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On the compactness of the $\overline{\partial}$-Neumann operator [(*)]

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RÉSUMÉ. — Dans cet article, nous étudions le problème de la compacité de l'opérateur de Neumann sur des domaines dans une variété hermitienne. En particulier, nous démontrons la compacité de cet opérateur sur des intersections q-convexes et sur des domaines strictement q-convexes à bord non-lisse.

ABSTRACT. — In this article, we study the problem of the compactness of the Neumann operator in subdomains of hermitian manifolds. In particular, we show the compactness of this operator in q-convex intersections and in strictly q-convex domains with nonsmooth boundary.

1. Introduction

It is a well-known fact that the $\overline{\partial}$-Neumann operator N_q on a smoothly bounded, relatively compact domain $D \subset\subset \mathbb{C}^n$ is a compact operator on $L^2_{0,q}(D)$ (the space of $(0,q)$-forms on D with square-integrable coefficients) if D satisfies Hörmander's condition $Z(q)$, i. e. if the Levi form of a smooth defining function of D has, at every boundary point of D, at least $n-q$ positive or at least $q+1$ negative eigenvalues (cf. the work of G. Folland and J. J. Kohn [4]). In particular, the operators N_s, $s \geqslant q$, are compact on every

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smoothly bounded, strictly q-convex domain $D \subset\subset \mathbb{C}^n$. Here, a smooth, strictly q-convex domain is a domain given by a $\mathcal{C}^\infty$-smooth function r as

$$D = \{z \in \mathbb{C}^n : r(z) < 0\} \subset\subset \mathbb{C}^n \,, \quad dr \neq 0 \text{ on } bD = \{z : r(z) = 0\}$$

such that the complex Hessian form of r at each $p \in \overline{D}$

$$L_p(r)(t) := \sum_{j,k=1}^{n} \frac{\partial^2 r(p)}{\partial z_j \partial \overline{z}_k} t_j \overline{t_k} \,, \quad t \in \mathbb{C}^n \,,$$

has at least $n-q+1$ positive eigenvalues. In particular, the operators N_s, $s \geqslant 1$, are compact for any bounded, smoothly bounded strictly pseudoconvex domain. In two independent papers G. M. Henkin and A. Iordan [9] as well as J. Michel and M.-C. Shaw [18] showed that the compactness property of N_s remains true for transversal intersections of strictly pseudoconvex domains, thus answering a question posed by Kohn. Michel and Shaw derived their result from a subelliptic $\frac{1}{2}$-estimate for N_s; this stronger statement was also proved (using a different method) by Henkin, Iordan and Kohn in [10].

It seems that the question whether the $\overline{\partial}$-Neumann operator is compact on bounded transversal intersections of q-convex domains in $\mathbb{C}^n$ cannot be answered using the methods of Henkin-Iordan and Michel-Shaw . There are some papers on this subject where compactness of the Neumann operator on q-convex intersections and nonsmooth q-convex domains, respectively, is shown under some (quite strong) additional assumptions, cf. the work of S. K. Vassiliadou [25], of M. Nieten [19] and of T. Hefer, L. Ma and S. K. Vassiliadou [8]. These works are based on the ideas of the papers [9], [18] mentioned above; the additional assumptions rely on an idea of L.-H. Ho [11]. However, the seemingly simple question whether N_q is compact on the transversal intersection of a strictly pseudoconvex and a strictly q-convex domain in $\mathbb{C}^n$ remained open. One of the difficult problems in generalizing the papers of Henkin-Iordan and Michel-Shaw, respectively, was the fact that on strictly q-convex intersections (which are not q-convex in the sense of Ho, cp. [11], [25] and [8]), there is no good control of the $\mathcal{C}^2$-norms of defining functions for smooth exhaustions of the intersection domains. As opposed to that, on transversal intersections of strictly pseudoconvex domains, the 1-convexity condition of Ho is automatic.

In this paper we prove compactness of the Neumann operator by combining two ideas:

1. If there are compact solution operators for the $\overline{\partial}$-equation on $(0,q)$- and $(0,q+1)$-forms, then the Neumann operator is compact on $(0,q)$-forms.

2. The Henkin-Ramirez type integral formulae provide compact solution operators.

The first idea already turns up in a paper by S. Fu and E. Straube [5]. Our method can in particular be applied to the open problems mentioned above using a result of Ma and Vassiliadou [16] on estimates for solutions of the $\overline{\partial}$-equation on q-convex intersections. It also gives a result on the nonexistence of integral solution operators for $\overline{\partial}$ with uniformly integrable kernels on certain convex domains. Finally, we present an application concerning the compactness of the Neumann operator in L^2-Sobolev spaces of higher order.

Most of the results which we derive from our general compactness theorems have previously been proved by other methods, but we think that our approach is simpler as it avoids some delicate density problems.

