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Phenomenological Electrodynamics in Accelerated Systems of Reference (*)

by

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ABSTRACT. — An experimental and theoretical study is made of some aspects of the constitutive equations of phenomenological electrodynamics for a stationary metric by measuring the frequency difference between the resonant clockwise and counter-clockwise traveling wave modes of a closed path Fabry-Perot cavity. An effective index of refraction for an observer in an accelerated frame of reference is developed in detail and is used to discuss the experimental results. The frequency difference is proportional to the moment of the energy flux rather than the angular momentum of the traveling waves and supports the asymmetry of the Minkowski form of the electromagnetic part of the energy-momentum tensor.

INTRODUCTION

This paper is primarily concerned with an experiment for testing some aspects of phenomenological electrodynamics for bodies in accelerated frames of reference. Motivated by the interference experiments [1] of Michelson, Sagnac and Pogany, one of us [2] suggested that the traveling electromagnetic wave modes of a cavity resonator are degenerate and that

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this degeneracy could be removed by rotating the cavity. These initial considerations indicated that the separation in frequency is given by $\Delta\nu/\nu = 2\Omega \cdot \mathbf{J}/h\nu$ where Ω is the angular velocity of the cavity and \mathbf{J} the orbital angular momentum of the photon. The name « Coriolis-Zeeman Effect » for a photon seemed appropriate. In subsequent papers Heer [3], [4] developed a detailed theory for this frequency difference for a cavity in vacuum and for a cavity partly filled with matter, and it became apparent that the moment of the energy flux was the quantity of importance. The advent of optical masers made possible experimental studies [3], [5], [6] of this frequency difference. A schematic diagram of the experimental apparatus which was used for the investigations of this paper is shown in Fig. 1.

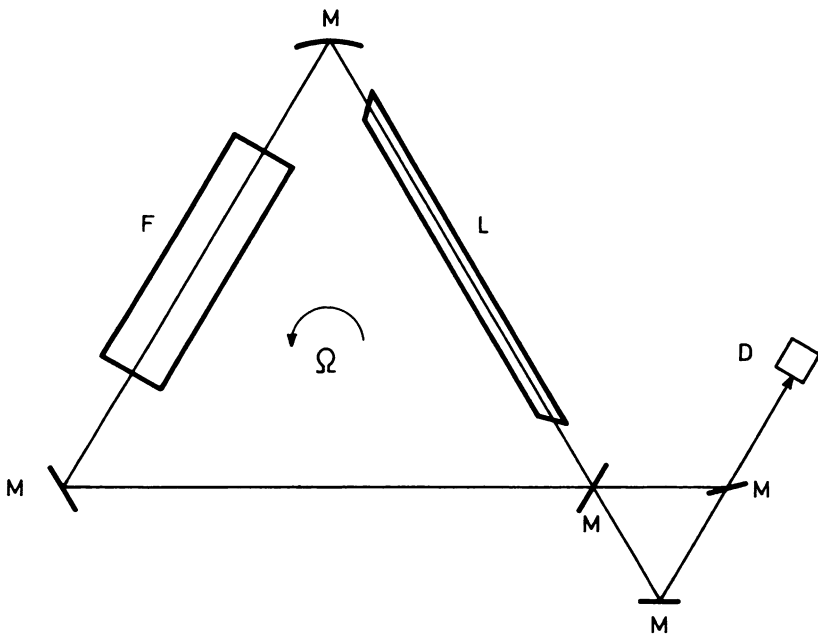


FIG. 1.

In order to emphasize the role of the moment of energy flux some aspects of these earlier considerations are discussed. If the structure or topology of an electromagnetic resonator is such that traveling wave modes are possible, then these modes are degenerate in the two directions of the traveling wave. This degeneracy can be removed by optically active media which give rise to Faraday rotation or by rotation of the cavity. If separation of space and time variables is used, then complex space modes

are required in the analysis. The effect of rotation can be included by formulating Maxwell's equations in a covariant manner. Since the source, observer, and boundaries are in the accelerated frame of reference, the resonant frequencies of the complex modes are determined for a time variable in the rotating system. If the cavity contains macroscopic media it is necessary to consider the constitutive equations for the media. Only constant K_e , K_m , and σ were considered by Heer in his analysis of the problem. More recently Post [7], Post and Yildiz [8], and Yildiz and Tang [9] have considered the fourth order tensor relating the fields $F_{\alpha\beta}$ and $H^{\alpha\beta}$.

