ANNALES DE L'I. H. P., SECTION A

NGUYEN VAN HIEU
NGUYEN NGOC THUAN
V. A. SULEYMANOV

On the shrinkage of the forward and backward diffraction peaks

On the shrinkage
of the forward and backward diffraction peaks

par

NGUYEN VAN HIEU, NGUYEN NGOC THUAN
and V. A. SULEYMANOV

SUMMARY. — Various upper bounds for the ratios of the differential to the total cross sections of the elastic and inelastic processes are established.

1. INTRODUCTION

It was shown in a series of papers [1] [4] that the general requirements of analyticity and unitary lead to some bounds on the asymptotic behaviour of cross sections of elastic and inelastic processes. Defining the width of the diffraction peak by the relation

$$W = \frac{1}{\dfrac{d}{dt}\left[\ln\dfrac{d\sigma}{dt}\right]_{t=0}}$$

Finn [5], Kowalski [6], Kinoshita [7] and Bessis [8] have shown that W cannot decrease arbitrarily as s increases. Other characteristics of the diffraction cone for two-body (elastic and inelastic) processes of the type

$$a + b \rightarrow c + d \tag{I}$$

are the ratios

$$\Delta_{\mathrm{I}} = \frac{1}{\sigma_{\mathrm{I}}} \frac{d\sigma_{\mathrm{I}}}{dt}\bigg|_{t=t_0} = \frac{1}{\sigma_{\mathrm{I}}} \frac{1}{2kk'} \frac{d\sigma_{\mathrm{I}}}{d\cos\theta}\bigg|_{\theta=0} \qquad (1)$$

$$\delta_{\mathrm{I}}(t) = \frac{1}{\sigma_{\mathrm{I}}} \frac{d\sigma_{\mathrm{I}}}{dt}, \qquad \delta'_{\mathrm{I}}(\theta) = \frac{1}{\sigma_{\mathrm{I}}} \frac{d\sigma_{\mathrm{I}}}{d\cos\theta};$$

where k and k' are the 3-dimensional momenta of the initial and final particles in the c. m. s. and t_0 is the momentum transfer at vanishing angle and $\Delta_{\mathrm{I}} = \delta_{\mathrm{I}}(t_0)$. At large s we have $2kk' = \frac{s}{2}$. For multiple production processes

$$a + b \rightarrow c + \ldots \qquad (\mathrm{II})$$

it is also possible to introduce quantities

$$\Delta_{\mathrm{II}} = \frac{1}{\sigma_{\mathrm{II}}} \frac{2}{s} \frac{d\sigma_{\mathrm{II}}}{d\cos\theta}, \qquad \delta'_{\mathrm{II}}(\theta) = \frac{1}{\sigma_{\mathrm{II}}} \frac{d\sigma_{\mathrm{II}}}{d\cos\theta}, \qquad (2)$$

where $\dfrac{d\sigma_{\mathrm{II}}}{d\cos\theta}$ is the cross section of the production of a particle « c » in the given angle θ, integrated over all the other variables. In the papers [4] [9] it was shown that at $s \rightarrow \infty$

$$\Delta_{\mathrm{I,II}} \leqslant \mathrm{const}\ \ln^2 s, \qquad \delta'_{\mathrm{I,II}}(\theta) \leqslant \mathrm{const}\ \frac{\sqrt{s}\ \ln s}{\sin\theta},$$

$$\delta_{\mathrm{I}}(t) \leqslant \mathrm{const}\ \frac{\ln s}{|t|^{1/2}}. \qquad (3)$$

In this paper we generalize these results and find the explicit expressions for the constants which were not determined in ref. [4] [9].

The experimental check of the relation of the form (3) is difficult, because the differential cross section at the vanishing angle must be measured for this purpose. In view of these difficulties we modify the relations of the type (3) and introduce inequalities containing only differential cross sections in some interval of angles.

For the backward scattering it is also possible to derive inequalities similar to that for $\Delta_{\mathrm{I,II}}$. However it is known from experimental data that the main contribution to the total cross sections $\sigma_{\mathrm{I,II}}$ comes from the interval of small angles, whereas cross sections at $\theta \sim 180°$ decrease quickly. Therefore the upper bound seems to be too high. We intend to show that even stronger bounds exist: in the denominator of the l. h. s. of the last formula σ_{I} and σ_{II} can be replaced by the cross section integrated over the backward hemisphere ($-1 \leqslant \cos\theta \leqslant 0$).

