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Section A :
Physique théorique.

Relation between the Convergence Function and the Electromagnetic Form Factor in a Composite Model

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ABSTRACT. — A convergence function F is introduced in a Fermi theory (self interacting nucleon antinucleon fields) in order to have a free divergency theory. The properties of the convergence function are studied. The structure functions of elementary and composite particles are calculated in terms of the convergence function F. The relation between F and the Dirac charge form factor G of the nucleon is derived in the time like region. The possibility of equating F and G is investigated and some implications of the assumption $F = G$ are given (for instance, the universality of the form factor of mesons and nucleons in the time like region is obtained).

RÉSUMÉ. — Nous introduisons une fonction de convergence F dans une théorie de Fermi (champ de fermions en auto-interaction) de façon à avoir une théorie ne contenant pas de quantités divergentes. Après avoir

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(1) The main of this work was done during his stay as associate member at the International Centre for Theoretical Physics, Trieste (Italy).

calculé les expressions des fonctions de structure des particules élémentaires et composées en fonction de F et étudié les propriétés de F, nous donnons la relation entre F et le facteur de forme électromagnétique G de Dirac du nucléon. Nous cherchons enfin s'il est possible de poser la relation $F \equiv G$, et nous donnons les conséquences d'une telle identification (nous obtenons par exemple l'universalité des facteurs de forme des mésons et des nucléons dans la région du genre temps).

1. INTRODUCTION

The theoretical study of the form factors has got a new interest from the composite models [1] mainly due to the fact that the existence of a structure for the particles may be taken into account.

Though the idea of deriving the electromagnetic form factors from strong interaction (in a composite model) is an old one [2], we think that a simple model gives a better insight without introducing any potentials or using Bethe-Salpeter equation. Here, we consider the case of a strongly self-interacting fermion field where a convergence function is put in order to have a free divergency Fermi model; the present work is to show the possibility of the identification of this convergence function with the electromagnetic form factors of the fermion (nucleon N) and the implications of such an identification. Though the non relativistic results thus obtained refrain from a realistic comparison with experimental data, we think however that a better understanding and the main features of the mechanism relating electromagnetic form factors and the criteria of compositness in strong interaction are obtained.

In the Chapter 2, we give explicitly all the structure functions appearing in the expansion of the physical particle operators in the third order approximation (elementary particles and composite vector boson operators). Then, we have the relations between the convergence function and the structure functions. The latter may be related to physical process, and we compute the amplitude of fermion-antifermion $(N - \overline{N})$ elastic scattering in terms of the structure functions.

In the Chapter 3, we derive the relation between the form factor and the convergence function from the usual unitarity condition of the $N - \overline{N} - \gamma$ process, assuming that the $N - \overline{N}$ dominate in the intermediate states. For the simplicity's sake, we consider only the Dirac form factor. The condition on the asymptotic behaviour of the convergence function leads to the fact that the Dirac form factor verifies a Mushkelishvili-Omnes equation in the time-like region; the phase of the Dirac form factor is obtained to be related in a simple manner with the phase of the $N\overline{N}$ elastic scattering amplitude.

In order to look for its dependence in terms of the convergence functions, we compute in the Chapter 4 the expression of the differential cross section of the N — $\bar{N}$ elastic scattering.

The Chapter 5 is devoted to look for the possibility of equating the Dirac form factor and the convergence function and investigating the implications of this assumption. The Dirac form factor is obtained to verify a non linear integral equation in the time-like region. The differential cross section of the N — $\bar{N}$ elastic scattering is also found to be proportional to the fourth power of the electromagnetic form factor for *any values* of $t = -\bar{q}^2$; this result is to be compared with Wu and Yang's conjecture concerning the proton-proton scattering.

2. DESCRIPTION OF THE MODEL

Let us consider the model described by the following Hamiltonian $H = H_0 + H_1$. H_0 is the free Hamiltonian of bare fermions (N) and antifermions (N); H_1 is the self-interaction of fermion-antifermion field,

$$(2.1) \qquad H = H_0 + H_1,$$

$$(2.2) \qquad H_0 = \sum_{S} \int d^3 \vec{p}\, E_{\vec{p}} \left\{ b^+_{\vec{p},S}\, b_{\vec{p},S} + d^+_{\vec{p},S}\, d_{\vec{p},S} \right\},$$

$$(2.3) \qquad H_1 = g_F \int d^3 \vec{p}\, d^3 \vec{q}\, d^3 \vec{l}\, \hat{F}(\vec{p}, \vec{q} - \vec{p})\, \hat{F}^*(\vec{q} - \vec{l}, \vec{l})$$

$$\times\, b^+_{\vec{p},r}\, 0^{r,s}_{\mu\rho}(\vec{p}, \vec{q} - \vec{p})\, d^+_{\vec{p}-\vec{q},s}\, d_{\vec{q}-\vec{l},r'}\, 0^{+r',s'}_{\mu\rho}(\vec{q} - \vec{l}, \vec{l})\, b_{\vec{l},s'}.$$

There is summation over repeated indices.