Energy-Momentum Tensor

The earlier discussion in terms of the electromagnetic energy momentum tensor [4] is considered further. If the energy momentum tensor suggested by Minkowski as

$$S_{\alpha}^{\beta} = F_{\alpha\mu} H^{\beta\mu} - \frac{1}{4} \delta_{\alpha}^{\beta} F_{\mu\nu} H^{\mu\nu} \quad (1)$$

whose covariant divergence $S_{\alpha}^{\beta}{}_{;\beta}$ yields the four-force $f_{\alpha} = F_{\alpha\beta} j^{\beta}$ is used, the various components in noncovariant vectors are

$$S_t^x = -c^{-1} (\mathbf{E} \times \mathbf{H})_x = (\text{energy flux}) \quad (2a)$$

$$S_x^t = c (\mathbf{D} \times \mathbf{B})_x = (\text{momentum density}) \quad (2b)$$

$$S_t^t = -\frac{1}{2} (\mathbf{E} \cdot \mathbf{D} + \mathbf{B} \cdot \mathbf{H}) = (\text{energy density}) \quad (2c)$$

In an instantaneous rest frame these quantities are interpreted as energy flux, momentum and energy density, and retain this character in every other reference frame. The energy density S_T^T in an instantaneous rest frame and the energy density in the cavity frame are related by

$$S_T^T = S_t^t + (g_{ta}/g_{tt}) S_t^a \quad (3)$$

For a stationary metric the coefficients of the metric tensor do not depend on the local time variable t or x^4 and complex normal mode solutions [4] of Maxwell's equations may be obtained which are independent of t . Inherent in the form of the Minkowski electromagnetic energy-momentum tensor is the assumption that a transparent or nondissipative media does not even locally exchange energy with the electromagnetic field. It is

not clear that this is true in general for accelerated frames of reference, but it does follow for $f_i = 0$ that

$$\partial S_i^\beta / \partial x^\beta = 0 + \mathcal{O}(\Omega^2) \quad (4a)$$

For nondissipative walls S_i^a or $(\mathbf{E} \times \mathbf{H})$ vanishes at the walls and

$$\frac{\partial}{\partial t} \int dv S_i^t \cong 0 \quad (4b)$$

Thus the integral of the energy density over the volume of the cavity, $\int dv S_i^t$, is a conserved quantity. A system which permits traveling waves is degenerate in the sense of permitting \pm values of S_i^a or $(\mathbf{E} \times \mathbf{H})$. If the cavity contains media which is not magnetically optically active, then $\int dv S_{\mathbf{T}}^t$ is the same for both modes and the frequency difference between resonant photons in the two modes is given by

$$\begin{aligned} \Delta\nu/\nu &= \left[2 \int dv (g_{ta}/g_{tt}) S_i^a \right] \left[\int dv S_i^t \right]^{-1} \\ &= \left[2c^{-2} \boldsymbol{\Omega} \cdot \int dv [\mathbf{r} \times (\mathbf{E} \times \mathbf{H})] \right] \left[\frac{1}{2} \int dv (\mathbf{E} \cdot \mathbf{D} + \mathbf{B} \cdot \mathbf{H}) \right]^{-1} \end{aligned} \quad (5)$$

Since the numerator is of $\mathcal{O}(\Omega)$, the frequency separation can be obtained correct to first order in the angular velocity by using the values of \mathbf{E} , \mathbf{H} , \mathbf{B} , and \mathbf{D} in a system with $\Omega = 0$. Equation (5) emphasizes the importance of energy flux and indicates that the frequency separation depends on the moment of the energy flux of the photons.

THEORY

Although Eq. (5) gives a detailed description of the frequency difference, it is not in a convenient form for use with dispersive media or with media moving with respect to the rotating cavity. A plane wave expansion is more convenient and perhaps even necessary for this type problem. Such an expansion is given in the next section. In subsequent sections this plane wave expansion is extended to apply to an optical maser medium.

Plane Wave Expansion

In the short wavelength approximation in which the approximation that an electromagnetic wave may be discussed in terms of its amplitude and phase, i. e. $E \exp i\varphi$, the propagation of waves in a dispersive medium which is moving with respect to an observer in an accelerated frame of reference may be discussed in terms of three invariants. If these invariants are known in one frame they are known in any other frame of reference. The first invariant is the phase

$$d\varphi = k_\alpha dx^\alpha \tag{6}$$

Physical content is provided by the invariant $k_\alpha k^\alpha$ and transformations between reference frames by the invariant $k_\alpha U^\alpha$. In the following discussion the index α is used to denote the covariant and contravariant quantities in the general accelerated frame of reference \mathcal{A} of the observer, the index Φ is used to denote a reference frame I instantaneously at rest with respect to \mathcal{A} , and Φ' is used to denote a reference frame I' which is at rest with respect to the moving media.