For simplicity we consider only the spinless particles.

2. ANALYTIC PROPERTIES
OF THE ELASTIC SCATTERING AMPLITUDE
AND THE NUMBER OF SUBTRACTIONS

The obtained inequalities are consequences of the analyticity of the elastic scattering amplitude (process I) $F(s, t) \equiv f(s, z)$ in the momentum trans_ fer t (or in $z = \cos \theta$). It follows, as is well know, from the general principles of the quantum field theory, that the function $f(s, z)$ for a number of processes is analytic in the topological product of the s plane with real cuts (and poles) and the circle $|t| \leqslant \gamma$ (see [10-12]). We assume the distributions in the local field theory to be linear functionals on the space of infinitely differentiable rapidly decreasing functions, i. e. tempered distributions [13, 14]. Then for all values of t in the circle $|t| \leqslant \gamma$ the amplitude $F(s, t)$ is polinomialy bounded and satisfies a dispersion relation in s with a finite number of substractions:

$$F(s, t) = \sum_{n=0}^{N} C_n(t) s^s + \frac{s^{N+1}}{\pi} \int_{-\infty}^{\infty} \frac{\text{Im } F(s', t)}{s'^{N+1}(s' - s)} ds', \qquad |t| \leqslant \gamma. \qquad (4)$$

Let us decompose the amplitude $F(s, t) \equiv f(s, z)$ in partial waves

$$f(s, z) = 8\pi \frac{\sqrt{s}}{k} \sum_{l=0}^{\infty} (2l + 1) a_l(s) p_l(z), \qquad (5)$$

where k is the three-dimensional momentum of particles in c. m. s. We put $z_0 = 1 + \frac{\gamma}{2k^2}$ and denote by E_{z_0} the ellipse with foci at $z = \pm 1$ and with the major semiaxis z_0. Then for all values of z in E_{z_0} and on its boundary the inequality $|p_l(z)| \leqslant p_l(z_0)$ holds. On the other hand, due to the unitarity condition

$$\text{Im } a_l(s) \geqslant |a_l(s)|^2 \geqslant 0. \qquad (6)$$

Since the function $f(s, z)$ is analytic in z up to the point $z = z_0$ the series for the imaginary part

$$\text{Im } f(s, z) = 8\pi \frac{\sqrt{s}}{k} \sum_{l=0}^{\infty} (2l + 1) \text{ Im } a_l(s) p_l(z) \qquad (7)$$

converges at $z = z_0$. Therefore this series must be convergent absolutely and uniformly in the ellipse E_{z_0} and on its boundary and it determines a function analytical in t in this closed ellipse and bounded uniformly and polynomial in s.

Following the method of Greenberg and Low [2] and using the condition (3) we can (on the basis of these properties of analyticity and polynomial boundedness) obtain the Froissart bounds

$$| F(s, 0) | \leqslant \text{const } ln^2 s \tag{8}$$

$$| F(s, t) |_{t < 0} \leqslant \text{const } s^{3/4} (lns)^{3/2}. \tag{9}$$

In the paper of Jin and Martin [15] the following statement was proved:

If $F(s, t)$ satisfies a dispersion relation with two substractions in s for those values of t that are in a circle $| t | \leqslant \alpha$, with a rather small but finite radius $\alpha(\alpha < \gamma)$, then it follows from (4) and the unitarity condition that it satisfies a dispersion relation with two substractions for all values of t from the circle $| t | \leqslant \gamma$.

We show now, that in fact $F(s, t)$ satisfies a dispersion relation with two substractions for all values of t in some circle $| t | \leqslant \alpha$.