$b^+_{\vec{p},s}$ and $b_{\vec{p},s}$ $\left(d^+_{\vec{p},s}\right.$ and $\left.d_{\vec{p},s}\right)$ are respectively the bare creation and annihilation operators of a fermion N (antifermion $\bar{N}$) having the momentum $\vec{p}$ and the quantum number s. The coupling of fermion-antifermion pair is through the following expression :

$$(2.4) \qquad 0^{r,s}_{\mu\rho}(\vec{p}, \vec{p}') = \bar{u}(\vec{p}, r)\, \gamma_\mu \otimes \tau_\rho\, v(\vec{p}', s),$$

$u(\vec{p}, r)$ and $v(\vec{p}', s)$ are the usual solutions of the Dirac equation. γ_μ ($\mu = 1, \ldots, 4$) are Dirac matrices, τ_ρ ($\rho = 1, \ldots, 4$) are isospin matrices $\left[\vec{\tau}\right.$ are Pauli matrices, and $\left.\tau_4 = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}\right]$.

$0^{r,s+}_{\mu\rho}$ is the hermitian conjugate of the expression defined in (2.4). For the sake of writing, we write these two expressions under the form $0^{r,s}$ and $0^{r,s+}$.

As is wellknown, a Fermi self-interaction theory has divergencies, we introduce a convergence function $\hat{F}(\vec{p}, \vec{q}-\vec{p})$. This complex function is not completely arbitrary. It has been shown to be symmetric and even with respect of the two arguments $\vec{p}$ and $\vec{q}-\vec{p}$ (these properties come from the conditions of the commutation relations of fermion and antifermion physical operators) [3]. Here, we take

$$\hat{F}(\vec{p}, \vec{q}-\vec{p}) = F\left(\omega_{\vec{q}}(\vec{p})\right),$$

where

$$\omega_{\vec{q}}(\vec{p}) = E_{\vec{p}} + E_{\vec{q}-\vec{p}}, \qquad E_{\vec{p}} = \sqrt{\vec{p}^2 + m^2},$$

m is the fermion (antifermion) mass.

It has been shown to belong to a class of entire function but this condition may be released. We have a bound for its asymptotic behaviour [3] :

$$\lim_{\omega \to \infty} \| F(\omega) \|^2 \leq \frac{c}{\omega^{2+\varepsilon}} \qquad (c = \text{const.},\ \varepsilon > 0). \tag{2.5}$$

A. Physical operators

The commutation condition :

$$\left[b^{\text{out}}_{\vec{p},s}, H \right]_- = \left(E_{\vec{p}} - i\,\varepsilon\right) b^{\text{out}}_{\vec{p},s} \tag{2.6}$$

(and similar relation for $d^{\text{out}}_{\vec{p},s}$) determines the outgoing physical operator $b^{\text{out}}_{\vec{p},s}$ of the fermion N (antifermion $\overline{\text{N}}$) [2].

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[2] The sign of the infinitesimal quantity ε is choosen in the following manner : when we look for the time dependent operator

$$b_{\vec{p},s}(t) = \exp\{-iHt\}\, b_{\vec{p},s} \exp\{iHt\},$$

we have

$$\underset{t \to +\infty}{\text{weak-Limit}}\; b_{\vec{p},s}(t) = b^{\text{out}}_{\vec{p},s},$$

$$\underset{t \to -\infty}{\text{weak-Limit}}\; b_{\vec{p},s}(t) = b^{\text{in}}_{\vec{p},s}.$$

In the third order approximation ([3]), we get

$$(2.7)\quad b^{\text{out}}_{\vec{p},s} = b_{\vec{p},s} + \int d^3\vec{l}\, d^3\vec{q}\, f(\vec{p},\vec{l},\vec{q})\, 0^{s,r} d^+_{\vec{q}-\vec{p},r}\, d_{\vec{q}-\vec{l},t}\, 0^{+t,m}\, b_{\vec{l},m} + \dots,$$

$d^{\text{out}}_{\vec{p},s}$ is obtained from $b^{\text{out}}_{\vec{p},s}$ by charge conjugaison. The fermion structure function $f(\vec{p},\vec{l},\vec{q})$ is found from (2.6) :

$$(2.8)\qquad f(\vec{p},\vec{l},\vec{q}) = \frac{N(\vec{p},\vec{l},\vec{q})}{D(\vec{p},\vec{l},\vec{q})},$$

$$(2.9)\qquad D(\vec{p},\vec{l},\vec{q}) = \omega_{\vec{q}}(\vec{l}) - \omega_{\vec{q}}(\vec{p}) + i\,\varepsilon,$$

$$(2.10)\qquad \omega_{\vec{q}}(\vec{p}) = E_{\vec{p}} + E_{\vec{q}-\vec{p}}, \qquad E_{\vec{p}} = \sqrt{\vec{p}^2 + m^2},$$

$$(2.11)\quad N(\vec{p},\vec{l},\vec{q}) = g_F \frac{F^*\left(\omega_{\vec{q}}(\vec{l})\right) F\left(\omega_{\vec{q}}(\vec{p})\right)}{1 + g_F \int d^3\vec{l}' \dfrac{\left\| F\left(\omega_{\vec{q}}(\vec{l}')\right)\right\|^2 \operatorname{Tr}\{\vec{l}', \vec{q}-\vec{l}'\}}{D(\vec{p},\vec{l}',\vec{q})}}.$$

The function $\operatorname{Tr}\{\vec{l}', \vec{q}-\vec{l}'\}$ defined in the denominator of $N(\vec{p},\vec{l},\vec{q})$ is the short writing of the coefficient of $\delta_{\mu\mu'}\,\delta_{\rho\rho'}$ in the expression :

$$(2.12)\qquad \operatorname{Tr}\left\{0_{\mu\rho}(\vec{l}', \vec{q}-\vec{l}')\, 0^+_{\mu'\rho'}(\vec{q}-\vec{l}', -\vec{l}')\right\}.$$

