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Composite propagator and the equivalence theorem in the $Z_3 = 0$ theory (*)

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SUMMARY. — The $Z_3 = 0$ theory for allo-composite particles is based on an equivalence theorem between a field theory with a Yukawa coupling in the limit when $Z_3 \to 0$ and a field theory with a Fermi coupling. The composite propagators are different in both theories. Here we study this problem in a suitable field theoretical model, relating it to the above mentioned theorem, which is shown to represent only a restricted equivalence.

RÉSUMÉ. — Dans la théorie des particules allo-composées, basée sur un théorème d'équivalence de la théorie des champs à couplage de Yukawa à la limite $Z_3 \to 0$ et des théories des champs à couplage de Fermi, les propagateurs des particules présentent certaines différences.

Nous étudions ici ces différences dans un modèle particulier et montrons que le théorème énoncé plus haut doit être pris dans un sens restreint.

INTRODUCTION

The field theoretical approach to allo-composite particles [*1*] is based on the $Z_3 = 0$ theory of B. Jouvet. This theory relies on an equivalence theorem [*2*] which states that for a given field theory with a Yukawa coupling

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there exists a field theory with a Fermi coupling yielding the same observable results of the former theory considered in the limit $Z_B = 0$ (where Z_B is the renormalization constant of the field associated to the composite particle).

The Yukawa coupling corresponds to the basic interaction $B \leftrightarrow A + A$ while the Fermi coupling corresponds to a theory where only a particle A appears explicity ($A + A \leftrightarrow A + A$) and particle B is composite.

The equivalence theorem has been studied in the Lee model [*3*]. The propagator of the composite particle has also been studied in this model by J. C. Houard [*4*], who proved that the composite propagator in the limiting Yukawa theory is different from that of the corresponding Fermi theory. This difference was related in this paper to the existence of a resonance whose position goes to infinity when $Z_3 \to 0$.

In Section I we show by an explicit calculation in the Lee model that this resonance completely accounts for the difference in both propagators.

In Section II we introduce a suitable zero dimensional model (i. e., a model in which the fields depend only on t) in order to examine more closely the meaning of the result of Section I. We find that Jouvet's equivalence theorem should not be interpreted in a strict sense, and that the difference between the propagators is a consequence of this limitation.

The result is shown to be related to the remarks on the singular character of the limit $Z_3 \to 0$ made by Sekine [*5*].

I. — This Section is based on the results of reference [*4*], and we shall use the same notation.

From the field equations we can deduce that in the limit $Z_3 \to 0$ the renormalized field $\psi_v(x)$ should be replaced by

$$\hat{\psi}_v(x) \equiv \frac{1}{\delta v_l} j(x) \equiv -\frac{g}{\delta v_l}\psi_N(x)\int d\vec{x}' f(\vec{x} - \vec{x}')A(x', t). \qquad (1.1)$$

The propagator of the (elementary) V-particle is

$$S'(x) = \langle 0 | T(\psi_v(x)\psi_v^+(0)) | 0 \rangle \qquad (1.2)$$

and the composite (limit) propagator $S_l'(x)$ is the limit of (1.2) when $Z_3 \to 0$.

We can also introduce in a natural way the matrix element corresponding to (1.2) in the Fermi model:

$$J(x) = \langle 0 | T(\hat{\psi}_v(x)\hat{\psi}_v^+(0)) | 0 \rangle.$$

Direct calculation shows that, in spite of equation (1.1),

$$J(x) \neq S_l'(x) = \lim_{Z_3 \to 0} S'(x),$$

the difference of these functions being equal to

$$(1/Z_3)\exp(i\delta v_l t/Z_3) = (g_0^2/g_l^2)\exp(ig_0^2 t/\lambda),$$

i. e. to a term which oscillates with infinite amplitude and infinite frequency. The corresponding Fourier transforms differ by the constant term $1/\delta v_l$ which is responsible for the different asymptotic behaviour.