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2. Definitions

Let us collect some definitions and notations which will be used throughout the rest of this paper. Let X be a complex manifold of dimension $n \geqslant 2$, equipped with a hermitian metric ds^2 on the tangent bundle TX. Then X is an orientable Riemannian manifold with respect to the metric $\operatorname{Re} ds^2$. We denote by dV a volume form on X and by $*$ the associated Hodge-operator. If $D \subset\subset X$ is a domain, we let $L^2_{p,q}(D)$ be the space of (p,q)-forms on D with square-integrable coefficients; this is a Hilbert space with respect to the inner product

$$(f,g) := \int_D f \wedge *\overline{g}\,.$$

The $\overline{\partial}$-operator is extended to $L^2_{p,q}(D)$ (in the sense of distributions) as the closed, densely defined operator

$$\overline{\partial} = \overline{\partial}_q : \operatorname{Dom}\overline{\partial}_q \subset L^2_{p,q}(D) \longrightarrow L^2_{p,q+1}(D)$$

with $f \in \operatorname{Dom}\overline{\partial}$ and $\overline{\partial} f = g$ if the equation

$$\int_D f \wedge \overline{\partial}\varphi = (-1)^{p+q+1} \int_D g \wedge \varphi$$

holds for all $\varphi \in C^{\infty}_{n-p,n-q-1}(D)$ with compact support; $\overline{\partial}^*$ denotes the Hilbert space adjoint of $\overline{\partial}$. The *complex Laplacian operator* $\Box$ is defined as

$$\Box := \overline{\partial}\overline{\partial}^* + \overline{\partial}^*\overline{\partial}$$

on the domain

$$\operatorname{Dom}\Box := \{f \in \operatorname{Dom}\overline{\partial} \cap \operatorname{Dom}\overline{\partial}^* : \overline{\partial}f \in \operatorname{Dom}\overline{\partial}^*, \overline{\partial}^* f \in \operatorname{Dom}\overline{\partial}\}.$$

The $\overline{\partial}$-Neumann problem consists in proving existence and regularity of solutions of the equation

$$\Box u = f \in L^2_{p,q}(D).$$

If the range of the complex Laplacian $\Box$ is closed on (p,q)-forms, the Neumann operator

$$N_{p,q} : L^2_{p,q}(D) \to \operatorname{Dom}\Box$$

is well-defined by the conditions

$$\Box(N_{p,q}f) = f - H_{p,q}f \quad \text{and} \quad H_{p,q}(N_{p,q}f) = 0 \quad \forall f \in L^2_{p,q}(D),$$

where $H_{p,q}$ is the orthogonal projection onto $\ker\Box$ in $L^2_{p,q}(D)$. In case the $\overline{\partial}$-equation $\overline{\partial}u = f$ (for $\overline{\partial}f = 0$) is always solvable on (p,q)-forms, we have $H_{p,q} = 0$. For example, if D is a smoothly bounded strictly q-convex domain in $\mathbb{C}^n$, we have $\ker\Box \cap L^2_{p,q}(D) = \{0\}$ and $\Box(N_{p,q}f) = f$.

If ξ is a point in $\mathbb{C}^n$, we denote by $B(\varepsilon, \xi)$ the euclidean ball of radius ε around ξ.

In this article, we will concentrate on results for $(0,q)$-forms. We use the abbreviation $N_q := N_{0,q}$, and we denote $\Box$ by $\Box_q$ if we wish to clarify its domain of definition.

3. Compactness of the Neumann Operator

In the following theorem we show that compactness of the Neumann operator N_q on a domain D can be checked by results on the solvability of the $\overline{\partial}$-equation on D which, in some cases, seem to be more accessible than direct consideration of the Neumann operator.

THEOREM 3.1.— *Let $D \subset\subset X$ be a relatively compact open set and let $q \in \{1,\dots,n\}$. Suppose there exist closed vector subspaces $V_q \subseteq \ker\overline{\partial}_q$ and $V_{q+1} \subseteq \ker\overline{\partial}_{q+1}$ of finite codimension and suppose there exist compact linear operators*

$$S_k : V_k \to L^2_{0,k-1}(D) \cap \operatorname{Dom}\overline{\partial}, \; k = q, q+1,$$

such that $\overline{\partial}_{k-1}S_kf = f$ *for all* $f \in V_k$, $k = q, q+1$. *Then the Neumann operator* N_q *is compact.*[1]

Proof. — For $k = q, q+1$, we have $V_k \subseteq \operatorname{im}\overline{\partial}_{k-1} \subseteq \ker\overline{\partial}_k$, and V_k is closed and of finite codimension, so the image of the operator $\overline{\partial}_{k-1}$ is closed in $L^2_{0,k}(D)$. By an argument of Hörmander [12], the same is true for the image of $\overline{\partial}^*_{k-1}$, and it is then easily checked that the image of $\Box_q$ is closed. This yields the existence of the operator N_q.

Now let U_q and U_{q+1} be finite dimensional orthogonal complements of V_q and V_{q+1} in $\operatorname{im}\overline{\partial}_{q-1}$ and $\operatorname{im}\overline{\partial}_q$ with bases $(f_{1,q}, \dots, f_{r_q,q})$ and $(f_{1,q+1}, \dots, f_{r_{q+1},q+1})$, respectively. Choose $u_{j,k}$ with $\overline{\partial}_{k-1}u_{j,k} = f_{j,k}$ and define the linear operators T_q and T_{q+1} on $\operatorname{im}\overline{\partial}_{q-1}$ and $\operatorname{im}\overline{\partial}_q$ by

$$T_kf = T_k(\sum_{j=1}^{r_k} \alpha_j f_{j,k} + g) := \sum_{j=1}^{r_k} \alpha_j u_{j,k} + S_k g \quad \text{for } g \in V_k\,.$$

Then T_k, $k = q, q+1$, are compact linear solution operators for $\overline{\partial}$ on the images of the respective $\overline{\partial}$-operators. We extend these operators to be zero on $(\operatorname{im}\overline{\partial})^{\perp}$.