In a reference frame at rest with respect to an isotropic dispersive medium physical content is provided by the invariant

$$k_\Phi k^{\Phi'} = k_T^2 [\mu^2(k_T) - 1] = k_\alpha k^\alpha \tag{7}$$

where μ is the index of refraction and $\omega' = ck_T$ expresses the frequency dependence. The four velocity has a single element $U^T = c$ and the transformation of frequencies is given by

$$k_\Phi U^{\Phi'} = k_T c = k_\Phi U^\Phi = k_\alpha U^\alpha \tag{8}$$

If the velocity of the moving medium with respect to the accelerated frame is given by $u^a = dx^a/dt$, then the space parts of the four velocity are given by $U^a = \Gamma u^a$ and the time part by $U^t = \Gamma c$. Γ is given on page 288 of Ref. [10]. $\Gamma = 1 + \mathcal{O}(2)$ where $\mathcal{O}(2)$ indicates quantities of order $(v/c)^2$.

For a stationary metric, that is a metric which does not depend on x^4 , the distance between space points is $d\sigma^2 = \gamma_{ab} dx^a dx^b$ and a frequency ω may be introduced. ω is defined by

$$\omega = ck_t = c(-g_{tt})^{1/2} k_T = (-g_{tt})^{1/2} \omega_0 \tag{9}$$

The spatial components of k_α are given by

$$k_a = [k_\alpha k^\alpha - k_t^2/g_{tt}]^{1/2} \gamma_{ab} n^b + (-g_{tt})^{-1} g_{ta} k_t \tag{10}$$

where $n^b = dx^b/d\sigma$ is the contravariant component in the path direction.

Equations (9) and (10) may be substituted into Eq. (6) to yield the phase. The total phase between two space time points is an invariant in the short wave length approximation and is stationary to a variation. Since

$$\delta \int dt = 0,$$

the classical path follows from

$$\delta \int \{ \mu_{\mathcal{A}} \} d\sigma = 0 \quad (11a)$$

where

$$\{ \mu_{\mathcal{A}} \} = (k_t)^{-1} k_a n^a \quad (11b)$$

A closed path is resonant if

$$c^{-1} v \oint \{ \mu_{\mathcal{A}} \} d\sigma = q \quad (12)$$

where q is a very large integer. These equations are rather awkward and in the approximation that terms of $\mathcal{O}(2)$ are neglected, the space is Euclidean. As shown in Appendix A the index of refraction appears to an observer on the rotating platform or reference system \mathcal{A} at, position \mathbf{r} as

$$\mu_{\mathcal{A}} = \mu(v) + c^{-1}(\boldsymbol{\Omega} \times \mathbf{r}) \cdot \hat{\mathbf{n}} + c^{-1}[(\mu^2 - 1) + \mu v d\mu/dv] \mathbf{u} \cdot \hat{\mathbf{n}} \quad (13)$$

\mathbf{u} is the velocity of the moving dispersive medium measured by an observer attached to the accelerated frame \mathcal{A} . $\hat{\mathbf{n}}$ is a unit vector in the direction of the ray and may be determined from Eq. (11a) which is Fermat's principle. The phase is computed along the path. For a closed path the line integral transforms to a surface integral and the resonant frequency follows from Eq. (12). If v_0 is evaluated at $\boldsymbol{\Omega} = 0$, the frequency separation between clockwise and counterclockwise traveling waves is given by

$$\Delta v/v_0 = 2c^{-1} \left\{ 2 \oint \boldsymbol{\Omega} \cdot d\mathbf{S} + \oint [(\mu^2 - 1) + \mu v d\mu/dv] \mathbf{u} \cdot \hat{\mathbf{n}} d\sigma \right\} \left\{ \int [\mu(v_0) + v_0 d\mu/dv] d\sigma \right\}^{-1} \quad (14)$$

This is in agreement with Eq. (5) for an almost plane wave expansion of a beam of finite cross-section and with the basic Eqs. (22), (32) and (41) of the earlier paper [4] for non-dispersive media. For the particular geometry of the cavity selected by Khromykh [11], the results obtained by him differ

from Eq. (14) in only the μ^2 term. This seems to be a misprint in his paper. For completely filled cavities and media attached to the cavity, this is in agreement with other recent papers [7] [8] [9]. Khromykh has recently criticized the application of these formula to a special case. This criticism is quite correct and Eq. (26) of Ref. [4] should read

$$\Delta v/v = (\Omega D/c)[1 - d/4D(1 - \mu)]^{-1}.$$

Effect of Optical Maser Media

If an almost plane wave approximation is valid for the description of the electromagnetic waves in the cavity, the electric field is given to a good approximation by

$$\mathbf{E}(\mathbf{r}, t) = \sum_{\alpha q} \hat{a}_{\alpha q} \{ \mathbf{A}_{\alpha q} \exp i(\varphi_{\alpha q A} - \omega_{\alpha q A} t) + \mathbf{B}_{\alpha q} \exp i(\varphi_{\alpha q B} + \omega_{\alpha q B} t) \} + c. c. \quad (15a)$$

where

$$\varphi_{\alpha q A} = c^{-1} \omega_{\alpha q A} \int_0^\sigma \mu_A d\sigma \quad (15b)$$