By means of a conformal mapping

$$\xi = z + \sqrt{z^2 - 1}$$

we transform the ellipse E_{z_0} into a ring with the centre at $\xi = 0$ and with the internal radius 1 and the external one R

$$R = z_0 + \sqrt{z_0^2 - 1}$$

and put $g(\xi, s) \equiv \text{Im} f(s, z)$. We denote by m and by M the values of $| g(\xi, s) |$ on the circle $| \xi | = 1$ and $| \xi | = R$ respectively. Since $| p_l(z) | \leqslant p_l(1) = 1$ in the interval $- 1 \leqslant z \leqslant 1$ and $\text{Im } a_l(s) \geqslant 0$, $| \text{Im} f(s, z) |$ for $- 1 \leqslant z \leqslant 1$ is always smaller than $\text{Im} f(s, 1)$. Also, it follows from the inequality (8) that

$$m \leqslant \text{const } s^{1 + \delta}$$

for any value of $\delta > 0$ small enough. On the other hand, because of the polynomial boundedness,

$$M \leqslant \text{const } s^N.$$

Now using Hadamard's theorem on three circles [17] we obtain for any value of z from the interval $1 \leqslant z \leqslant z_0$

$$\ln | \operatorname{Im} f(s, z) | \leqslant \left(1 - \frac{\ln r}{\ln R}\right) \ln m + \frac{\ln r}{\ln R} \ln M$$

$$\leqslant \left[\left(1 - \frac{\ln r}{\ln R}\right)(1 + \delta) + \frac{\ln r}{\ln R} N\right] \ln s, \qquad r = z + \sqrt{z^2 - 1}. \quad (12)$$

First we consider the case when $N \geqslant 2$. It can easily be seen on the basis of the relation (12) that

$$| \operatorname{Im} f(s, z) | \leqslant \operatorname{const} s^{2-\varepsilon} \qquad (13)$$

for $\varepsilon > 0$ if

$$\frac{\ln r}{\ln R} < \frac{1 - \delta - 2}{N - 1 - \delta}.$$

This condition is satisfied, in particular, when $1 \leqslant r \leqslant r_0$

$$\frac{\ln r_0}{\ln R} < \frac{1}{2N}.$$

Thus for all z from the interval

$$1 \leqslant z \leqslant 1 + \frac{\beta}{2k^2}, \qquad \beta = \frac{\gamma}{4N^2}$$

the inequality (13) holds. Because of the unitary condition and the properties of the Legendre polynomials this inequality holds also for any z in the ellipse Ez_1 with foci at $z = \pm 1$ and with the major semiaxis $z_1 = 1 + \beta/2k^2$ in particular for all z from the circle $| z - 1 | \leqslant \beta/2k^2$. Thus we have for any $| t | \leqslant \beta$ a dispersion relation

$$F(s, t) = \sum_{n=0}^{N} d_n(t)s^n + \frac{s^2}{\pi} \int \frac{\operatorname{Im} F(s', t)}{s'^2(s' - s)} ds' \qquad (14)$$

where the $d_n(t)$ are analytic in this circle. The relation (9) shows that $d_n(t) \equiv 0$ where $n \geqslant 2$ for all t from some interval $t_2 \leqslant t \leqslant t_1 < 0$. They must vanish indentically for any t in the circle $| t | \leqslant \beta$. It follows from these results and from the Jin and Martin theorem [15] that a dispersion relation in s with two subtractions holds true for all $| t | \leqslant \gamma$.

If, however, the constant N in (11) is smaller than 2, the later statement

follows immediatly from the expression (14) for any $|t| \leqslant \gamma$. Thus the integral

$$\int \frac{\text{Im } F(s, t)}{s'^2(s' - 1)} ds' \tag{15}$$

converges absolutely for all values of t in the circle $|t| \leqslant \gamma$ and, hence, for all t in the ellipse $E_\gamma^{(t)}$ with foci at $t = 0$ and $t = -4k^2$ and with the major semiaxis $2k^2 + \gamma$. In particular

$$|\text{Im } F(s, t)| \leqslant \text{const } s^2 \tag{16}$$

for all t from $E_\gamma^{(t)}$.

In the following we shall assume also that $F(s, t)$ satisfies a dispersion relation in s with a finite number of subtractions for all t from the ellipse $E_\gamma^{(t)}$. Then, due to the absolute convergence of the integral (15), we can write a dispersion relation (14) for all t in this ellipse. On the basis of (9) we conclude that a dispersion relation in s with two subtractions holds then for all t from the ellipse $E_\gamma^{(t)}$ if $F(s, t)$ is analytical in $E_\gamma^{(t)}$. In particular,

$$|F(s, t)| \leqslant \text{const } s^2 \tag{17}$$

for all t from $E_\gamma^{(t)}$.