We prove now that the contribution of the resonance found by Houard is precisely equal to $1/\delta v_l$. From reference [4] we know that

$$S'(p) = \mathscr{F}\{S'(x)\} = \frac{i}{(2\pi)^4}\int \frac{da\rho(a)}{p_0 - a + i\varepsilon} \tag{1.3}$$

$$S_l'(p) = \mathscr{F}\{S_l'(x)\} = \frac{i}{(2\pi)^4}\left[\int \frac{da\rho_l(a)}{p_0 - a + i\varepsilon} + \frac{1}{\delta v_l}\right] \tag{1.4}$$

$$J(p) = \mathscr{F}\{J(x)\} = \frac{i}{(2\pi)^4}\int \frac{da\rho_l(a)}{p_0 - a + i\varepsilon} \tag{1.5}$$

where $\rho(a)$ and $\rho_l(a)$ are the spectral functions, and for any finite a we have $\lim_{Z_3\to 0} \rho(a) = \rho_l(a)$. The function $\rho(a)$ contains a resonance whose position $a = \alpha$ goes to infinity when $Z_3 \to 0$:

$$\lim_{Z_3\to 0} \alpha = \lim_{Z_3\to 0}\left(-\frac{\delta v_l}{Z_3}\right) = \infty \tag{1.6}$$

(we recall that δv_l is negative). Both spectral functions contain the term $\delta(a - M)$ which is conveniently removed by the substitution

$$\bar{\rho}(a) = \rho(a) - \delta(a - M)$$

and an analogous one for $\bar{\rho}_l(a)$. We can then write [4]

$$\bar{\rho}(a) = \frac{1}{\pi}\frac{I(a)}{R(a)^2 + I(a)^2}, \tag{1.7}$$

with

$$R(a) = Z_3(a - M) + \frac{g^2}{4\pi^2}(a - M)\int \frac{f(\omega)^2 K(\omega) d\omega}{(\omega + m - M)(\omega + m - a)}, \tag{1.8}$$

$$I(a) = \frac{g^2}{4\pi} f(a - m)^2 K(a - m)\theta(a - m - \mu), \tag{1.9}$$

where M, m, and μ are the masses of the V, N, and θ particle respectively; and $K(\omega) = (\omega^2 - \mu^2)^{1/2}$.

The resonance occurs at the value $a = \alpha$ for which $R(\alpha) = 0$. The height of the resonance is given by

$$h = [(g^2/4)\alpha f(\alpha)^2]^{-1} \tag{1.10}$$

and its width, which is defined by the condition $R(\alpha + \Gamma) = I(\alpha + \Gamma)$, is

$$\Gamma = -\frac{g^2}{4\pi\delta v_l}\alpha^2 f(\alpha)^2. \tag{1.11}$$

Equations (1.10) and (1.11) are actually valid only asymptotically, they can be deduced from equations (1.6), (1.7), (1.8) and (1.9) taking into account the shrinking of the peak for $\alpha \to \infty$ as a consequence of the asymptotic behaviour of $f(\alpha)$ (which vanishes faster then α^{-1}).

We introduce now two new functions, $\tilde{\rho}(a)$ and $\rho_r(a)$, defined by

$$\rho_r(a) = \frac{\Gamma^2 h}{(a-\alpha)^2 + \Gamma^2} \tag{1.12}$$

and

$$\bar{\rho}(a) = \tilde{\rho}(a) + \rho_r(a). \tag{1.13}$$

The functions $\rho_r(a)$ represents the contribution of the resonance to $\bar{\rho}(a)$ and it is easy to verify that we have

$$\lim_{Z_3 \to 0} \tilde{\rho}(a) = \bar{\rho}_l(a) \tag{1.14}$$

and

$$\lim_{Z_3 \to 0} \rho_r(a) = 0 \tag{1.15}$$

since $\lim_{Z_3 \to 0} (\Gamma^2 h/\alpha^2) = 0$. The essential point here is that in the limit $Z_3 \to 0$ the contribution to the spectral integral of $\rho_r(a)$ is exactly $(1/\delta v_l)$. To show this we evaluate the spectral integral for $\rho_r(a)$ using equation (1.9):

$$\int_{m+\mu}^{\infty} \frac{\rho_r(a)da}{p_0 - a} = -\frac{\Gamma^2 h}{(a-p_0)^2 + \Gamma^2}$$

$$\left\{ \text{Log} \frac{\sqrt{(m+\mu-\alpha)^2 + \Gamma^2}}{m+\mu-p_0} + \frac{\alpha - p_0}{\Gamma}\left(\frac{\pi}{2} - \text{arc tg}\,\frac{m+\mu-\alpha}{\Gamma}\right) \right\}.$$