The annihilation (creation) operator $A^{\text{out}}_{\vec{q}}$ $\left(A^{\text{out}+}_{\vec{q}}\right)$ of the composite boson having the momentum and the quantum number indexed uniquely by the momentum $\vec{q}$ may be found by a relation similar to (2.6) :

$$(2.13)\qquad \left[A^{\text{out}}_{\vec{q}}, H\right]_- = (\Omega_{\vec{q}} - i\,\varepsilon)\, A^{\text{out}}_{\vec{q}},$$

$\Omega_{\vec{q}}$ is the energy of the composite boson. In the third order approximation, we have

$$(2.14)\qquad A^{\text{out}}_{\vec{q}} = \int d\vec{l}\, \lambda(\vec{q},\vec{l})\, d_{\vec{q}-\vec{l},s}\, 0^{+s,r}\, b_{\vec{l},r} + \dots.$$

([3]) We recall that the order of the approximation is the maximum number of field operators appearing in the expansion. A general method to solve the eigenvalue equation (2.6) is given in the reference [4].

The eigenvalue $\Omega_{\vec{q}}$ is given by the root of the compatibility condition :

$$1 + g_F \, S\left(\vec{q}, \Omega_{\vec{q}}\right) = 0, \tag{2.15}$$

where

$$S\left(\vec{q}, \Omega_{\vec{q}}\right) = \int d\vec{l}' \frac{\left\| F\left(\omega_{\vec{q}}\left(\vec{l}'\right)\right)\right\|^2 \mathrm{Tr}\left\{\vec{l}', \vec{q} - \vec{l}'\right\}}{\omega_{\vec{q}}\left(\vec{l}'\right) - \Omega_{\vec{q}} + i\varepsilon} \quad (4), \tag{2.16}$$

$$\lambda\left(\vec{q}, \vec{l}\right) = \frac{1}{\sqrt{I(\vec{q})}} \frac{F^*\left(\omega_{\vec{q}}\left(\vec{l}\right)\right)}{\omega_{\vec{q}}\left(\vec{l}\right) - \Omega_{\vec{q}} + i\varepsilon}. \tag{2.17}$$

The normalization function $I\left(\vec{q}\right)$ is choosen so that

$$\left[A^{\text{out}}_{\vec{q}}, A^{\text{out}+}_{\vec{q}'}\right]_- = \delta_{\vec{q},\vec{q}'} \quad (5),$$

$$I\left(\vec{q}\right) = \int d\vec{l} \frac{\left\| F\left(\omega_{\vec{l}}\left(\vec{l}\right)\right)\right\|^2 \mathrm{Tr}\left\{\vec{l}, \vec{q} - \vec{l}\right\}}{\left[\omega_{\vec{q}}\left(\vec{l}\right) - \Omega_{\vec{q}}\right]^2}. \tag{2.18}$$

We have checked that the conditions (2.6) and (2.13) entails that $b^{\text{out}}_{\vec{p},s}$, $d^{\text{out}}_{\vec{p},s}$, $A^{\text{out}}_{\vec{q}}$ and their hermitian conjugates *verify free field commutation relations*. The total Hamiltonian H, expressed in terms of the physical operators of elementary fermion, elementary antifermion, and composite bosons is put into free form.

This lengthy recall is necessary to understand what will follow. We shall now study the behaviour of the function $N\left(\vec{p}, \vec{l}, \vec{q}\right)$ defined in (2.11) for $\vec{q} \neq 0$ and for $\vec{q} = 0$. The case $\vec{q} \neq 0$ is interesting because it enables us to see explicitly the relation between the Fermi-type interaction theory with compound bosons and the Yukawa type interaction theory in which the compound bosons are considered as elementary particles. The case $\vec{q} = 0$ will be studied in much more details because we need it when the relation between the convergence function and the form factor will be considered.

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(4) The infinitesimal quantity $i\varepsilon$ in (2.16) may be dropped if the mass μ of the composite boson is less than $2m$ (stable boundstate). Up to the third order approximation, the " in " and " out " operators of composite boson are not distinguished. This does not hold for higher order approximation.

(5) We consider only the part in $\delta_{\mu\mu'}\, \delta_{\rho\rho'}$ in the expression (2.12).

B. Study of the function $N(\vec{p}, \vec{l}, \vec{q})$.

Let us give a different form of $N(\vec{p}, \vec{l}, \vec{q})$ defined in (2.11) if we take account of (2.15). Substracting (2.15) in the denominator of (2.11), we find

$$(2.19)\quad N(\vec{p}, \vec{l}, \vec{q}) = \frac{F^*\left(\omega_{\vec{q}}(\vec{l})\right) F\left(\omega_{\vec{q}}(\vec{p})\right)}{\left(\omega_{\vec{q}}(\vec{p}) - \Omega_{\vec{a}} + i\varepsilon'\right) \displaystyle\int \frac{d^3\vec{l}' \,\| F(\omega_{\vec{q}}(l'))\|^2 \operatorname{Tr}\{l', q\}}{\left\{\begin{array}{l}\left[\omega_{\vec{q}}(\vec{l}') - \omega_{\vec{q}}(\vec{q}) + i\varepsilon''\right] \\ \times \left[\omega_{\vec{q}}(l'') - \Omega_{\vec{a}} + i\varepsilon'''\right]\end{array}\right\}}}.$$