Consider the orthogonal decompositions

$$\begin{aligned} L^2_{0,k}(D) &= \ker\overline{\partial} \oplus (\ker\overline{\partial})^{\perp}\,,\ k = q,\, q+1\,, \\ L^2_{0,q}(D) &= \operatorname{im}\overline{\partial} \oplus \operatorname{im}\overline{\partial}^* \oplus \ker\Box_q \ \text{ and} \\ L^2_{0,q+1}(D) &= \operatorname{im}\overline{\partial} \oplus \operatorname{im}\overline{\partial}^* \oplus \ker\Box_{q+1}\,. \end{aligned}$$

For $k \in \{q, q+1\}$, let $P_k : L^2_{0,k-1}(D) \to (\ker\overline{\partial})^{\perp}$ and $Q_k : L^2_{0,k}(D) \to \operatorname{im}\overline{\partial}$, respectively, be the orthogonal projections on these closed subspaces. Then we define

$$K_k := P_k\, T_k\, Q_k \quad \text{for } k = q, q+1\,.$$

We will show that

$$N_q = K_q^* K_q + K_{q+1}K_{q+1}^*\,. \tag{3.2}$$

Observe that this is not obvious since we don't have existence of the operator N_{q+1} (in that case one can show that $K_k = \overline{\partial}^* N_k$). Denote the right hand side of (3.2) by M_q.

Firstly, if $\alpha \in \ker\Box$, then we have $\alpha \in \ker\overline{\partial} \cap \ker\overline{\partial}^*$, so $Q_q\alpha = 0$ and $P_{q+1}\alpha = 0$ which gives $M_q\alpha = N_q\alpha = 0$. Now suppose $\alpha \perp \ker\Box$, i. e.

[1] In the case $q = n$, we only suppose the existence of S_n, of course.

$\alpha = \overline{\partial}\overline{\partial}^* N_q\alpha + \overline{\partial}^*\overline{\partial} N_q\alpha$. Then, dropping indices for convenience, we get

$$\begin{aligned} K_q^* K_q \alpha &= QT^*PTQ\alpha &&= QT^*PT\overline{\partial}\overline{\partial}^* N\alpha \\ &= QT^*P\overline{\partial}^* N\alpha &&= QT^*\overline{\partial}^* N\alpha \\ &= Q(\overline{\partial}T)^* N\alpha &&= QN\alpha\,, \end{aligned}$$

since $\overline{\partial}T = Q$ by construction. Similarly, we obtain

$$\begin{aligned} K_{q+1} K_{q+1}^* \alpha &= PTQT^*P\alpha &&= PTQT^*\overline{\partial}^*\overline{\partial} N\alpha \\ &= PTQ(\overline{\partial}T)^*\overline{\partial} N\alpha &&= PTQ\overline{\partial} N\alpha \\ &= PT\overline{\partial} N\alpha &&= PN\alpha\,, \end{aligned}$$

which shows $M_q\alpha = Q_q N_q\alpha + P_{q+1} N_q\alpha = N_q\alpha$ also in this case. But M_q is obviously compact, so the same is true for N_q.[2] □

In particular, the Neumann operator N_n is compact on any bounded domain for which the Bochner-Martinelli integral formula holds (cf. R. M. Range [22]), and N_q is compact on any bounded domain in $\mathbb{C}^n$ for which the $\overline{\partial}$-equation on $(0,q)$-forms and $(0,q+1)$-forms can be solved by integral operators of the form

$$S_k f(z) = \int\limits_{\zeta\in D} f(\zeta)\wedge \mathcal{S}_k(\zeta,z)\,,\ k\in\{q,q+1\}\,,$$

the kernels of which satisfy the hypotheses of one of the two following theorems.

THEOREM 3.3. — *Let $D \subset\subset \mathbb{C}^n$ be open, and let $\mathcal{S}$ be Lebesgue-measurable on $D\times D$. Suppose there exists a positive function $C(\delta)$ defined for $\delta > 0$ with $\lim_{\delta\to 0} C(\delta) = 0$ such that for all $\delta > 0$*

1. $\int\limits_{B(\delta,y)\cap D} |\mathcal{S}(x,y)|dV(x) \leqslant C(\delta)$ *for all* $y\in D$,
2. $\int\limits_{B(\delta,x)\cap D} |\mathcal{S}(x,y)|dV(y) \leqslant C(\delta)$ *for all* $x\in D$ *and*
3. $|\mathcal{S}(x,y)|$ *is bounded on* $\{(x,y)\in D\times D : |x-y|\geqslant\delta\}$.

Then the linear integral operator $S : L^p(D)\to L^p(D)$ defined by

$$Sf(y) := \int\limits_{x\in D} f(x)\,\mathcal{S}(x,y)\,dV(x)$$

is compact for all p with $1 < p < \infty$.

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(2) The idea of representing N_q as $\overline{\partial}^* N_{q+1}(\overline{\partial}^* N_{q+1})^* + (\overline{\partial}^* N_q)^*\overline{\partial}^* N_q$ is due to R. M. Range [21].

Proof. — This theorem was proved by Range [22], Appendix C. $\square$

THEOREM 3.4. — *Let $D \subset\subset \mathbb{C}^n$ be open, and let S be Lebesgue-measurable on $D \times D$. Suppose S satisfies the estimates*

1. $\int_D |S(\zeta, z)|^\gamma dV(\zeta) \leqslant N^\gamma < \infty$ *a. e. in z and*
2. $\int_D |S(\zeta, z)|^\gamma dV(z) \leqslant N^\gamma < \infty$ *a. e. in ζ*

for some $\gamma > 1$. Then the integral operator defined by S is compact on $L^2(D)$.