σ is along the optical path and $\omega_{\alpha q A}$ is given by Eq. (12) for the resonance condition for a closed path. $A_{\alpha q}$ is the coefficient for waves traveling in the $+\sigma$ direction and is a slowly varying function of time or of σ . The index α denotes the two canonical states of polarization of the photons. $B_{\alpha q}$ is the amplitude of waves traveling in the $-\sigma$ direction. The transverse dependence of A and B confines the beam to a finite cross-section. Proper cavity modes [12] of the type introduced by Fox and Li and extended to cavities with traveling waves by Collins and by Clark could be used, but such modes would make the analysis considerably difficult. These proper modes have the general character of Eq. (15a), and this equation is adequate for the subsequent analysis.

μ_A is the index of refraction of the wave traveling in the $+\sigma$ direction and is given by Eq. (13) for media for which the dispersive index of refraction $\mu(v)$ is known. The maser medium is more complex and a nonlinear electric polarization \mathbf{P} occurs. This source term compensates for cavity losses and introduces a highly dispersive power dependent term into the dielectric constant or index of refraction. Lamb [13] has given a method for treating the problem of a gaseous maser. Heer and Graft [14] have extended this method to obtain the electric polarization \mathbf{P} for atoms with

angular momentum which are interacting with traveling electromagnetic waves of the kind given by Eq. (15a) and with a static magnetic field. \mathbf{P} is related to \mathbf{E} and \mathbf{E}^3 by second and fourth order tensors [14].

For single mode operation with Brewster angle windows and with a transverse magnetic field the electric vector is linearly polarized throughout the maser medium. The tensor relationships become much simpler and the polarization and the electric field are related by [14]

$$-\varepsilon_0^{-1}\mathbf{P}(\omega_A) = \mathbf{A}[a(\omega_A) - b(\omega_A)|\mathbf{A}|^2 - c(\omega_A, \omega_B)|\mathbf{B}|^2] \quad (16a)$$

$$-\varepsilon_0^{-1}\mathbf{P}(-\omega_B) = \mathbf{B}[a^*(\omega_B) - b^*(\omega_B)|\mathbf{B}|^2 - c^*(\omega_B, \omega_A)|\mathbf{A}|^2] \quad (16b)$$

Since the coefficients are small an index of refraction can be introduced by the use of the relationship

$$\varepsilon_0^{-1}\mathbf{P} = (\mu^2 - 1)\mathbf{E} \quad \text{or} \quad \mu = 1 + \frac{1}{2}\varepsilon_0^{-1}\mathbf{P}/\mathbf{E}.$$

Thus the approximate index of refraction is

$$\mu(\omega_A) = \frac{1}{2}[-a'(\omega_A) + b'(\omega_A)|\mathbf{A}|^2 + c'(\omega_A, \omega_B)|\mathbf{B}|^2] \quad (17)$$

where the coefficients are written as $a = a' + ia''$, etc. A similar expression follows from Eq. (16b) for $\mu(\omega_B)$. The general form of $a(\omega)$ is given by Eq. (28) of Ref. [14]. $a(\omega)$ may be related to experimental data by noting that the linear gain per meter is $2\pi\lambda^{-1}a''$ and that

$$a(\omega) = \text{const } Z(\omega - \omega_{ab}, \Gamma_{ab}, D) \quad (18)$$

The linear gain for the 1.15 micron maser line of Ne is approximately [15] $0.1 m^{-1}$. The shape of the line is given by the Doppler-broadening integral Z and for a 900 MHz Doppler width has a Gaussian shape. Z has a maximum value of $i\pi^{1/2}$ and $a'' \cong 10^{-8}$. For large Doppler broadening b and c have the approximate form [14]

$$b = (2\Gamma_{ab})^{-1}C \quad (19a)$$

$$c = [i(2\omega_{ab} - \omega_A - \omega_B) + 2\Gamma_{ab}]^{-1}C \quad (19b)$$

where C includes constant terms and a broad Gaussian term similar to Eq. (18). Oscillation implies the term $(b''|\mathbf{A}|^2 + c''|\mathbf{B}|^2)$ is comparable to a'' . Different approaches may be used to analyze maser action in this cavity. Heer [15] [14] has examined the ideal problem and shown that the two traveling waves can coexist away from the center of the atomic line and the beat frequency between these oscillations is proportional to the angular velocity. Reference [15] discusses non-reciprocal back-scatter and the tendency for one traveling wave to entrain the other. Non-

reciprocal coupling between the traveling waves or any other effect which makes $|A|^2 \neq |B|^2$ causes $\mu_A \neq \mu_B$ and introduces a frequency separation or « bias beat ». At very low rates of angular rotation non-ideal conditions cause the two waves to entrain each other and « locking of modes » occurs. A recent experimental study of this effect has been reported by Aronowitz and Collins [17].