Then, in the $\omega_{\vec{q}}(\vec{p})$-plane ($\vec{q}$ is considered as a parameter), under the conditions, the boundstate mass $\mu < 2m$ and $g_F < -|g_{\min}|$ with

$$(2.20)\qquad |g_{\min}|^{-1} = \int d\vec{l}' \frac{\left\| F\left(\omega_{\vec{q}}(\vec{l}')\right)\right\|^2 \operatorname{Tr}\{\vec{l}', -\vec{l}'\}}{2E_{\vec{l}} - 2m}$$

it is easily seen that $\Omega_{\vec{q}}$ is a *simple* pole of $N(\vec{p}, \vec{l}, \vec{q})$. Then, expanding $N(\vec{p}, \vec{l}, \vec{q})$ in the $\omega_{\vec{q}}(\vec{p})$ plane in the neighbourhood of the pole $\Omega_{\vec{a}}$, we have

$$N(\vec{p}, \vec{l}, \vec{q}) = F^*\left(\omega_{\vec{q}}(\vec{l})\right) F\left(\omega_q(\vec{p})\right) \times \left\{ \frac{1}{\left(\omega_{\vec{q}}(\vec{p}) - \Omega_{\vec{q}} + i\varepsilon\right)} \frac{1}{\displaystyle\int \frac{d\vec{l}' \,\| F(\omega_q(l'))\|^2 \operatorname{Tr}\{\vec{q}\ \vec{l}'\}}{\left[\omega_{\vec{q}}(\vec{l}') - \Omega_{\vec{a}}\right]^2}} + \frac{1}{\displaystyle\int \frac{d^3\vec{l}' \,\| F(\omega_q(l'))\|^2 \operatorname{Tr}\{l', q\}}{(\omega_q(l') - \Omega_{\vec{q}})^2}} + \left(\omega_q(p) - \Omega_{\vec{q}}\right) \Phi(\Omega_{\vec{a}}) + \dots \right\} \ (^6),$$

$\Phi(\Omega_{\vec{q}})$ is a regular function of $\Omega_{\vec{a}}$.

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(6) The formula

$$\frac{1}{f(x)} = \frac{1}{f(x_0)} + \frac{(x - x_0)}{1!} \frac{(-f'(x_0))}{f^2(x_0)} + \dots$$

may be used.

Let us express the right handside taking account of the expressions of $\lambda\left(\vec{q}, \vec{l}\right)$ given in (2.17) and of $\mathrm{I}\left(\vec{q}\right)$ given in (2.18) :

$$\mathrm{N}\left(\vec{p}, \vec{l}, \vec{q}\right) = \frac{\mathrm{F}\left(\omega_{\vec{q}}\left(\vec{l}\right)\right)\lambda\left(\vec{q}, \vec{l}\right)}{\sqrt{\mathrm{I}\left(\vec{q}\right)}} + \mathrm{F}^*\left(\omega_{\vec{q}}\left(\vec{l}\right)\right)\mathrm{F}\left(\omega_{\vec{q}}\left(\vec{p}\right)\right)\left\{\frac{1}{\mathrm{I}\left(\vec{q}\right)} + \left(\omega_{\vec{q}}\left(\vec{p}\right) - \Omega_{\vec{q}}\right)\Phi\left(\Omega_q\right) + \ldots\right\}.$$

Let us define

$$\frac{g_{\mathrm{Y}}}{\sqrt{4\pi}} = \frac{1}{\sqrt{\mathrm{I}\left(\vec{q}\right)}}. \tag{2.21}$$

Then

$$\mathrm{N}\left(\vec{p}, \vec{l}, \vec{q}\right) = \mathrm{F}\left(\omega_q\left(\vec{l}\right)\right)\lambda\left(\vec{q}, \vec{l}\right)\frac{g_{\mathrm{Y}}}{\sqrt{4\pi}} + \mathrm{F}^*\left(\omega_q\left(\vec{l}\right)\right)\mathrm{F}\left(\omega_q\left(\vec{p}\right)\right) \times\left\{\frac{g_{\mathrm{Y}}^2}{4\pi} + \left(\omega_q\left(\vec{p}\right) - \Omega_{\vec{q}}\right)\Phi\left(\Omega_q\right) + \ldots\right\}. \tag{2.22}$$

Putting the expression (2.22) in the expression (2.7) of $b^{\text{out}}_{\vec{p},s}$, we see that $\frac{g_{\mathrm{Y}}}{\sqrt{4\pi}}$ defined in (2.21) is the renormalized coupling constant of the Yukawa interaction in which the composite bosons are considered as elementary ones. The evident relation

$$\mathrm{I}\left(\vec{q}\right) = \frac{\partial \mathrm{S}}{\partial \Omega_{\vec{q}}}$$

between $\mathrm{I}\left(\vec{q}\right)$ and $\mathrm{B}\left(\vec{q}, \Omega_{\vec{q}}\right)$ defined in (2.16) (7) shows that the renormalization constant Z_3 of the composite boson vanishes. It is also easily seen that $\frac{g_{\mathrm{Y}}}{\sqrt{4\pi}}$ is the residue of the scattering amplitude within a multiplicative factor (8). The dependence on $\vec{q}$ of $\frac{g_{\mathrm{Y}}^2}{4\pi}$, of $\mathrm{I}\left(\vec{q}\right)$ and of $\mathrm{S}\left(\vec{q}, \Omega_{\vec{q}}\right)$ is not a difficulty, it is inherent to the fact that the model is

(7) It is easily seen that $\mathrm{S}\left(\vec{q}, \Omega_{\vec{q}}\right)$ is proportional to the self-energy of the Yukawa boson in the non relativistic limit.