Proof. — The proof is a slight modification of an idea of J. Michel [17]. By the theorem in Appendix B of [22], S is bounded from $L^\gamma(D)$ to $L^q(D)$ for $\frac{1}{q} = \frac{1}{\gamma} + \frac{1}{\gamma} - 1 < \frac{1}{\gamma}$; so, for $k \in \mathbb{N}$, S^k is bounded from $L^\gamma(D)$ to $L^{p_k}(D)$, where $\frac{1}{p_k} = \frac{1}{\gamma} + k(\frac{1}{\gamma} - 1)$. Consider the *normal* operator $T := S^*S$ on $L^2(D)$. By the regularity of S, applied to the kernel function itself, there exists an integer $m \in \mathbb{N}$ such that the kernel of T^m belongs to $L^2(D \times D)$. Therefore, T^m is a Hilbert-Schmidt operator, hence compact. This implies that T^m satisfies the following two conditions:

1. The spectrum $\sigma(T^m)$ has no limit point except possibly 0.
2. If $\lambda \neq 0$, then $\dim\ker(T^m - \lambda \mathrm{id}) < \infty$.

But then the operator T, too, has these two properties. By Theorem (12.30) in [23], T is compact. This shows that the operator S is also compact since

$$\|Sf\|^2 = (Tf, f) \leqslant \|Tf\|\|f\|$$

which implies that for each bounded sequence (f_ν) for which (Tf_ν) is a Cauchy sequence, (Sf_ν) is also a Cauchy sequence. $\square$

Theorem 3.1 can of course be applied to convex domains of finite type using the kernels of K. Diederich and J. E. Fornæss constructed in [1] – see also [2], [3] and [7], but these domains are perhaps more easily handled by Catlin's subelliptic estimates which also give the general case of finite type pseudoconvex domains. Moreover, Fu and Straube have given the following beautiful necessary and sufficient geometric condition for the compactness of N_q in the case of convex domains.

THEOREM 3.5. — *Let $D \subset\subset \mathbb{C}^n$ be a convex domain, and let $1 \leqslant q \leqslant n$. Then the following conditions are equivalent:*

1. *There is an L^2-compact solution operator for $\overline{\partial}$ on $(0, q)$-forms.*

2. *The boundary $b\Omega$ contains no affine variety of dimension $\geqslant q$.*

3. *The boundary $b\Omega$ contains no analytic variety of dimension $\geqslant q$.*

4. *The $\overline{\partial}$-Neumann operator N_q is compact.*

Proof. — This is Theorem 1.1 in [5]. □

In fact, in order to prove the implication 1. ⇒ 2. of this theorem, Fu and Straube use an explicit construction involving estimates on the Bergman kernel of a convex domain and an extension theorem of T. Ohsawa and K. Takegoshi [20] to show that if $b\Omega$ contains an affine variety of dimension q, then there cannot exist a compact solution operator for $\overline{\partial}$ on $(0,q)$-forms. Before, S. G. Krantz [14] had already given examples of convex domains with noncompact Neumann operators.

As a consequence we get the following negative result on – necessarily – infinite type domains.

COROLLARY 3.6. — *Let $D \subset\subset \mathbb{C}^2$ be a convex domain with noncompact Neumann operator N_1. Then there exists no compact linear solution operator for the $\overline{\partial}$-equation on $L^2_{0,1}(D)$. In particular, there exists no integral solution operator for $\overline{\partial}$ with a uniformly integrable kernel (in the sense of Theorems 3.3 or 3.4).*

Proof. — There exists a compact (integral) solution operator for the $\overline{\partial}$-equation on $(0,2)$-forms on D, given by the Bochner-Martinelli kernel. By Theorem 3.1 there cannot exist any compact linear solution operator for $\overline{\partial}$ on $(0,1)$-forms. □

In order to give our principal applications of Theorem 3.1, we will describe two methods of how to pass from certain locally defined compact solution operators for $\overline{\partial}$ to global compact solution operators. The first method — a variation of the bump method — is inspired by an article of N. Kerzman [13].

THEOREM 3.7. — *Let $D \subset\subset \mathbb{C}^n$ be a domain, and let $q \in \{1,\dots,n\}$. Suppose there exists a neighborhood basis $\mathcal{V}$ of $\overline{D}$ such that for every $V \in \mathcal{V}$ there is a continuous, linear operator*

$$T_V : L^2_{0,q}(V) \to L^2_{0,q-1}(V)$$

such that $\overline{\partial} T_V f = f$ for $f \in L^2_{0,q}(D) \cap \ker \overline{\partial}$ and such that for any $\zeta \in C_0^\infty(V)$ the operator $f \mapsto \zeta T_V f$ is compact. Suppose furthermore that there exist an $\varepsilon > 0$ and finitely many points $\xi_1,\dots,\xi_M \in bD$ such that the following two conditions are fulfilled:

1. $bD \subset\subset \bigcup_{j=1}^{M} B(\frac{\varepsilon}{2}, \xi_j)$.

2. *Let* $D_0 := D$ *and* $D_j := D \cup \bigcup_{k=1}^{j} B(\frac{\varepsilon}{2}, \xi_k)$, $j = 1, \ldots, M$. *Suppose there exists a linear, compact solution operator* T_{j+1} *for the* $\overline{\partial}$*-equation on* $D_j \cap B(\varepsilon, \xi_{j+1})$ *for* $j = 1, \ldots, M-1$.

Then there exists a compact solution operator

$$T : L^2_{0,q}(D) \to L^2_{0,q-1}(D)$$

for the $\overline{\partial}$*-equation on* D.