Equation (19b) is dependent on the natural width or the pressure broadened width [18] near the center of the atomic line. For the Doppler broadened line the dispersive term $vd\mu/dv$ or its counterpart for an intensity dependent index of refraction is less than 10^{-3} . Near line center the index changes by an amount comparable to 10^{-8} in a bandwidth comparable to the natural width and $vd\mu/dv$ can become as large as 0.3. This highly dispersive region is the equivalent of the « Lamb dip » for a traveling wave system [15]. The effect of this term can be reduced by operating at fixed power, by operating well away from line center, or by using an isotopic mixture of Ne^{20} and Ne^{22} . The linear gain of a mixture of equal parts of Ne^{20} and Ne^{22} is shown in Fig. 2. Since the mode spacing of the apparatus is 112 MHz, single mode operation away from line center is not possible. Constant power or isotopic mixtures were used.

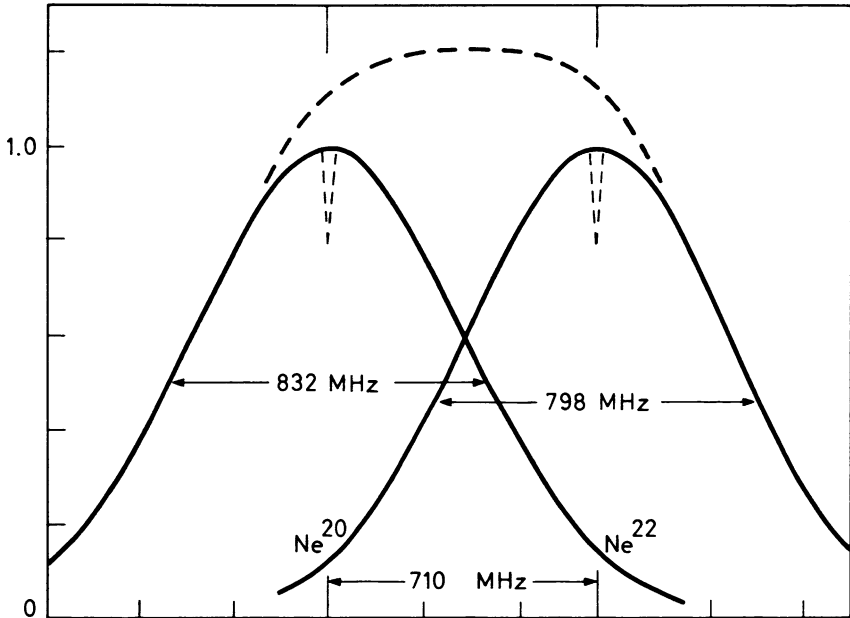


FIG. 2.

The boundaries of the cavity and the detector are in the frame of reference of the cavity. During the interaction of an atom with the radiation field gaseous atoms do not normally undergo collisions and cannot in this sense be regarded as attached to the cavity. The average value of the atomic velocities is zero relative to the frame of reference attached to the cavity and it is this average value which determines the macroscopic electric polarization. If $v_z = u^{\sigma}$ is the velocity of the atom relative to the cavity the usual Lorentz transformation implied by Eq. (8) may be used to find the frequency of the electric field seen by the atom and the contribution of the atom to the macroscopic polarization. This is equivalent to $\mathcal{O}(\Omega^2)$ to measuring v_z relative to the cavity in the previous determination [14] of P.

EXPERIMENTAL APPARATUS

The important components of the experimental apparatus are shown in Fig. 1. The cavity resonator is a triangular Fabry-Perot formed by two plane mirrors and a 10 m radius of curvature spherical mirror. Dielectric coated mirrors were made by the Electro-Optical Division of the Perkin-Elmer Corp. for 99.9 % reflectivity at an angle of incidence of 30° for 1.153μ radiation which is polarized perpendicular to the plane of the cavity. These mirrors are of average optical maser quality. The sides of the equilateral triangle are 0.875 m. This choice of mirrors and mirror spacing meets the low loss condition given by Collins [19].