In the limit $Z_3 \to 0$, i. e. $\alpha \to \infty$, the logarithmic term vanishes; the argument of the inverse tangent function tends to $-\alpha/\Gamma$, i. e. to infinity. Neglecting p_0 and Γ as compared with α we finally obtain

$$\lim_{Z_3 \to 0} \int_{m+\mu}^{\infty} \frac{\rho_r(a)da}{p_0 - a} = -\frac{h\Gamma\pi}{\alpha} = \frac{1}{\delta v_l}.$$

This proves, as we have already stated, that the resonance completely accounts for the observed difference between the propagators of the Yukawa and the Fermi models. We remark that in coordinate space the difference is given by

$$S'_l(x) - J(x) = \delta(\vec{x})\theta(t) \lim_{Z_3 \to 0} \frac{1}{Z_3} e^{-i\alpha t}$$

The discussion we have just made shows that limit (1.14) can be interchanged with an spectral integral of the type (1.3), while this is not the case for the limit $\lim_{Z_3 \to 0} \rho(a) = \rho_l(a)$, the difference between both cases is due to the $\rho_r(a)$ part of the spectral function which, in spite of (1.15), gives a contribution $(1/\delta v_l)$ to the spectral integral in the limit $Z_3 \to 0$.

II. — We shall now study the equivalence theorem and the asymptotic behaviour of the composite propagator in zero dimensional models [6].

We start with a Yukawa type model defined by the unrenormalized Lagrangian

$$I_y = \varphi_0^+\left(i\frac{\partial}{\partial t} - \mu_0\right)\varphi_0 + \psi^+\left(i\frac{\partial}{\partial t} - m\right)\psi - g_0(\varphi_0^+\psi\psi + \psi^+\psi^+\varphi_0). \qquad (2.1)$$

In the interaction representation one has

$$\varphi_0(t) = b \exp[-i\mu_0 t], \qquad \psi(t) = a \exp[-imt],$$

together with the commutation relations $[a, a^+] = [b, b^+] = 1$.

Ig Z_3 is the renormalization constant of the field φ_0 we define

$$\varphi = \sqrt{Z_3}\varphi_0, \qquad g = \sqrt{Z_3}g_0, \qquad \delta_\mu = \mu - \mu_0, \qquad \delta v = Z_3\delta\mu,$$

where φ, g and μ are the renormalized field, coupling constant and mass, respectively. Ig we impose the condition $Z_3 = 0$ in the field equation for φ we find that φ should be replaced by the current $j(t) = (g_l/\delta v_l)\psi(t)\psi(t)$, where the index l refers to the limiting values of the corresponding quantities when $Z_3 \to 0$. Replacement of the field $\varphi(t)$ by $j(t)$ in the original Lagrangian yields the Lagrangian of the equivalent Fermi theory.

$$L_F = \psi^+\left(i\frac{\partial}{\partial t} - m\right)\psi - \lambda\psi^+\psi^+\psi\psi \qquad (2.2)$$

where $\lambda = g_l^2/\delta v_l = \lim_{Z_3 \to 0} (-g_0^2/\mu_0)$.

Let us precise the meaning of the limit $Z_3 \to 0$ of the Yukawa theory. This theory is characterized by three independent parameters m, μ_0, and g_0. The limit $Z_3 \to 0$ will be taken here (this should become clear later, after we give the expression for Z_3 in terms of the parameters of the theory) keeping m fixed, while $g_0^2 \to \infty$. μ_0 being a function of g_0^2 such that

$$\lim\left(-g_0^2/\mu_0(g_0^2)\right) = \lambda + 0(g_0^{-N}), \qquad N \geqslant 3.$$