Proof. — Let $P : L^2_{0,q}(D) \to \ker \overline{\partial}$ be the orthogonal projection onto the kernel of $\overline{\partial}$. We choose functions $\chi_j \in C_0^\infty(B(\varepsilon, \xi_j))$, $j = 1, \ldots, M$, such that $\chi_j \equiv 1$ on a neighborhood of $\overline{B(\frac{\varepsilon}{2}, \xi_j)}$. Given $f \in L^2_{0,q}(D)$, we define $f_1 \in L^2_{0,q}(D_1)$ as follows. Let $f^{(1)}$ be the restriction of Pf to $D_0 \cap B(\varepsilon, \xi_1)$. Set

$$f_1 := \overline{\partial}(T_1 f^{(1)} - \chi_1 T_1 f^{(1)}) \quad \text{and} \quad \psi_0 := \chi_1 T_1 f^{(1)} .$$

We extend ψ_0 trivially outside of $D \cap B(\varepsilon, \xi_1)$. The form f_1 obviously can be extended to Pf in D outside of $B(\varepsilon, \xi_1)$, and it vanishes on $D \cap B(\frac{\varepsilon}{2}, \xi_1)$. Therefore, we can extend f_1 to $D_1 = D \cup B(\frac{\varepsilon}{2}, \xi_1)$, and we have the equation

$$f - f_1 = \overline{\partial}\psi_0 \quad \text{on } D .$$

It is evident from the construction that $\overline{\partial} f_1 = 0$, that the map $f \mapsto f_1$ is linear and continuous in L^2 and that the map $f \mapsto \psi_0$ is linear and compact (by hypothesis on T_1). Now let $f^{(2)}$ be the restriction of f_1 to $D_1 \cap B(\varepsilon, \xi_2)$. Set

$$f_2 := \overline{\partial}(T_2 f^{(2)} - \chi_2 T_2 f^{(2)}) \quad \text{and} \quad \psi_1 := \chi_2 T_2 f^{(2)}$$

as above; again, we see that $f_2 \in L^2_{0,q}(D_2)$. Then we have $f_1 - f_2 = \overline{\partial}\psi_1$ on D_1 and $\overline{\partial} f_2 = 0$. We continue this procedure until we arrive at a $\overline{\partial}$-closed form $f_M \in L^2_{0,q}(D_M)$ and at $\psi := \psi_0 + \cdots + \psi_{M-1}$ with

$$f - f_M = \overline{\partial}\psi \quad \text{on } D .$$

The result of this operation is a continuous linear operator

$$f \mapsto f_M = f_M(f)$$

and a *compact* linear operator

$$f \mapsto \psi = \psi(f) .$$

Now let $V \in \mathcal{V}$ be a neighborhood of $\overline{D}$ as in the hypothesis of the theorem with

$$D \subset\subset V \subset\subset D_M ,$$

and let T_V be the associated solution operator for $\overline{\partial}$. Let $\zeta \in \mathcal{C}_0^\infty(V)$ be a function which is identically 1 in a neighborhood of $\overline{D}$. Then, by hypothesis on T_V, the operator

$$Tf := \zeta T_V(f_M(f)|_V) + \psi(f)$$

is a compact solution operator for $\overline{\partial}$ on D. □

The second method has the advantage of being purely local (this time with respect to the entire domain, not only its boundary).

THEOREM 3.8.— *Let $D \subset\subset X$ be a relatively compact domain in the hermitian manifold X.*

1. *Suppose there are compact operators P_k, S_k and T_k on* $\operatorname{Dom}\overline{\partial}$ *such that the following homotopy formula holds for $k = q$, $q+1$:*

$$f = P_k f + \overline{\partial} S_k f + T_k \overline{\partial} f \quad \text{for } f \in L^2_{0,k}(D) \cap \operatorname{Dom}\overline{\partial} . \tag{3.9}$$

Then the Neumann operator N_q is compact.

2. *Suppose there is a finite covering of $\overline{D}$ by open sets $U_1, \dots, U_m$ such that for each $j \in \{1, \dots, m\}$ there exists a homotopy formula*

$$f = \overline{\partial} S_k^{(j)} f + T_k^{(j)} \overline{\partial} f , \ f \in L^2_{0,k}(D \cap U_j) \cap \operatorname{Dom}\overline{\partial} ,$$

for the $\overline{\partial}$-equation with compact operators $S_k^{(j)}$ and $T_k^{(j)}$. Then there exist compact operators P_k, S_k and T_k such that (3.9) holds.

Proof.— Let $\chi_1, \dots, \chi_m$ be smooth, nonnegative functions on X such that $\operatorname{supp}\chi_j \subset\subset U_j$ and $\sum_{j=1}^{m} \chi_j \equiv 1$ on $\overline{D}$. Define linear operators S_k, T_k and P_k by the formulas

$$S_k f := \sum_{j=1}^{m} S_k^{(j)}(\chi_j f) ,$$

$$T_k g := \sum_{j=1}^{m} T_k^{(j)}(\chi_j g) \quad \text{and}$$

$$P_k f := \sum_{j=1}^{m} T_k^{(j)}(\overline{\partial}\chi_j \wedge f) .$$

By hypothesis on $S_k^{(j)}$ and $T_k^{(j)}$, these operators are compact. This gives formula (3.9). For $k = q$, $q+1$, let F_k be the restriction of $\mathrm{id} - P_k$ to the closed subspace $Z_k := \ker \overline{\partial}_k$ of $L^2_{0,k}(D)$. Then F_k is a Fredholm operator on Z_k, so $V_k := \operatorname{im} F_k$ is a closed subspace of finite codimension in Z_k. If we denote by G_k the restriction of F_k to the closed, finitely codimensional subspace $(\ker F_k)^{\perp} \subset Z_k$, we see that the operator

$$\tilde{S}_k := S_k \circ G^{-1} : V_k \to L^2_{0,k-1}(D) \cap \operatorname{Dom} \overline{\partial}$$

is, in fact, a compact solution operator for $\overline{\partial}$ on V_k, so Theorem 3.1 applies. □

It is possible to find local integral homotopy operators for the $\overline{\partial}$-equation (as in the last theorem) in quite general situations. In most of the cases we know, such formulas are proved, initially, only for forms which are $\mathcal{C}^1$-smooth up to the boundary of the domain. This is usually sufficient if the domain in question is smoothly bounded. For the nonsmooth cases, we add a theorem on the extension of such homotopy formulas to $L^2_{0,q}(D) \cap \operatorname{Dom} \overline{\partial}$. The hypotheses of the next theorem can be easily verified for the local homotopy formulas in our later applications.