Diffraction, mirror, scattering, and other cavity losses are compensated by the introduction of a He-Ne maser at 1.153μ . The He-Ne maser is of conventional design with anti-parallel Brewster windows at the ends and with r. f. excitation at 14 MHz. A 8 mm i. d. quartz tube which was filled with a 1 : 5 natural neon and helium mixture at 1.5 Torr was used for one series of experiments. A 6 mm i. d. quartz tube which was filled with a 1 : 1 : 10 mixture of Ne^{20} , Ne^{22} , and He at a total pressure of 1.3 Torr was used in the second series of experiments. Both maser tubes have an active length of 0.6 m. Oscillation occurs at 1.15μ and the $\pm \sigma$ or clockwise and counterclockwise traveling waves are coupled out of the cavity and recombined as a beam in a single direction as shown in Fig. 1. The resultant radiation is incident on a Kodak Ektron N-2 lead sulfide infrared detector. Detection occurs and the resultant difference frequency is simultaneously observed with an oscilloscope and an electronic counter. The oscilloscope trace yields a measure of the instantaneous angular velocity and the reading

of the electronic counter a measure of the angle of rotation. Angular rotation at various rates is provided by a variable speed drive.

The most accurate experimental data occurs for rotation through a fixed angle at an almost constant rate of angular rotation. The total number of counts for rotation through angle θ is related to the difference frequency by

$$N = \int_0^t dt \Delta v = g \int_0^t \Omega dt = g\theta \tag{20}$$

where g is the coefficient of Ω and is a geometry factor for the system. For a fixed angle of rotation of 89.5° in the laboratory a count of 684,000 is expected for an empty lossless cavity.

Air currents are minimized during the observations and the motion of the air through the system is due to rotation. Then the velocity $\mathbf{u} = \boldsymbol{\Omega} \times \mathbf{r}$. Since $\mu^2 - 1$ and $\mu v d\mu/dv$ are of the order of or less than 3×10^{-4} , this correction may be neglected in the experimental data.

Material media is placed into the cavity by inserting the media in one side of the triangle. 1.26 cm thick homosil quartz flats which are flat to $\lambda/30$ and parallel to 0.2 s are placed in the path as pairs at anti-parallel Brewster angles as shown in Fig. 3. One to six pairs are inserted in the path and the insertion loss remains small. Long Schott glass rods with anti-reflecting ends were tried, but the quality of the commercial anti-reflecting coatings was such as to introduce a loss of more than 15 % and maser action was not possible.

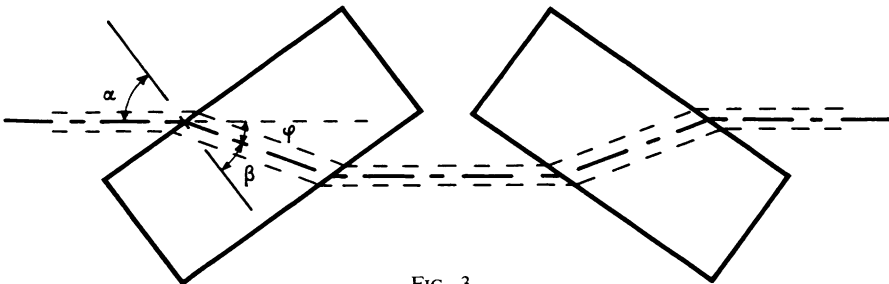


FIG. 3.

EXPERIMENTAL RESULTS

In the absence of dispersive media and media with an index of refraction Eq. (14) reduces to

$$\Delta v = (\Omega D / 3^{1/2} \lambda) = 0.438 \times 10^6 \Omega \tag{21}$$

For a fixed angle of rotation of 89.5° or 1.562 radians Eqs. (20) and (21) yield a total of 684,000 counts. The insertion of a pair of Brewster angle windows as shown in Fig. 3 changes the basic equation to

$$\Delta v = (\Omega D / 3^{1/2} \lambda) \left[1 + (3D)^{-1} (0.6415) \sum_i m_i d_i \right]^{-1} \quad (22a)$$

or

$$N = 684,000 \left(1 + 0.2443 \sum_i m_i d_i \right)^{-1} \quad (22b)$$

The index of refraction of fused quartz has been taken as [19] $\mu = 1.449$ and the dispersion $v d\mu/dv$ as $+0.014$. m_i is the number of Brewster angle windows with thickness d_i . The Brewster angle windows on the maser tube have a thickness of 3×10^{-3} m and with this correction the number of counts for rotation through 89.5° is reduced to 683,000 counts. This may be compared with the experimental value of 683,000 counts shown in Fig. 4. Since the angle of 89.5° is measured in the laboratory and the apparatus measures the angle in this degree of approximation relative to