3. BEHAVIOUR OF DIFFRACTION PEAKS OF THE ELASTIC AND INELASTIC PROCESSES

Following Greenberg and Low we apply now, to the function $\text{Im} f(s, z)$ analytic in the ellipse E_{z_0} the Cauchy formula

$$\text{Im } f(s, z) = \frac{1}{2\pi i} \oint_{\partial E_{z_0}} \frac{\text{Im } f(s, z')}{z' - z} dz', \tag{18}$$

where ∂E_{z_0} denotes the boundary of the ellipse E_{z_0}. Hence, using the familiar formula

$$\frac{1}{z' - z} = \sum_{l=0}^{\infty} (2l + 1) P_l(z) Q_l(z'), \tag{19}$$

we obtain

$$\text{Im } a_l(s) = \frac{k}{16i\pi^2 \sqrt{s}} \oint_{\partial E_{z_0}} \text{Im } f(s, z') Q_l(z') dz'. \tag{20}$$

Since on the contour ∂E_{z_0}

$$|Q_l(z')| \leqslant \left(\frac{\pi}{l}\right)^{1/2} \frac{\left(z_0 + \sqrt{z_0^2 - 1}\right)^{-(l+1)}}{\left(1 - \frac{1}{\left(z_0 + \sqrt{z_0^2 - 1}\right)^2}\right)^{1/2}}$$

the inequality

$$\operatorname{Im} a_l(s) \leqslant \frac{R(s)}{l^{1/2}} \left[1 + 2\sqrt{\frac{\gamma}{s}}\right]^{-l} \qquad R(s) = \text{const. } s^{9/4} \qquad (21)$$

follows from the formula (20).

We have, due to the unitarity condition

$$|a_l(s)| \leqslant \text{const} \frac{s^{9/8}}{l^{1/4}} \left[1 + 2\sqrt{\frac{\gamma}{s}}\right]^{-1/2}. \qquad (22)$$

We denote by L that value of l for which $R(s)\left[1 + 2\sqrt{\frac{\gamma}{s}}\right]^{-l}$ equals unity

$$L = \frac{\ln R(s)}{\ln\left[1 + 2\sqrt{\frac{\gamma}{s}}\right]} \approx \frac{1}{2}\sqrt{\frac{s}{\gamma}} \ln R(s), \qquad s \to \infty, \qquad (23)$$

and consider the series (5) for $z = 1$. Further, we decompose it into two parts

$$\left[\sum_{l=0}^{(1+v)L-1} + \sum_{l=(1+v)L}^{\infty}\right][(2l+1)a_l(s)]$$

where v is a suitable positive number for which $(1 + v)L$ is an integer. The following estimate for the second sum can easily be derived

$$\left|\sum_{l=(1+v)L}^{\infty}(2l+1)a_l(s)\right| \leqslant \frac{1}{R^{v/2}L^{1/4}}\left[2(1+v)L\sqrt{\frac{s}{\gamma}} + \frac{2s}{\gamma}\right]. \qquad (24)$$

We use the Schwartz inequality to get a bound for the first sum. We have thus

$$\left|\sum_{l=0}^{(1+v)L-1}(2l+1)a_l(s)\right|^2 \leqslant (1+v)^2L^2\sum_{l=0}^{\infty}(2l+1)|a_l(s)|^2 = (1+v)^2L^2\frac{s}{16\pi}\sigma_{\text{el}},$$

$$(25)$$

where σ_{el} is the total cross section for the elastic scattering. It is to be remembered that the differential cross section is

$$\frac{d\sigma_{\text{el}}}{dt} = \frac{1}{64\pi sk^2} |F(s,t)|^2.$$

We choose v such that the inequality

$$\sigma_{\text{el}}^{1/2} \gg \frac{1}{R^{1/2}L^{1/4}} \tag{26}$$

holds. Then the second sum can be neglected as compared to the first one. We assume $\sigma_{\text{el}} > \text{const } s^{-\rho}$, $\rho > 0$. In this cas it follows from (26) that

$$s^{-\rho} \ll s^{-\frac{9}{4}v - \frac{1}{4}}.$$

Hence we obtain a restriction on v:

$$v > \frac{4}{9}\rho - \frac{1}{9}.$$

Chosing

$$v = \frac{4}{9}(\rho + \varepsilon) - \frac{1}{9}$$

where ε is a positive number small enough, we obtain

$$\frac{1}{\sigma_{\text{el}}}\frac{d\sigma_{\text{el}}}{dt}\bigg|_{t=0} \leqslant \left[1 + \frac{\rho + \varepsilon}{2}\right]^2 \frac{1}{\gamma}\ln^2\left(\frac{s}{s_0}\right)\left[1 + 0\left(\frac{s_0^\varepsilon}{s^\varepsilon}\right)\right] \tag{27}$$

A relation of this type containing an unknown constant was first derived in paper [9].