THEOREM 3.10 *Let $D \subset\subset X$ be a relatively compact domain. Suppose there exist homotopy operators S_q and T_q for the $\overline{\partial}$-equation on D such that*

$$f = \overline{\partial} S_q f + T_q \overline{\partial} f \tag{3.11}$$

for all $f \in \mathcal{C}^1_{0,q}(\overline{D})$ and such that S_q and T_q are linear and continuous on $L^2_{0,q}(D)$, $L^1_{0,q+1}(D)$ and $L^2_{0,q+1}(D)$, respectively. Let $S \subset\subset bD$ be a subset of the boundary of D such that $bD - S$ is smooth and such that there is a neighborhood basis $(V_\varepsilon)_{\varepsilon>0}$ of S (relative to X) satisfying the following conditions:[3]

1. *$V_\varepsilon \subset\subset V_\delta$ for $\varepsilon < \delta$.*
2. *The volume of V_ε satisfies the estimate $\mathrm{vol}(V_\varepsilon) \leqslant C\varepsilon^2$ for a constant C independent of ε.*
3. *There are smooth functions ψ_ε such that $\operatorname{supp} \psi_\varepsilon \subset\subset V_\varepsilon$, $\psi_\varepsilon \equiv 1$ on $V_{\frac{\varepsilon}{2}}$, $0 \leqslant \psi_\varepsilon \leqslant 1$ and $\sup |\overline{\partial} \psi_\varepsilon| \leqslant \frac{C}{\varepsilon}$ for the constant C above.*
4. *For each $\varepsilon > 0$ and for each $f \in L^2_{0,q}(D) \cap \operatorname{Dom} \overline{\partial}$ which vanishes identically in V_ε, the homotopy formula (3.11) remains valid.*

(3) In case $q = n$, we have $T_q = 0$, and the problem of extending the homotopy formula to L^2 is trivial. Thus, in our later applications, the hypotheses of this theorem (concerning the singular subset of the boundary) have to be checked only in case $q < n$.

Then the homotopy formula is valid (in the sense of distributions) on $L^2_{0,q}(D) \cap \operatorname{Dom} \overline{\partial}$.

Proof. — Let $\chi_\varepsilon := 1 - \psi_\varepsilon$, and let $f \in L^2_{0,q}(D) \cap \operatorname{Dom} \overline{\partial}$ be given. By hypothesis, the homotopy formula is valid for $\chi_\varepsilon f$, so we have

$$\chi_\varepsilon f = \overline{\partial} S_q(\chi_\varepsilon f) + T_q(\chi_\varepsilon \overline{\partial} f) + T_q(\overline{\partial} \chi_\varepsilon \wedge f).$$

Now, $\chi_\varepsilon f$ and $T_q(\chi_\varepsilon \overline{\partial} f)$ converge to f and $T_q \overline{\partial} f$ in $L^2_{0,q}(D)$ as $\varepsilon \to 0$, respectively, whereas $\overline{\partial} S_q(\chi_\varepsilon f)$ converges to $\overline{\partial} S_q f$ in the sense of distributions. We only have to show that the remaining term $T_q(\overline{\partial} \chi_\varepsilon \wedge f)$ converges to zero. To see this, it suffices to show that $\overline{\partial} \chi_\varepsilon \wedge f$ converges to zero in $L^1_{0,q+1}(D)$. But we have, by the Cauchy-Schwarz inequality,

$$\int_D |\overline{\partial} \chi_\varepsilon \wedge f| \, dV \leqslant \int_{V_\varepsilon} \frac{C}{\varepsilon} |f| \, dV \leqslant C^2 \left(\int_{V_\varepsilon} |f|^2 \, dV \right)^{\frac{1}{2}} \to 0$$

for $\varepsilon \to 0$. $\square$

To later verify the fourth condition of this theorem, we prove

LEMMA 3.12. — *Let $D \subset\subset \mathbb{C}^n$ be a domain and let $S \subset\subset bD$ be a subset of the boundary such that $bD - S$ consists of smooth points. Let V be an open neighborhood of S. Suppose there are L^2-continuous operators S_q and T_q such that*

$$f = \overline{\partial} S_q f + T_q \overline{\partial} f \tag{3.13}$$

for all $f \in C^1_{0,q}(\overline{D})$. Let $f \in L^2_{0,q}(D) \cap \operatorname{Dom} \overline{\partial}$ be given with $f \equiv 0$ in $V \cap D$. Then (3.13) also holds for f.