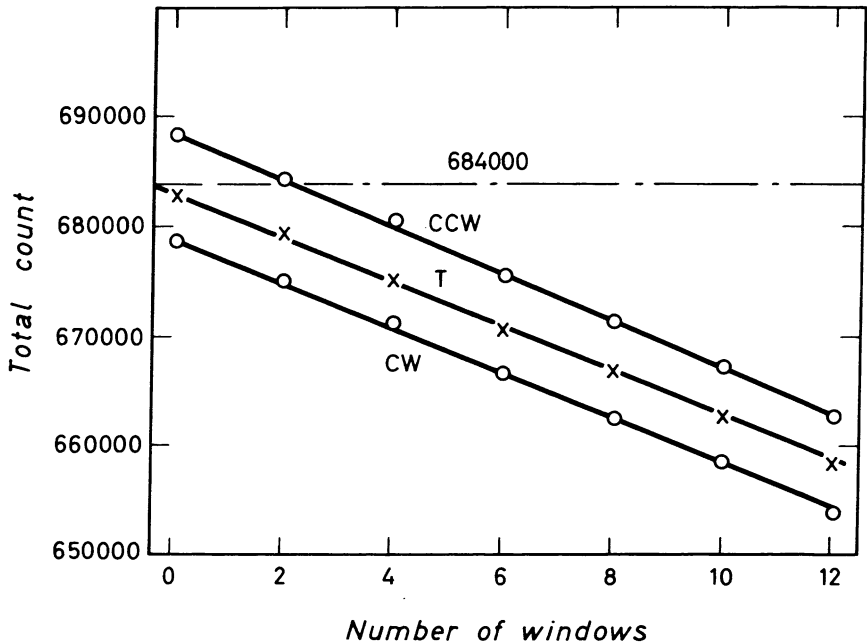


FIG. 4.

the fixed stars, it is necessary to include the rotation of the Earth in the calculation. The upper point corresponds to a rotation in the sense of the Earth's rotation and the lower point is rotation in the opposite sense. For equal times of rotation in the clockwise and in the counterclockwise directions the average of the two points corresponds to a measure of the laboratory angle of rotation and is in excellent agreement with the theoretical value.

The angular velocity Ω_E of the Earth along the axis of the local vertical which is the axis of rotation of the system is $7.26 \times 10^{-5} \cos 50^\circ$ radians per second and a beat frequency of

$$\Delta\nu_E = 20.5 \text{ Hz}$$

is expected. Due to the « locking » of the two modes discussed in an earlier section, this low rate of rotation is not observed [20]. In Fig. 4 the system is rotated in the c. w. direction in 136 s and in the c. c. w. direction in 148 s. A correction 5820 counts is needed for the Earth's rotation. The total separation between c. w. and c. c. w. counts is approximately 10,000. This residual of 4000 counts is due to a steady bias of the system in one sense of rotation. Either nonreciprocal back-scatter or a small residual Faraday rotation could account for this undesirable feature.

Since the maser media can become highly dispersive, the total number of counts as a function of power or intensity is shown in Fig. 5. A shift of the order of 10 per cent occurs for natural neon as the maser intensity is adjusted over a limited range. In contrast the isotopic mixture is quite insensitive to the intensity. As the flats are inserted into the system it is necessary to keep the intensity constant for the natural neon maser. The Q of the cavity is changed and the degree of r. f. excitation of the maser is held constant. No particular problems were encountered with the isotopic mixture as long as the degree of excitation of the maser limited the number of longitudinal modes to three or less. Only for the isotopic mixture was the empty cavity count in excellent agreement with the theoretically expected value.

The shift in frequency as a function of the number of flats is shown in Fig. 4. A suitable average of the two curves is compared with Eq. (22b). Agreement is excellent. The error is largest for twelve flats or windows. A theoretical value of 659,000 counts is expected and a value of 658,000 is observed. This is comparable to the experimental scatter of the order of ± 1000 counts. Due to the unknown dispersion of the maser medium for natural neon, the data cannot be compared with the absolute theoretical value. The slope of the shift in the number of counts with the number