The Hamiltonians of the theories are given by

$$H_y = ma^+a + \mu_0 b^+ b + g_0(b^+a^2 + a^{+2}b), \tag{2.3}$$

$$H_F = ma^+a + \lambda a^{+2}a^2. \tag{2.4}$$

The unrenormalized propagator is calculated from

$$S(t) = \langle 0 | T(\varphi_0(t)\varphi_0^+(0)) | \rangle = \theta(t)\langle 0 | \varphi_0(t)\varphi_0^+(0) | 0 \rangle$$

(see for instance reference [6]) and its Fourier transform is given by

$$S(p) = \mathscr{F}\{S(t)\} = \frac{1}{p - \mu_0 - \dfrac{2g_0^2}{p - 2m}}. \tag{2.5}$$

It is easily seen that $S(p)$ has two poles on the real axis. One of them goes to infinity like g_0 while the other remains at a finite position; the latter is obviously the one which corresponds to the renormalized mass μ, and the residue at this pole is Z_3. We get

$$Z_3 = \frac{1}{1 + \dfrac{2g_0^2}{(2m - \mu)^2}} = \frac{1}{1 + \dfrac{(\mu - \mu_0)^2}{2g_0^2}}, \tag{2.6}$$

$$\mu = \frac{\mu_0}{2} + m - \sqrt{\frac{(\mu_0 - 2m)^2}{4} + 2g_0^2}. \tag{2.7}$$

In the limit $Z_3 \to 0$ we have

$$\mu \to \mu_l + 0(g_0^{-2}), \qquad g^2 \to g_l^2 + 0(g_0^{-2}), \qquad \delta v \to \delta v_l + 0(g_0^{-2}),$$

with

$$\mu_l = 2(m + \lambda),\ g_l^2 = (2m - \mu_l)^2/2 = 2\lambda^2,\ \delta v_l = g_l^2/\lambda = 2\lambda.$$

We shall now introduce the renormalized propagator

$$S'(p) = Z_3^{-1}S(p) = \mathscr{F}\{\langle 0 | T(\varphi(t)\varphi^+(0)) | 0 \rangle\}.$$

It will be meaningful to compare it with the corresponding function constructed with the « current » $j(t)$ (or Fermi propagator). One gets

$$S_l'(p) = \lim_{Z_3 \to 0} S'(p) = \frac{1}{p - 2(m + \lambda)} + \frac{1}{\delta v_l}, \tag{2.8}$$

while a direct calculation gives

$$S_F(p) = \mathscr{F}\{\theta(t)\langle 0|j(t)j^+(0)|0\rangle\} = \frac{1}{p - 2(m + \lambda)}. \tag{2.9}$$

Thus we find again the same difference between the propagators as in Sect. 1.

In order to understand the origin of the $1/\delta v_l$ term we shall now solve the eigenvalue problem for the Hamiltonian (2.3). One gets two eigenvelues $E_\pm$:

$$E_\pm = \frac{\mu_0}{2} + m \pm \sqrt{\frac{(\mu_0 - 2m)^2}{4} + 2g_0^2}. \tag{2.10}$$

One of them, E_-, is the renormalized mass $E_- = \mu = 2(m + \lambda) + 0(g_0^{-2})$. The other eigenvalue is found at a position which in the limit $Z_3 \to 0$ goes to infinity, and will be later identified as the source of the difference between both models (i. e. it corresponds to the resonance found in the three-dimensional model). One has

$$\lim_{Z_3 \to 0} E_+ = \mu_0 - 2\lambda - \frac{2\lambda^2}{g_0^2} E_F + 0(g_0^{-4}). \tag{2.11}$$

The orthonormalized eigenvectors are:

$$|E_-(t)\rangle = e^{-iE_-t}|E_-\rangle = \sqrt{Z_3}e^{-iE_-t}\left(b^+ + \frac{\mu - \mu_0}{2g_0}(a^+)^2\right)|0\rangle, \tag{2.12}$$

$$|E_+(t)\rangle = e^{-iE_+t}|E_+\rangle = \sqrt{A}e^{-iE_+t}\left(b^+ + \frac{E_+ - \mu_0}{2g_0}(a^+)^2\right)|0\rangle, \tag{2.13}$$

with

$$A = \left[1 + \frac{(E_+ - \mu_0)^2}{2g_0^2}\right]^{-1} = \left[1 + \frac{2g_0^2}{(E_+ - 2m)^2}\right]^{-1}.$$