Proof. — Let $W \subset\subset V$ be a neighborhood of S. We deform bD inside W to obtain a *smooth* domain $G \subset D$ with the property $\overline{bG - bD} \subset\subset V$. Let r be a defining function for G with $|\nabla r| = 1$ on bG. For a point $z \in bG$ let $\nu(z) = -\nabla r(z)$ be the unit inner normal to bG at z. If $z_0 \in bG$ is given, we may choose a ball $B_0 = B(\eta, z_0)$ around z_0 such that for any $z \in \overline{B_0 \cap G}$ we have

$$z + \delta \nu(z_0) + \tau \in G \tag{3.14}$$

for all sufficiently small $\delta > 0$ and all τ with $|\tau| < \delta^2$. Cover bG with a finite number $B_1, \ldots, B_{m-1}$ of such balls and denote the corresponding normal vectors by $\nu_1, \ldots, \nu_{m-1}$. Choose δ_0 so small that condition (3.14) is satisfied in each $B_j \cap G$ for $0 < \delta < \delta_0$. Choose an open set $B_m \subset\subset G$ such that

$$\overline{G} \subset\subset \bigcup_{j=1}^{m} B_j \,,$$

set $\nu_m := 0$, and let $\chi_1, \ldots, \chi_m$ be smooth functions with $\operatorname{supp} \chi_j \subset B_j$ such that

$$\sum_{j=1}^{m} \chi_j \equiv 1$$

on $\overline{G}$. Now let φ be a smooth nonnegative, radially symmetric function on $\mathbb{C}^n$, supported in $B(1,0)$, such that

$$\int_{\mathbb{C}^n} \varphi(\tau)\, d\tau = 1\,,$$

and let $\varphi_\delta(\tau) := \delta^{-2n}\varphi(\frac{\tau}{\delta})$. We define an approximation f^δ for the given form f by

$$f^\delta(z) := \sum_{j=1}^{m} \int_{\mathbb{C}^n} (\chi_j f)(z + \delta\nu_j + \tau)\, \varphi_{\delta^2}(\tau)\, d\tau$$

where these integrals are to be interpreted in the obvious manner. This is well-defined because of condition (3.14). Clearly the forms f^δ are smooth on $\overline{G}$ and satisfy $f^\delta \to f$ in $L^2_{0,q}(G)$ for $\delta \to 0$. But for every $j \in \{1, \ldots, m\}$ we also have

$$\overline{\partial} \int_{\mathbb{C}^n} (\chi_j f)(z + \delta\nu_j + \tau)\, \varphi_{\delta^2}(\tau)\, d\tau = \int_{\mathbb{C}^n} \overline{\partial}((\chi_j f)(z + \delta\nu_j + \tau))\, \varphi_{\delta^2}(\tau)\, d\tau$$

since ν_j does not depend on z and since $f \in \operatorname{Dom} \overline{\partial}$ by hypothesis. Therefore we get $\overline{\partial} f^\delta \to \overline{\partial} f$ in $L^2_{0,q+1}(G)$. Now let $Y \subset\subset V$ be an open neighborhood of $\overline{bG - bD}$ and let z_0 be a point in $Y \cap B_j$ for a $j \leqslant m$. We claim that $f^\delta(z_0) = 0$ for all δ with $0 < \delta < \delta_0$ if δ_0 is sufficiently small. In fact, if $\delta_0 < 1$ is so small that $z + \delta\nu_j + \tau \in V$ for all τ with $|\tau| < \delta^2 < \delta_0^2$, the definition of f^δ and the hypothesis on f immediately prove the claim. Note that δ_0 can be chosen independently of $z_0 \in Y$. Thus, we may extend f^δ trivially to a smooth form on $\overline{D}$, and we obtain

$$f^\delta \to f \quad \text{and} \quad \overline{\partial} f^\delta \to \overline{\partial} f$$

in $L^2_{0,q}(D)$ and $L^2_{0,q+1}(D)$, respectively. Since $f^\delta \in C^1_{0,q}(\overline{D})$, and since the operators S_q and T_q are continuous with respect to L^2-norm, the assertion of the lemma is now obvious. □

The above proof is essentially due to Kerzman [13]. Theorem 3.10 and Lemma 3.12 imply that in the following examples of domains D with non-smooth boundaries the smooth forms on $\overline{D}$ are dense in $\operatorname{Dom} \overline{\partial}$ with respect to the graph norm $\|f\| + \|\overline{\partial} f\|$. It is well-known that such a density theorem holds for fairly general boundaries, but we preferred to include a direct proof for the domains we consider.

As a first application, we now study q-convex intersections as in the following definition (which is due to Ma and Vassiliadou [16]).

DEFINITION 3.15. — *A domain $D \subset\subset X$ is called a C^3 q-convex intersection if there exist an open neighborhood U of bD and a finite number of real C^3-functions $r_1, \dots, r_N$, $n \geqslant N+2$, defined on U such that the following three conditions are fulfilled:*

1. *$D \cap U = \{z \in U : r_j(z) < 0$ for $j = 1, \dots, N\}$.*
2. *For $1 \leqslant i_1 < i_2 < .. < i_\ell \leqslant N$ the 1-forms $dr_{i_1}, .., dr_{i_\ell}$ are $\mathbb{R}$-linearly independent on $\bigcap_{j=1}^{j=\ell}\{r_{i_j} \leqslant 0\}$.*
3. *For $1 \leqslant i_1 < \dots < i_\ell \leqslant N$, for every $z \in \bigcap_{j=1}^{j=\ell}\{r_{i_j} \leqslant 0\}$, if we set $I = (i_1, .., i_\ell)$, there exists a complex linear subspace T_z^I of T_zX of complex dimension at least $n - q + 1$ such that for $i \in I$ the complex Hessian forms $L(r_i)$ restricted to T_z^I are positive definite.*

We obtain the following theorem on the compactness of the Neumann operator on such intersections.