(8) We show below that the function $\mathrm{N}\left(\vec{p}, \vec{l}, \vec{q}\right)$ is related in a simple manner to the $\mathrm{N} - \overline{\mathrm{N}}$ scattering amplitude [*see* relation (2.29)].

non-relativistic. This difficulty does not appear in the relativistic model, in this case, $\frac{g_\mathcal{Y}^2}{4\pi}$, $I(\vec{q})$ and $S(\vec{q}, \Omega_{\vec{q}})$ do not depend on $\vec{q}$ and $\Omega_{\vec{q}}^2 = \vec{q}^2 + \mu^2$, μ being the composite boson mass. So we may put $\vec{q} = 0$ in all these expressions without trouble.

We shall need the expression of the complex function

$$(2.23) \qquad N(\vec{k}, \vec{k}, 0) \equiv N(\omega), \qquad \text{with} \quad \omega = \omega_0(\vec{k}) = 2\,E_{\vec{k}}.$$

It may be written as

$$(2.24) \qquad N(\omega) = g_F \frac{\| F(\omega) \|^2}{H(\omega)},$$

where

$$(2.25) \qquad \begin{cases} H(\omega) = A(\omega) - i\,B(\omega), \\ A(\omega) = 1 + g_F\, PV \int_{\omega_0}^{\infty} h(\omega') \dfrac{\| F(\omega')^2 \| \operatorname{Tr}(\omega')}{\omega' - \omega}\, d\omega', \\ B(\omega) = -\pi\, g_F\, h(\omega) \| F(\omega)^2 \| \operatorname{Tr}(\omega), \\ \omega_0 \quad = 2\,m, \qquad h(\omega) = \dfrac{\pi}{2}\,\omega \sqrt{\omega^2 - 4\,m^2}, \end{cases}$$

PV means the principal value.

The phase $-\delta(\omega)$ of $N(\omega)$ is given by

$$(2.26) \qquad \operatorname{tg} \delta(\omega) = \frac{B(\omega)}{A(\omega)}.$$

For $\omega < \omega_0$, $H(\omega)$ is real ($B(\omega) = 0$), $\delta(\omega) = \pi$. If $g_F \geq 0$, $H(\omega) \geq 1$ is a non-decreasing function of ω and has no zero. There are no composite bosons in this case.

If $g_F < 0$, $H(\omega)$ is a non increasing function of ω,

$$(2.27) \qquad \begin{cases} \qquad\qquad \text{For} \quad | g_F | < - | g_{\min} |, \\ \text{with} \quad | g_{\min} |^{-1} = \displaystyle\int_{\omega_0}^{\infty} h(\omega') \frac{\| F(\omega') |^2}{\omega' - 2\,m} \operatorname{Tr}(\omega')\, d\omega'. \end{cases}$$

$H(\omega)$ has *one* zero μ (and only *one*). $H(\mu) = 0$ is the compatibility condition (2.15) of the existence of composite boson the mass of which is μ. $\omega - \mu = 2\,E_k - \mu$ which is positive is the binding energy. In what follows, we assume that the condition (2.27) is fulfilled.

For $\omega > \omega_0 = 2\,m$, $H(\omega)$ is complex.

The variations of the different functions are given in the table I.

TABLE I

ω	0		μ		$\omega_0 = 2m$		ω'		$+\infty$
$B(\omega)$			0		0		> 0		0
$A(\omega)$		+	0	+		−	0	+	1^+
$\operatorname{tg}\delta(\omega)$			0		0^-	↘	$-\infty$ ‖ $+\infty$	↘	0^+
$\delta(\omega)$			π		π	↘	$\frac{\pi}{2}$	↘	0^-

If we assume A (ω) to be a continuous function of ω, then A (ω) has at least a zero $\omega' > \omega_0$ and ω' corresponds to $\delta = \frac{\pi}{2}$. As $\left(\frac{d\delta}{d\omega}\right)$ for $\delta = \frac{\pi}{2}$ is negative, ω' does not correspond to a resonance, it corresponds to a ghost state [5].

The asymptotic behaviour of $\operatorname{tg} \delta = \frac{B(\omega)}{A(\omega)}$ for ω going to infinity is easily obtained

$$\operatorname{tg}_{\omega \to \infty} \delta(\omega) \approx \omega^2 \| F(\omega) \|^2. \tag{2.28}$$

To conclude this chapter, we compute the N — $\overline{\text{N}}$ scattering amplitude of fermion N (k_1, α_1) and antifermion $\overline{\text{N}}(\overline{k}_2, \alpha_2)$ from the expressions of $b^{\text{out}+}_{\vec{p},s}$, $d^{\text{out}}_{\vec{p},s}$, $b^{\text{in}+}_{\vec{p},s}$, $d^{\text{in}+}_{\vec{p},s}$ given in (2.7) ([9]). The final state is a fermion N $(\overline{l}_1, r)$ and an antifermion $\overline{\text{N}}(\overline{l}_2, s)$. The notations are obvious. The computation may be done using the time dependence of the operators and the definition of S-matrix in terms of outgoing and ingoing states ([10]). We obtain for the scattering amplitude :

$$\begin{aligned} \langle \overline{\text{N}}(\overline{l}_2, s)\, \text{N}(l_1, r) \mid \text{T} \mid \overline{\text{N}}(\overline{k}_2, \alpha_2)\, \text{N}(\overline{k}_1, \alpha_1) \rangle \\ = - \text{N}\left(\vec{k}_1, \vec{l}_1, \vec{k}_1 + \vec{k}_2\right) 0^{r,s}\left(\vec{l}_1, \vec{l}_2\right) 0^{+\alpha_2,\alpha_1}\left(\vec{k}_1, \vec{k}_2\right). \end{aligned} \tag{2.29}$$

The expressions in the right hand side have been defined in (2.11), and in (2.4).