Let us consider now the eigenvalue problem for the Fermi model. One gets only one eigenvalue $E_F = 2(m + \lambda)$ and the corresponding eigenvector is

$$|E_F(t)\rangle = e^{-E_Ft}|E_F\rangle = -\frac{1}{\sqrt{2}}e^{-iE_Ft}(a^+)^2|0\rangle. \tag{2.15}$$

One easily verifies that

$$\lim_{Z_3 \to 0} E_- = E_F + 0(g_0^{-2}), \tag{2.16}$$

$$\lim_{Z_3 \to 0} |E_-\rangle = |E_F\rangle + 0(g_0^{-1}) b^+ |0\rangle, \tag{2.17}$$

$$\lim_{Z_3 \to 0} |E_+(t)\rangle = e^{-iE_+ t} \left\{ \left[1 - \frac{\lambda^2}{g_0^2} + 0(g_0^{-4}) \right] b^+ + \left[\frac{\lambda}{g_0} + 0(g_0^{-3}) \right] (a^+)^2 \right\} |0\rangle, \tag{2.18}$$

while the limiting value of E_+ is given by (2.11). This shows that for any finite energy ($E_+ \to \infty$) the spectra of both models coincide. However, complete equivalence could be reached only if the second eigenstate dissappears, which is not the case.

We are now able to calculate the spectral functions for both theories. For the Yukawa model we have

$$\rho(E) = \delta(E - E_-) + \frac{A}{Z_3} \delta(E - E_+), \tag{2.19}$$

and in the limit $Z_3 \to 0$ we get

$$\lim_{Z_3 \to 0} \rho(E) = \delta(E - E_-) + \left(\frac{g_0^2}{2\lambda^2} - \frac{2\lambda^2}{g_l^2} + 0(g_0^{-2}) \right) \delta(E - E_+). \tag{2.20}$$

For the propagator we have

(2.21)
$$S_l'(p) = \lim_{Z_3 \to 0} S'(p) = \lim_{Z_3 \to 0} \int_0^\infty \frac{\rho(E) dE}{p - E},$$
$$S_l'(p) = \frac{1}{p - E_F} + \lim_{Z_3 \to 0} \frac{1}{2\lambda^2} \frac{g_0^2}{p + \frac{g_0^2}{\lambda}} = \frac{1}{p - E_F} + \frac{1}{\delta v_l}.$$

In the Fermi theory the spectral function $\rho_F(E)$ is given by

$$\rho_F(E) = \sum_n |\langle 0 | j(0) | E_n \rangle|^2 \delta(E \pm E_n) = \delta(E - E_F) \tag{2.22}$$

and the Fermi propagator is

$$S_F(p) = \int \frac{\rho_F(E) dE}{p - E} = \frac{1}{p - E_F}. \tag{2.23}$$

These calculations clearly show that the difference between both propagators arises from the contribution of the state $|E_+\rangle$ to the spectral function. It is also easily seen from (2.21) that the difference in the asymptotic behaviour is due to the interchanging of the limit $Z_3 \to 0$ and $p \to \infty$. Thus we see the correspondence with the results of the three-dimensional theory, the resonance being represented here by $|E_+\rangle$. We find again $\lim_{Z_3 \to 0} \rho_y(E) = \rho_F(E)$ for any finite E, but $\rho_y(E)$ contains a δ-function (the contribution of $|E_+\rangle$) whose position and coefficient both tend to infinity, and which gives a contribution to the propagator. Notice the fact that the source of the difference between the limiting Yukawa theory and the Fermi theory is perfectly singled out in the zero-dimensional model, because of the absence of the continuum.