We assume that the total cross section tends to the constant for $s \to \infty$ i. e. $\rho = 0$. Then

$$\frac{1}{\sigma_{\text{el}}}\frac{d\sigma_{\text{el}}}{dt}\bigg|_{t=0} \leqslant \frac{1}{\gamma}\ln^2\left(\frac{s}{s_0}\right).$$

In the case of the scattering on a nonvanishing angle, using the inequality for the Legendre polynomials

$$|p_l(\cos\theta)| \leqslant \frac{1}{\sqrt{\pi_l \sin\theta}} \qquad \theta \neq 0, \pi,$$

the following relations can be derived

$$\frac{1}{\sigma_{\text{el}}}\frac{d\sigma_{\text{el}}}{d\cos\theta}\bigg|_{\theta \neq 0, \pi} \leqslant \frac{s^{1/2}\ln(s/s_0)}{\pi\sin\theta\sqrt{\gamma}}\left[1 + \frac{\rho + \varepsilon}{2}\right] \tag{28}$$

$$\frac{1}{\sigma_{el}} \frac{d\sigma_{el}}{dt}\bigg|_{t \neq 0} \leqslant \frac{\ln s/s_0}{\pi \sqrt{\gamma |t|}} \left[1 + \frac{\rho + \varepsilon}{2} \right]. \tag{29}$$

We consider now the process

$$a + b \rightarrow c + d. \tag{II}$$

Decompose the amplitude $T(s, z)$ into partial waves

$$T(s, z) = 8\pi \sqrt{\frac{s}{kk'}} \sum_{l=0}^{\infty} (2l + s) b_l(s) p_l(z), \tag{30}$$

where k and k' are three-dimensional momenta of the initial and final particles.

The unitary condition reads

$$\text{Im } a_l(s) = |a_l(s)|^2 + |b_l(s)|^2 + \dots \tag{31}$$

From (31) and (21) we obtain

$$|b_l(s)| < \sqrt{\text{Im } a_l(s)} < \frac{R^{1/2}}{L^{1/4}} \left[1 + 2\sqrt{\frac{\gamma}{s}} \right]^{-1/2}. \tag{32}$$

By repeating the calculations used for the elastic processes we get

$$\frac{1}{\sigma_{inel}} \frac{d\sigma_{inel}}{dt}\bigg|_{t=0} \leqslant \frac{1}{\gamma} \left[1 + \frac{\rho' + \varepsilon}{2} \right]^2 \ln^2 (s/s_0) \tag{33}$$

$$\frac{1}{\sigma_{inel}} \frac{d\sigma_{inel}}{dt}\bigg|_{t \neq 0} \leqslant \frac{\ln (s/s_0)}{\pi \sqrt{\gamma |t|}} \left[1 + \frac{\rho' + \varepsilon}{2} \right] \tag{34}$$

$$\frac{1}{\sigma_{inel}} \frac{d\sigma_{inel}}{d \cos \theta}\bigg|_{\theta \neq 0, \pi} \leqslant \frac{s^{1/2} \ln (s/s_0)}{\pi \sin \theta \sqrt{\gamma}} \left[1 + \frac{\rho' + \varepsilon}{2} \right]. \tag{35}$$

Here ρ' is a constant, such that for $s \rightarrow \infty$ $\sigma_{inel} > \text{const } s^{-\rho'}$.