([9]) The ingoing operators are obtained from outgoing operators by the change of $i\varepsilon$ in $-i\varepsilon$.

([10]) It may be also done using the expansion of Umezawa in terms of physical operators [6].

3. UNITARITY CONDITION. RELATION BETWEEN THE FORM FACTOR AND THE CONVERGENCE FUNCTION

We have now all the elements to study our problem. We look for the relation between the absorptive part of the form factor and the convergence function. For this purpose, let us consider the vertex $N\bar{N}\gamma$ (*fig.* 1). $\bar{l}_i$ and $\bar{k}_i$ ($i = 1, 2$) are the quadrimomenta, α_1, α_2, r, s are the quantum number indices. We assume that in the inter-

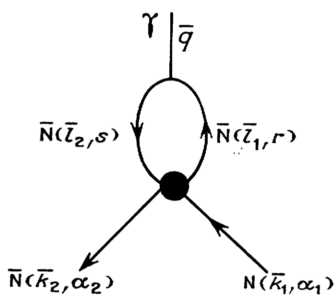

Fig. 1

mediate states, $N - \bar{N}$ contribution dominates. In this approximation, the usual unitarity condition for $N - \bar{N}\gamma$ process may be written as :

$$
\begin{aligned}
(3.1)\quad & \frac{e}{\pi}\sqrt{\frac{m^2}{E_{\vec{k}_1} E_{\vec{k}_2}}}\, \bar{v}\left(\vec{k}_2, \alpha_2\right) \\
& \times \left\{ \operatorname{Im} G(q^2)\,\gamma_\mu + \frac{\mu}{2m} \operatorname{Im} G'(q^2)\,\sigma_{\mu\nu}\, q_\nu \right\} u\left(\vec{k}_1, \alpha_1\right) \\
= & - i \sum_{r,s} \int d^3\vec{l}_1\, d^3\vec{l}_2\, \delta^4\left(\bar{l}_1 + \bar{l}_2 - \bar{k}_1 - \bar{k}_2\right) \\
& \times \left\langle 0\, j_\mu(0) \,|\, \bar{N}(\bar{l}_2, s)\, \bar{N}(\bar{l}_1, r) \right\rangle \\
& \times \left\langle \bar{N}(\bar{l}_2, s)\, N(\bar{l}_1, r) \,|\, T \,|\, \bar{N}(\bar{k}_2, \alpha_2)\, N(\bar{k}_1, \alpha_1) \right\rangle,
\end{aligned}
$$

$G(q^2)$ and $G'(q^2)$ are the nucleon form factors, $\bar{q} = \bar{k}_1 + \bar{k}_2$, μ is the anomalous magnetic moment, m the nucleon mass, $j_\mu(0)$ the electromagnetic current. $\bar{v}\left(\vec{k}_2, \alpha_2\right)$ and $u(k_1, \alpha_1)$ are the usual Dirac spinors.

The right hand side of (3.1) contains the $N - \bar{N}$ scattering amplitude which has been already calculated in the model, its expression is given in (2.29). In the considered Fermi self interaction with vector coupling, we obtain composite vector bosons which may be interpreted as vector mesons (for instance ρ and ω mesons). We have dropped the tensor term $q_\mu q_{\mu'}$ in (2.12); to remain in the same approximation, we consider only the Dirac form factor $G(\bar{q}^2)$ ([11]).

Putting the expression (2.29) in (3.1), and after straight forward calculation, we get in the center of mass system :

$$\bar{k}_1 = \left(\vec{k}, E_{\vec{k}}\right); \qquad \bar{k}_2 = \left(-\vec{k}, E_{\vec{k}}\right),$$
$$\vec{q}^2 = -4 E_{\vec{k}}^2 = -t,$$

$$(3.2)\quad \operatorname{Im} G(\bar{q}^2) = \frac{1}{4\pi}\sqrt{\frac{\bar{q}^2 + 4m^2}{\bar{q}^2}}\,\frac{(2m^2 - q^2)}{3}\,N\left(\vec{k}, \vec{k}, 0\right) G^*(\bar{q}^2),$$

where $N\left(\vec{k}, \vec{k}, 0\right)$ is given in (2.23), $N\left(\vec{k}, \vec{k}, 0\right)$ depends on the convergence function $F(\omega)$ through the relation (2.24). We get then the relation between the Dirac form factor and the convergence function. From (3.2), we deduce that the *phase of the Dirac form factor* (which is complex in the time like region) is equal to the phase $\delta(\omega)$ of $N\left(\vec{k}, \vec{k}, 0\right)$ defined in (2.26). The phase δ which is related to the $N - \bar{N}$ scattering amplitude has been already studied in the Chapter 2 ([12]). Let us look for the asymptotic behaviour of $G(q^2)$ for $\bar{q}^2$ going to infinity. The asymptotic behaviour of tg δ and the convergence function $F(\omega)$ have been respectively given in (2.28) and (2.5).