We have stated that in the limit the field $\varphi(t)$ should be replaced by the current $j(t)$. Let us then study the vectors $\varphi^+(t)|0\rangle$ of the Yukawa theory and $j^+(t)|0\rangle$ of the Fermi theory. One has

$$\varphi^+(t)|0\rangle = |E_-\rangle\langle E_-|\varphi^+(t)|0\rangle + |E_+\rangle\langle E_+|\varphi^+(t)|0\rangle, \quad (2.24)$$

$$j^+(t)|0\rangle = |E_F\rangle\langle E_F|j^+(t)|0\rangle. \quad (2.25)$$

Taking the limit $Z_3 \to 0$ in (2.24) we get

$$\lim_{Z_3 \to 0} \varphi^+(t)|0\rangle = j^+(t)|0\rangle + \left[\frac{1}{\sqrt{Z_3}} + 0(g_0^{-1})\right] e^{iE_+ t}|E_+\rangle. \quad (2.26)$$

This last equation shows that the vectors $\varphi^+(t)|0\rangle$ and $j^+(t)|0\rangle$ differ by the vector associated with the « resonant » state $|E_+\rangle$, with a coefficient which diverges as $Z_3^{-1/2}$. The fact that the difference is proportional to $|E_+\rangle$ only allows us to say that the projection of $\varphi^+(t)|0\rangle$ on the subspace which is orthogonal to $|E_+\rangle$ (in our case this space is one-dimensional and its unit vector is $|E_-\rangle$ will tend to $j^+(t)|0\rangle$.

In the following paragraph we shall consider the problem from a different point of view: that of the time evolution of the state vectors.

Let us consider an arbitrary vector $|\psi(t)\rangle$ belonging to the sector AA of the Hilbert space of the Yukawa model, which is a two dimensional space. In the basis $(b^+|0\rangle, (a^+)^2|0\rangle)$ we have

$$|\psi(t)\rangle = \alpha(t)b^+|0\rangle + \beta(t)(a^+)^2|0\rangle.$$

The time evolution of $|\psi(t)\rangle$ is determined by the Schrodinger equation

$$i\frac{\partial}{\partial t}|\psi(t)\rangle = H_y|\psi(t)$$

which gives a system of two differential equations for the coefficients $\overline{\alpha}(t) = (Z_3)^{-1/2}\alpha(t)$ and $\beta(t)$:

$$\frac{i\sqrt{Z_3}}{g_0}\frac{d\overline{\alpha}}{dt} = \frac{\mu_0\sqrt{Z_3}}{g_0}\overline{\alpha} + 2\beta, \quad (2.27)$$

$$i\frac{d\beta}{dt} = g_0\sqrt{Z_3}\bar{\alpha} + 2m\beta. \tag{2.28}$$

The general solution is of the form

$$\bar{\alpha}(t) = Ae^{-iE_+t} + Be^{-iE_-t},$$
$$\beta(t) = Ce^{-iE_+t} + De^{-iE_-t},$$

with

$$A = \frac{\sqrt{Z_3}\bar{\alpha}_0(E_- - \mu_0) - 2g_0\beta_0}{\sqrt{Z_3}(E_- - E_+)},$$

$$B = \frac{2g_0\beta_0 - \sqrt{Z_3}(E_+ - \mu_0)\bar{\alpha}_0}{\sqrt{Z_3}(E_- - E_+)},$$

$$C = \frac{\sqrt{Z_3}}{2g_0}(E_+ - \mu_0)A,$$

$$D = \frac{\sqrt{Z_3}}{2g_0}(E_- - \mu_0)B.$$

If we want $|\psi(t)\rangle$ to coincide with the vector $\varphi^+(-t)|0\rangle$ of the preceding paragraph we must choose $\alpha_0 = 1/\sqrt{Z_3}$, $\beta_0 = 0$. We get for

$$\varphi^+(-t)|0\rangle = |\Phi(t)\rangle$$

the expression

$$\lim_{Z_3\to 0} |\Phi(t)\rangle$$
$$= \left\{\frac{1}{\sqrt{Z_3}}\left[1 - \frac{2\lambda^2}{g_0^2} + 0(g_0^{-4})\right]e^{-iE_+t} + [\sqrt{Z_3} + 0(g_0^{-3})]e^{-iE_-t}\right]b^+|0\rangle$$
$$+ \left\{\left[\frac{1}{\sqrt{2}} + 0(g_0^{-2})\right]e^{-iE_+t} - \left[\frac{1}{\sqrt{2}} + 0(g_0^{-2})\right]e^{-iE_-t}\right\}(a^+)^2|0\rangle \tag{2.29}$$

which coincides with (2.24) if we express it in the basis $(|E_-\rangle, |E_+\rangle)$.