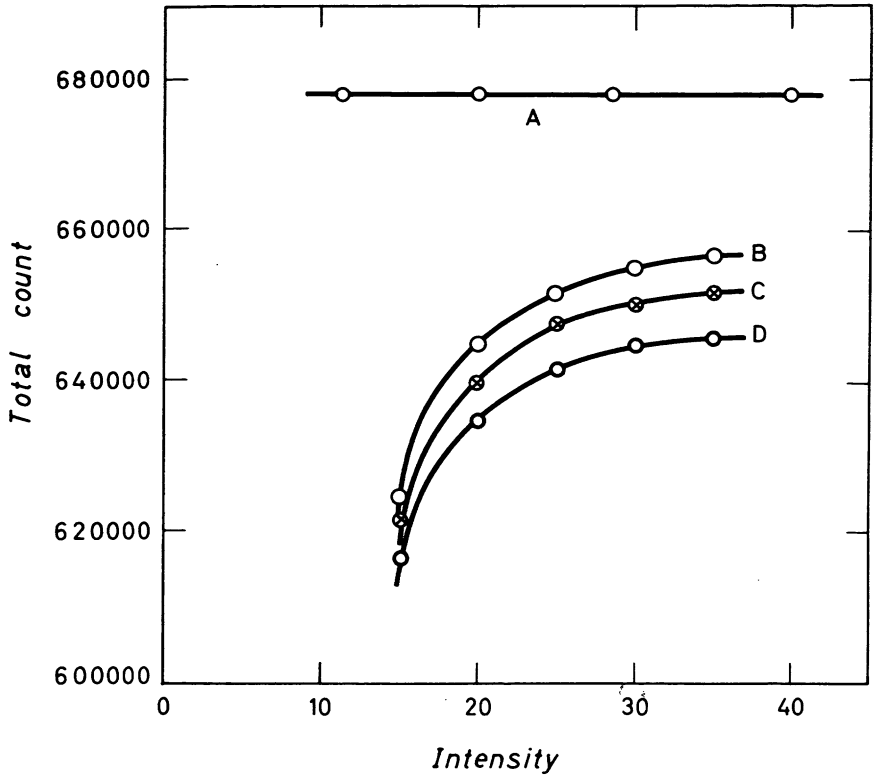


FIG. 5.

of flats was determined [21]. The results of observations at ten different rates of rotation and for a number of observations at each rate yielded a slope of average value of $-3.7 \pm 0.1\%$ and are in good agreement with the data using the isotopic mixture.

CONCLUSIONS

The experimental data for the separation in frequency of the two traveling wave modes of a cavity resonator in an accelerated frame of reference are in excellent agreement with the theoretical discussion given in earlier sections of this paper. Insertion of macroscopic media into the cavity causes a frequency separation in agreement with that predicted by Eq. (14) and by Eq. (5). If Eq. (5) is used it is necessary to take into account the

finite cross-section of the beam and the change in cross-section as the beam passes through the Brewster angle flats. Suitable corrections for dispersion are also needed in Eq. (5). Both Eqs. (5) and (14) are derived in similar manner. Equation (5) assumes the stress-energy tensor is known in an instantaneous rest frame and since it is written in a covariant form, the stress-energy tensor is known in every other frame of reference. Equation (14) assumes the phase of a wave and the index of refraction are known in an instantaneous rest frame and if written in a covariant manner may be found in any other reference frame. If Maxwell's equations are used to find the frequency separation [4], the transformation of the constitutive equations between frames of reference may be derived in a similar manner. Agreement between these theoretical developments and the experimental data indicate that through terms proportional to Ω a covariant formalism is adequate for the discussion of problems in an accelerated frame of reference. In problems concerning electrodynamics of moving or accelerated bodies it is adequate to know the experimental properties in an instantaneous rest frame. In a certain sense this could be regarded as an extension of the formulation of Minkowski for the phenomenological electrodynamics of moving bodies to that for accelerated bodies.

The asymmetry of the electromagnetic part of the stress-energy tensor of Minkowski has been a subject of discussion for some years. I. Tamm [23] examined the asymmetry of $S_{z\rho}$ for Cerenkov radiation. M. V. Laue [24] has shown that only the form of the energy-momentum tensor given by Minkowski is correct for a phenomenological description of moving bodies. Both M. v. Laue and Moeller [10] emphasize that the addition theorem of velocities for the ray velocity is in agreement only with the unsymmetric form of the tensor as given by Eq. (1). The similarity is quite apparent for the invariants $d\varphi = k_\alpha dx^\alpha$ and $k_\alpha k^\alpha$. In the short wave length approximation k_a has the directional properties of the momentum S_a^i and the ray direction k^a has the directional properties of the energy flux S_i^a . The asymmetry of the energy momentum tensor is apparent in this experiment. Equation (5) yields a frequency separation proportional to the moment of the energy flux and this is in accord with the experimental data shown in Fig. 4. Only for vacuum is the result proportional to angular momentum. Again this is in accord with the Minkowski formulation in that transparent or nondissipative bodies do not even locally exchange energy with the electromagnetic field. Equations (4a) and (4b) are equivalent to this statement. In the short wavelength approximation the divergence of the energy flux is zero and the energy flux of the beam remains constant along the path. The experimental data reported in this paper supports

the asymmetric formulation of the electromagnetic part of the energy momentum tensor of Minkowski for bodies in accelerated as well as Lorentz frames of reference.

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Note added in proof: HUTCHINGS, WINCOUR, DURRETT, JACOBS and ZINGERY, *Phys. Rev.*, **152**, 1966, 467 discuss an experimental investigation of the dispersion of isotopic mixtures of the type shown in Figure 2.