From the relation $\operatorname{Im} G(\omega) = \operatorname{Re} G(\omega)\, \mathrm{tg}\, \delta$, we get

$$(3.3)\qquad \operatorname*{Im}_{\omega \to \infty} G(-\omega^2) \simeq \operatorname{Re} G(-\omega^2)\, \omega^2 \frac{C}{\omega^{2+\varepsilon}}\cdot$$

If $\operatorname*{Re}_{\omega\to\infty} G(-\omega^2) \simeq \frac{A'}{\omega^2}$, A' being a constant (it will be shown to hold, effectively), we may write for the form factor $G(q^2)$ an unbsubstracted dispersion relation

$$G(\bar{q}^2) = \frac{1}{\pi}\int_{4m^2}^{\infty} \frac{\operatorname{Im} G(-\sigma^2)}{\sigma^2 + q^2 - i\varepsilon}\, d\sigma^2,$$

where $\operatorname{Im} G(-\sigma^2) = \mathrm{tg}\,\delta(\sigma^2) \operatorname{Re} G(\sigma^2)$.

([11]) This approximation is not an essential point. Terms in $q_\mu q_\nu$ may be taken into account. On one hand, one is led to complicated calculations; on the other hand, a system of coupled relations in $G(\bar{q}^2)$ and $G'(\bar{q}^2)$ is obtained. Here, for the sake of simplicity, we drop tensor terms.

([12]) There is another possibility used by Goldberger-Treiman, they introduce the mean of ingoing and outgoing states in the intermediate states [7].

The form factor $G(q^2)$ verifies a Muskhekishvili-Omnes equation. One type of solution is [8]

$$(3.4)\qquad \begin{cases} G(q^2) = G \exp[u(q^2)], \\ u(q^2) = \rho(q^2) + i\,\delta(q^2), \\ \delta(q^2) = \dfrac{1}{\pi}\,\mathrm{PV}\displaystyle\int_{4m^2}^{\infty} d\sigma^2 \frac{\delta(-\sigma^2)}{\sigma^2 + q^2}; \end{cases}$$

(3.4) shows that the assumption (3.3) is true. $G(\bar{q}^2)$ is asymptotically proportional to q^{-2} for $q^2 \to \infty$, which is a behaviour not expected if we compare with the latest experimental behaviour in q^{-4}.

4. DEPENDENCE OF THE $N - \bar{N}$ SCATTERING DIFFERENTIAL CROSS SECTION IN TERMS OF THE CONVERGENCE FUNCTION

Let us compute the differential cross section $\frac{d\sigma}{d\Omega}$ of the $N - \bar{N}$ scattering in order to see its dependence in terms of the convergence function $F(\omega)$ (and then in terms of the form factor G when we shall put $F = G$ as we shall do in the next chapter).

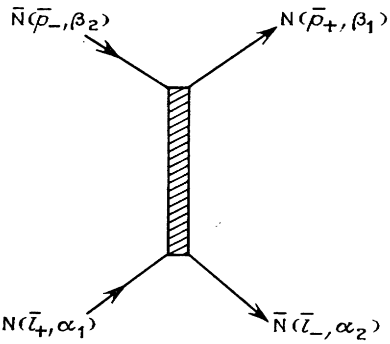

Fig. 2

All the notations are given in the figure 2. In the center of mass system, the momentum-energy transfert q is equal to $(0, 2E_{\vec{p}})$. As usually, we define :

$$s = -(\bar{l}_- - \bar{p}_-)^2 = -2p^2(1 - \cos\varphi),$$
$$t = -(\bar{l}_+ + \bar{l}_-)^2 = 4E_{\vec{l}}^2 = -\bar{q}^2,$$

φ is the scattering angle $(\vec{l}_-, \vec{p}_-)$. The scattering amplitude is given in (2.29). A straightforward calculation gives for the vector coupling the following expression of the differential cross section in the center of mass system :

$$\frac{d\sigma}{d\Omega} = \frac{1}{(2\pi)^2} \left\| \mathrm{N}(\vec{p}, \vec{l}, 0) \right\|^2 \frac{1}{t} \left\{ \frac{1}{2} t^2 + st + (s - 2m^2)^2 \right\}, \tag{4.1}$$

$$\left\| \mathrm{N}(\vec{p}, \vec{l}, 0) \right\|^2 = g_{\mathrm{F}}^2 \, \| \mathrm{F}(-t) \|^4 / \mathrm{A}^2(-t) + \mathrm{B}^2(-t), \tag{4.2}$$

F (— t) is the convergence function [13]. A (— t) and B (— t) are defined in (2.25).

We do the two following remarks. First, the expression (4.1) is different from one obtains in the one elementary boson exchange approximation in the intermediate state. However, if we consider *only* the contribution of the pole μ of $\mathrm{N}(\vec{p}, \vec{l}, 0)$ [*see* the expansion (2.22) of $\mathrm{N}(\vec{p}, \vec{l}, \vec{q})$], this contribution may be exactly equated to the differential cross section obtained in the one elementary vector boson exchange approximation, provided the vector boson-nucleon renormalized coupling constant of the corresponding Yukawa interaction is $\frac{g_{\mathrm{Y}}^2}{4\pi}$ given by (2.21).

The second remark is that (4.1) is not the first approximation of a perturbative expansion in the Fermi-coupling constant g_{F} [*see* for instance the expressions of A (— t) and B (— t). It is the result of the sum of infinite one pair contributions (chain approximation).