Finally we consider a process of multiple production

$$a + b \rightarrow c + A, \tag{III}$$

where A denotes all possible systems of hadrons. It was shown in ref. [4] that the total cross section of the inelastic processes of the type (III) with the production of a partical « c » on the give angle θ is of the form

$$\frac{d\sigma^c}{d \cos \theta} = \frac{2\pi}{k^2} \sum_{ll'} (2l + 1)(2l' + 1) p_l(\cos \theta) p_{l'}(\cos \theta) c_{ll'}, \tag{36}$$

where, due to the unitary condition, the coefficients $c_{ll'}$ are related to the imaginary parts of the partial amplitudes of the amplitudes of the elastic scattering (I) by the inequality

$$|c_{ll'}| \leqslant \sqrt{\operatorname{Im} a_l(s) \operatorname{Im} a_{l'}(s)}. \tag{37}$$

Substituting (21) into (37) we get

$$|c_{ll'}| \leqslant \text{const. } s^{9/4} \left[1 + 2\sqrt{\frac{\gamma}{s}} \right]^{-\frac{l+l'}{2}}. \tag{38}$$

Using again the beforementioned arguments we get on the basis of (38) and (37) the bounds

$$\frac{1}{\sigma^c} \frac{d\sigma^c}{d\cos\theta}\bigg|_{\theta=0} \leqslant \frac{s}{2}\frac{1}{\gamma} \left[1 + \frac{\rho'' + \varepsilon}{2} \right]^2 \ln^2 (s/s_0) \tag{39}$$

$$\frac{1}{\sigma^c} \frac{d\sigma^c}{d\cos\theta}\bigg|_{\theta\neq0,\pi} \leqslant \frac{s^{1/2} \ln (s/s_0)}{\pi \sin\theta\sqrt{\gamma}} \left[1 + \frac{\rho'' + \varepsilon}{2} \right], \tag{40}$$

where ρ'' is a constant, such that $\sigma^c > \text{const } s^{-\rho''}$ for $s \to \infty$.

4. GENERALIZATIONS

Formula (27) holds also for the backward scattering (at the angle $\theta = 180°$). Since the main contribution—according to the experimental data—comes from an interval of angles close to $\theta = 0°$, this formula practically is of no interest for the backward scattering. To study the character of the backward diffraction peak we introduce the notion of the total cross section on the backward hemisphere $\sigma_{\text{back}}(s)$

$$\sigma_{\text{back}}(s) = \int\limits_{-1}^{0} \frac{d\sigma}{dz} dz \tag{41}$$

instead of the total cross section of the elastic scattering. We show now that the quantity $\dfrac{1}{\sigma_{\text{back}}} \dfrac{d\sigma}{dt}\bigg/ t = -4k^2$ also satisfies the inequality of the type (27).

We assume, instead of the rigorously proved analytic properties, the $F(s, t)$ is analytic in the topological product of the s-plane with the cuts

and the ellipse $E_\gamma^{(t)}$. Then, as it has been shown, in sec. 2, the inequality (17) holds.

We denote, by E' the ellipse with focci at $z = -1$ and $z = 0$ and with the major semiaxis $z_0' = \frac{1}{2} + \frac{2\gamma}{s}$, $s \to \infty$. It can be shown that this ellipse is contained in the Martin ellipse E_{z_0}. Therefore $f(s, z)$ is an analytic function in E'.

Introducing the new variable

$$W = 2\left(z + \frac{1}{2}\right)$$

we map E' into the Martin ellipse in the W plane with foci at $W = \pm 1$ and the major semiaxes $W_0 = 1 + \frac{4\gamma}{s}$, $s \to \infty$. We put $f(s, z) \equiv g(s, w)$ and decompose $g(s, w)$ into Legendre polynomials

$$g(s, w) = \frac{8\pi\sqrt{s}}{h} \sum (2l + 1)a_l'(s)p_l(W) \tag{42}$$

It follows from the analyticity of $g(s, w)$ and the condition (17) that

$$|a_l'(s)| \leqslant \frac{R'(s)}{L^{1/2}}\left[1 + 2\sqrt{\frac{2\gamma}{s}}\right]^{-l}, \tag{43}$$

where

$$R'(s) = \text{const } s^{9/4}. \tag{44}$$

Repeating now the arguments used in section 3 we get

$$\frac{1}{\sigma_{\text{back}}} \frac{d\sigma}{dt}\bigg|_{t=-4k^2} \leqslant \frac{1}{\gamma}\left[1 + \frac{\tau + \varepsilon}{4}\right]^2 \ln^2 (s/s_0)\left[1 + 0\left(\frac{s_0^2}{s^\varepsilon}\right)\right]. \tag{45}$$

