is evidence in favour of an entropic association, even though there is no well-defined temperature in this case. The fact that even when viscous processes cause a departure from the condition $\dot{A} \geqq 0$ the generalized second law of thermodynamics remains valid is strong support for the interpretation of event horizon area as entropy.

If this position is accepted, it then becomes of interest to speculate whether some of the formalism that has been applied to black holes will extend to the cosmological case too, for example, the membrane paradigm of Thorne *et al.* [14]. Interestingly, it is possible to prove similar area theorems for cosmological *particle* horizons. Application of the membrane paradigm here might lead to practical techniques for the investigation of the early universe.

The discussion of horizon entropy given here should be placed in the context of the longstanding search for a generalization of the Hawking black hole entropy to a universal « gravitational entropy » [15]. As we have seen, cosmological horizons are generally not in thermal equilibrium at a well-defined temperature, so these studies provide another set of systems in which to test non-equilibrium gravitational thermodynamics along the lines of the work of Candelas and Sciama [16].

The search for a generalized gravitational entropy could receive a clue from the fact that the horizon area of a cosmological model that contracts to a final singularity decreases with time. This is a clear violation of the generalized second law. (One could choose pure radiation as the cosmological fluid; its entropy will not change during the approach to the singularity.) One might argue that the concept of horizon entropy simply fails in this case. Alternatively, one could propose to modify the definition of the gravitational entropy so as to save the generalized second law of thermodynamics. This would entail adding to the horizon area another gravitational entropy term that increases as the universe collapses to compensate for the loss of horizon area. Such a term would accord with the general observation that the gravitational entropy ought to provide a measure of the « clumpiness » of a self-gravitating system [15].

An elegant way to formulate this proposal would be to define the total gravitational entropy in a certain spatial volume in terms of a *volume* integral plus a surface integral over the boundaries of the system, i. e. the event horizon. For a static spacetime, such as that of a black hole, the volume integral would yield only a constant, which could be defined away by rescaling the (arbitrary) zero of entropy. But in the case of a collapsing cosmological model the volume integral would be time-dependent and represent a rise of gravitational entropy.

If a generalized second law extended to cosmological horizons is accepted it can be used to rule out certain theories. For example, there has been much speculation about « hyperinflation », i. e. inflation in which $a(t)$ increases faster than an exponential. As this corresponds to decreasing event horizon area, one might argue that such models are in conflict with the second law of thermodynamics and so are unacceptable.

Hyperinflation occurs in gravitational theories with quadratic Lagrangians $R + \varepsilon R^2$. This theory possesses the solution [*17*]

$$a(t) = \exp\left[At + t^2/72\varepsilon\right]. \qquad (2.17)$$

For $\varepsilon > 0$ the horizon area decreases with time. Hence one may wish to rule out such theories as unphysical.

Whether one should rule out only the *solution* (2.17) or the entire Lagrangian is an interesting issue. A case could be made that any theory (i. e. Lagrangian) which permits a violation of the second law of thermodynamics, *even in principle*, is unacceptable, on the basis that a sufficiently resourceful intelligence could then contrive an entropy-decreasing solution and thereby construct an effective perpetuum mobile. The issue is *not* the same for gravitational entropy as for conventional entropy, where it is well known that a Maxwell demon who attempts such a strategy is inevitably stymied. Solutions such as (2.17) require the establishment only of a gross boundary condition, not the detailed manipulation of all the individual microscopic components of the system. This reflects the essentially *non-statistical* nature of gravitational entropy which, as has been noted by Hawking [*18*], sets it apart from ordinary entropy, giving it a more « objective » character. Thus, horizon area decreases can be achieved without a Maxwell-demon style information gathering exercise.

If one therefore adopts this stronger approach to horizon-decreasing theories, it could be used to filter out not only a large class of gravitational Lagrangians, but a large class of matter Lagrangians too. Thus might a restriction on the global structure of spacetime become a very general regulator of *local* quantum field theories through the *nonlocal* character of quantum states.

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