5. POSSIBILITY OF IDENTIFYING THE CONVERGENCE FUNCTION AND THE FORM FACTOR. SOME IMPLICATIONS OF THIS ASSUMPTION. CONCLUDING REMARKS

Let us recall the main results of the preceding chapters. In the approximation of $\mathrm{N} - \overline{\mathrm{N}}$ dominance in the intermediate state, we have found that the phase δ of the Dirac form factor may be equated to the phase of the function $\mathrm{N}(\vec{k}, \vec{k}, 0)$ which is related to the amplitude of $\mathrm{N} - \overline{\mathrm{N}}$ elastic scattering through the relation (2.29). The condition on the

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(13) F (ω), A (ω), B (ω) are assumed to depend on ω through ω^2.

convergence function F (ω) entails that δ goes to 0 for ω going to infinity. If the Fermi coupling constant g_F is less than the value — $| g_{\min} |$, we obtain composite vector bosons. If we consider the latter as elementary vector mesons and make the approximation of taking only into account the contribution corresponding to the pole (i. e. composite boundstate contribution), the results thus obtained leads in a natural way to the well-known vector meson dominance model. The condition that the square modulus of the Fermi convergence function $\| F(\omega) \|^2$ decreases faster than $\omega^{-2-\varepsilon}$ $(\varepsilon > 0)$ for large values of ω yields the conclusion that the Dirac form factor verifies an *unsubstracted* dispersion relation and is a solution of a Muskhelishvili-Omnes equation.

On one hand, if we look at the expression (2.3) of the Fermi self interaction with the vector coupling, the convergence function F (ω), introduced in order to have a free divergency Fermi theory, is associated with the nucleon current (strong interaction). On the other hand, the Dirac form factor is also associated with the nucleon current (electromagnetic current). Then, we are led to look for the possibility of identifying the convergence function and the Dirac form factor.

For this purpose, we have first to modify a restrictive assumption done on the convergence function and to look for the hypothesis which make the properties of the convergence function and those of the form factor compatible.

The convergence function F (ω) was assumed to be an entire function in order to introduce no other singularities [3] : this may be also interpreted in assuming that F (ω) does not add or cancel any singularities other than those already contained in the model. If this condition excludes the possibility of F (ω) to have poles, it allows cuts, for instance the cut from 4 m^2 to infinity which appears in the model and also in the form factor. The decrease at infinity of F (ω) is compatible with the decrease in q^{-2} at infinity of the form factor G (ω). Then, we may conclude to the possibility of identifying F (ω) and G (ω) as far as we know about form factor.

According to this identification, and after having replaced the expression of $N(\vec{k}, \vec{k}, 0)$ given in (2.23), the relation (3.2) gives the non linear integral equation

$$\operatorname{Im} G(\omega^2) = \frac{\left(\frac{1}{2} + \frac{m^2}{\omega^2}\right) \pi\, g_F\, h(\omega) \, \| G(\omega^2) \|^2}{12\, \pi^2 \left[A^2(\omega) + B^2(\omega)\right]^{\frac{1}{2}}}.$$

Let us now look for the implications of $F(\omega^2) \equiv G(\omega^2)$ on the differential cross section of the $N - \overline{N}$ scattering. A glance at its expression (4.1) may lead to a wrong conclusion. We may be tempted saying that it depends on the fourth power of the form factor $G(\omega^2)$. In fact, this is not completely true because the functions $A^2(-t)$ and $B^2(-t)$ in the denominator of (4.2) depend indeed on the form factor $G(\omega^2)$ [*see* relations (2.25)]. But if we take only (and only in this approximation) account of the first term in the expansion (2.22) of $N(\vec{p}, \vec{l}, 0)$ in the neighbourhood of the pole (i. e. the composite vector boson contribution) then we obtain that the differential cross section of the $N - \overline{N}$ elastic scattering is proportional to the fourth power of the form factor and this result holds for *any values of* $\vec{q}^2$. The latter has something to do with the Wu and Yang's hypothesis concerning the proton-proton scattering [9]. These two authors predict that the proton-proton differential cross section for scattering angle near 90° be proportional to the fourth power of the electromagnetic form factor for large values of $\vec{q}^2$. In order to test this hypothesis, some phenomenological models have been used and they show agreement with such a possibility [10]. This hypothesis for proton-proton scattering holds also for proton-antiproton elastic scattering because one may define the form factor for a particle and its antiparticle and invoque one of the Pomeranchuk theorems. Our result is that the contribution of the pole (i. e. the composite vector boson) in the differential cross section of $N - \overline{N}$ elastic scattering is proportional to the fourth power of the form factor for *any* values of $t = -\vec{q}^2$ (not only for large values of $\vec{q}^2$).

In the assumption $F(\omega) = G(\omega)$, the electromagnetic form factor is associated to the nucleon current whatever the interaction is strong or electromagnetic.

The expression of the structure function $\lambda(\vec{q}, \vec{p})$ of the composite bosons in (2.17) contains the convergence function $F(\omega)$ which may be considered as their form factor. In the case of $\gamma_5 \otimes \tau_\rho$ ($\rho = 1, 4$) and $\gamma_\mu \otimes \tau_\rho$ ($\mu, \rho = 1, \ldots, 4$) Fermi coupling, we obtain composite bosons (pseudoscalar-isovector, pseudoscalar-isoscalar, vector-isovector and vector-isoscalar nucleon-antinucleon boundstates) which may be interpreted as representing $\pi - \eta$, ρ and ω-mesons. The assumption $F(\omega) = G(\omega)$ entails that the form factors of the π, η, ρ and ω-mesons are all equal to the charge form factor of the nucleon in the time-like region. This result is to be related to the experimental results which indicate that there are no essential differences between the electromagnetic form factors of the π-mesons and of nucleon in the space-like region [11]. The assumption that the convergence function in the Fermi theory may be equated to the electromagnetic charge form factor of the nucleon leads to the universality of the form factors in the time-like region.

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