An analogous calculation can be performed in the Fermi model. Now the Hilbert space has only one dimension, and an arbitrary vector is

$$|\psi_F(t)\rangle = \beta_F(t)(a^+)^2|0\rangle.$$

The Schrödinger equation leads to

$$\frac{d\beta_F}{dt} = -iE_F\beta_F \tag{2.30}$$

whose general solution is $\beta_F(t) = \beta_{F_0}e^{-iE_Ft}$.

Let us compare equation (2.30) with the system (2.27) and (2.28). Taking the limit $Z_3 \to 0$ in these last equations we get

$$\bar{\alpha} - \frac{2\lambda}{g_l}\beta = 0,$$

$$\frac{d\beta}{dt} = - ig_l\bar{\alpha} - 2im\beta.$$

Elimination of $\bar{\alpha}$ gives

$$\frac{d\beta}{dt} = - 2i(m - \lambda)\beta = - iE_F$$

which is just equation (2.30). I. e., in the limit $Z_3 \to 0$ the differential equations for $\beta(t)$ are the same in both models, but, in spite of this fact, the two cases are not mathematically equivalent because in the first one $\bar{\alpha}_0$ can be chosen arbitrarily (the solutions of a system of two differential equations of the first order being determined by two initial conditions), while in the second $\bar{\alpha}(t)$ is necessarily given in terms of $\beta(t)$ for any value of t. This problem has been carefully examined by Sekine [5] who pointed out the singular character of the limit $Z_3 \to 0$.

Let us now consider the state vector $|\Phi_1(t)\rangle$ which at $t = 0$ coincides with the Fermi state vector

$$|\Phi_F(t)\rangle = - (1/\sqrt{2})e^{-iE_F t}(a^+)^2|0\rangle,$$

i. e. let us choose

$$\bar{\alpha}_0 = 0 \qquad \text{and} \qquad \beta_0 = - 1/\sqrt{2}.$$

We now get

$$\lim_{Z_3 \to 0} |\Phi_1(t)\rangle = |\Phi_F(t)\rangle = j^+(-t)|0\rangle$$

for any value of t. More generally, if we take as our initial conditions for the Yukawa state vector

$$\bar{\alpha}_0 = 0, \qquad \beta_0 \neq 0 \qquad \text{and} \qquad \lim_{Z_3 \to 0} \beta_0 = \beta_{F_0},$$

then it follows that the vector $|\psi(t)\rangle$ of the Yukawa theory so constructed tends in the limit to $|\psi_F(t)\rangle$.

Therefore, it appears from this analysis that the actual source of the difference between the two models in the present case is related to the different initial conditions imposed on the state vectors $|\Phi(t)\rangle$ and $|\Phi_F(t)\rangle$

which, in their turn, come from the difference between the Hilbert spaces (which are two and one dimensional in the Yukawa model and in the Fermi model, respectively). Let us remark that the difference between these two vectors directly accounts for the $1/\delta\nu_l$ term in the limit propagator. This is easily verified by showing that $S'(t)$ and $S_F(t)$ can be written as $S'(t) = \theta(t) \langle \Phi(0) | \Phi(t) \rangle$ and $S_F(t) = \theta(t) \langle \Phi_F(0) | \Phi_F(t) \rangle$.

We finally conclude from our study that both models are not strictly equivalent. This fact manifests itself in results such as the difference between $j^+(t)|0\rangle$ and $\lim_{Z_3 \to 0} \varphi^+(t)|0\rangle$, the difference between the limit spectrum of H_y and that of H_F (the eigenvector of H_y at infinity can give finite contributions to matrix elements), or the $1/\delta\nu_l$ term in the limit propagator. All these effects are interrelated, as we have shown, and reflect the non strict equivalence.

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RÉFÉRENCES

[1] For an up to date survey including recent developments of the theory and a list of references, see Jouvet's lectures (« The field theoretical approach to composite particles ») in *The ninth Latin American School of Physics*, edited by I. Saavedra, Benjamin, New York, 1968.

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