Here τ is a positive constant, such that for $s \to \infty$

$$\sigma_{\text{back}}(s) \geqslant \text{const } s^{-\tau}.$$

The experimental determination of the differential cross section at $\theta = 0^\circ$ and $\theta = 180^\circ$ is very difficult. To facilate the task we slightly modify the obtained relations (27) and (45). Again, like at the beginning of this section, we assume analyticity of $F(s, t)$ in the topological product of the s-plane with cuts and the ellipse $E_\gamma^{(t)}$. For definiteness we consider the scattering at a small angle. We denote by t_1 and t_2 some fixed negative values of t, $t_2 < t_1 < 0$. It can easily be shown that the Martin

ellipse contains the ellipse E'' with foci at t_1, t_2 and major semiaxis

$$t_0 + \frac{|t_2 - t_1|}{2}$$

By means of the mapping

$$u = \frac{2t}{t_1 - t_2} - \frac{t_1 + t_2}{t_1 - t_2}, \qquad t_1 \neq t_2,$$

we transform the ellipse E'' into the ellipse E_{u_0} in the u plane with the foc at $u = \pm 1$ and the major semiaxis $u_0 = \text{const} > 1$. Using the method presented above we can derive the following inequality

$$\frac{1}{\Delta_t \sigma} \frac{d\sigma}{dt}\bigg|_{t = t_1} \leqslant \text{const } \ln^2 (s/s_0), \tag{46}$$

where

$$\Delta_t \sigma = \int\limits_{t_1}^{t_2} \frac{d\sigma}{dt} \, dt.$$

The relevant inequality for the backward scattering can be established in a similar way

$$\frac{1}{\Delta_t \sigma_{\text{back}}} \frac{d\sigma}{dt}\bigg|_{t = -4k^2 + |n_1|} \leqslant \text{const } \ln^2 (s/s_0), \tag{47}$$

where

$$\Delta_t \sigma_{\text{back}} = \int\limits_{t = -4k^2 + |t_1|}^{t = -4k^2 + |t_2|} \frac{d\sigma}{dt} \, dt.$$

REFERENCES

[1] M. FROISSART, *Phys. Rev.*, t. **123**, 1961, p. 1053.

[2] O. W. GREENBERG and F. LOW, *Phys. Rev.*, t. **124**, 1961, p. 2047.

[3] A. MARTIN, *Phys. Rev.*, t. **129**, 1964, p. 1432.

[4] A. A. LOGUNOV, M. A. MESTVIRISHVILI, NGUYEN VAN HIEU, *Phys. Lett.*, t. **25** B, 1967, p. 611.

[5] A. C. FINN, *Phys. Rev.*, t. **132**, 1963, p. 2, 836.

[6] K. L. KOWALSKI, *Phys. Rev.*, t. **134**, 1965, B 1350.

[7] T. KINOSHITA, *Lectures in Theoretical Physics*, vol. **7**-B, 1965.

[8] T. D. BESSIS, *Nuovo Cim.*, t. **45**, 1966, p. 974.

[9] A. A. LOGUNOV, NGUYEN VAN HIEU, Report if the Topical Conference on High Energy Collision of Hadron, CERN, Geneva, 1968; Preprint JINR, 1968.

[10] A. MARTIN, *Nuovo Cim.*, t. 42 A, 1966, p. 930; t. 44 A, 1966, p. 1219.

[11] J. D. BESSIS and V. GLASSER, *Nuovo Cim.*, t. 52, 1967, p. 568.

[12] G. SOMMER, *Nuovo Cim.*, 52 A, 1967, p. 373.

[13] Н. Н. Боголюбов, Д. В. Ширков, Введение в теорию квантованных полей, гиттл, 1958.

[14] R. F. STREATER and A. S. WIGHTMAN, CPT, Spin, Statistic and all That, Benjamin, 1964.

[15] I. S. JIN and A. MARTIN, *Phys. Rev.*, t. 135, 1964, B 1375.

[16] F. CERULUS and A. MARTIN, *Phys. Lett.*, t. 8, 1964, p. 89.

[17] E. C. TICHTMARSH, *The theory of Functions*, Oxford University Press, 1939.