THEOREM 3.16. — *Let $D \subset\subset X$ be a q-convex intersection in the hermitian manifold X. Then the Neumann operator N_s is compact for $s \geqslant q$.*

In particular, let $D \subset\subset \mathbb{C}^n$ be the transversal intersection of strictly q_j-convex domains, $j = 1, \dots, N \leqslant n-2$, and let $q := q_1 + \dots + q_N - N + 1$. Then the Neumann operator N_s on D is compact for $s \geqslant q$.

Proof. — As an illustration of Theorem 3.7, we first present a proof in case $X = \mathbb{C}^n$. It was shown by Ma and Vassiliadou in [16] that the $\overline{\partial}$-equation on q-convex intersections in $\mathbb{C}^n$ can be solved on forms of type $(0, s)$, $s \geqslant q$, by the method described in Theorem 3.7 with local compact integral homotopy operators that satisfy the hypotheses of Theorem 3.4. The corresponding homotopy formulas extend to L^2 by Theorem 3.10 (the set S being the singular subset of the boundary). It was also shown in [16] that a q-convex intersection in $\mathbb{C}^n$ has a neighborhood basis consisting of smoothly bounded strictly q-convex domains V (the domains in [16] are only smooth of class C^3, but it is easily seen that we can obtain C^∞-neighborhoods by the same method; compare also [19] where a C^∞-smooth so-called *regularized max-function* is constructed which can be used to construct the smooth neighborhoods we need). On such domains the canonical solution operator $T_V = \overline{\partial}^* N_q$ to the $\overline{\partial}$-equation exists by [4], and if we take $\zeta \in C_0^\infty(V)$, the operator ζT_V is compact by Proposition (3.1.16) in [4] and by Rellich's lemma. Therefore, all the hypotheses of Theorem 3.7 are satisfied. An application of Theorem 3.1 then yields the first assertion for $X = \mathbb{C}^n$. If D is a q-convex intersection in a general manifold X, we can still use the local construction of Ma-Vassiliadou at the boundary of D. Since $\overline{D}$ is compact and since we clearly have local compact homotopy operators for $\overline{\partial}$ in the interior of D, we can apply Theorem 3.8 and Theorem 3.10 to get the result in full generality.

For the second assertion, it suffices to remark that in [19] it was shown in detail that an intersection of the form mentioned above *is* a q-convex intersection in the sense of Ma and Vassiliadou, where $q = q_1 + \cdots + q_N - N + 1$.

In particular, this yields a new proof for the compactness of the Neumann operator on intersections of strictly pseudoconvex domains.[4] □

As a further application, we show the compactness of the Neumann operator on nonsmooth, strictly q-convex domains as in the following definition.

DEFINITION 3.17. — *A domain $D \subset\subset X$ is called strictly q-convex if there exists an open neighborhood U of bD and a function $r \in C^2(U)$ such that r is strictly q-convex on U (compare the introduction) and such that $D \cap U = \{z \in U : r(z) < 0\}$. We do not suppose $dr \neq 0$ on bD.*

In her paper [15], Ma has shown the existence of local integral homotopy operators for the $\overline{\partial}$-equation on $(0,s)$-forms, $s \geqslant q$, on such domains.[5] These integral operators satisfy the hypotheses of Theorem 3.3 and Theorem 3.8. By the following theorem, we have a good local description of the singular set of the boundary of a strictly q-convex domain which will allow us to verify the hypotheses of Theorem 3.10 (note that this is trivial in the case $q = n$).

THEOREM 3.18. — *Let $D \subset\subset X$ be strictly q-convex, $q \leqslant n$. Let r be a defining function for D as in Definition 3.17, and let $S := \{z : dr(z) = 0\}$ be the singular subset of the boundary of D. Then for every $z \in S$ there exists a neighborhood U of z in X and a smooth real submanifold Y of U of real dimension $n - 1 + q$ such that $S \cap U \subseteq Y$. In particular, the real codimension of Y is greater or equal to 2 if $q < n$.*

Proof. — This was proved by G. Schmalz, see Lemma 1.3 in [24]. □

(4) See also the note added in proof.

(5) Ma only considers the case $X = \mathbb{C}^n$, but again, the local construction carries over to any complex manifold X.

Thus, if $\varepsilon > 0$ is given and if $q < n$, we may construct local tubular neighborhoods of the singular set S of bD with a volume dominated by ε^2. Since the boundary is compact, the union of a finite number of such neighborhoods satisfies the hypotheses imposed on the sets V_ε in Theorem 3.10. This yields the following result.

THEOREM 3.19. — *Let $D \subset\subset X$ be strictly q-convex. Then the Neumann operator N_s on D is compact for $s \geqslant q$.* □

In the case $q = 1$, this theorem was proved by Henkin and Iordan who consider also less regular boundaries.

Finally, we will give an application concerning the Neumann operators on Sobolev spaces other than $L^2_{0,q}(D)$. Let $D \subset\subset \mathbb{C}^n$ be a smoothly bounded domain. Consider the Sobolev spaces $L^{2,k}_{0,q}(D)$ of $(0,q)$-forms on D the coefficients of which have weak derivatives in L^2 up to order $k \in \mathbb{N}$. This is a Hilbert space with respect to the interior product

$$(f,g)_k := \sum_{|\alpha|+|\beta|\leqslant k} (D^\alpha \overline{D}^\beta f, D^\alpha \overline{D}^\beta g)$$

where D^α and $\overline{D}^\beta$ denote the differential operators (acting on the coefficients of f and g) defined by

$$\frac{\partial^{|\alpha|}}{\partial z_1^{\alpha_1} \cdots \partial z_n^{\alpha_n}} \quad \text{and} \quad \frac{\partial^{|\beta|}}{\partial \overline{z}_1^{\beta_1} \cdots \partial \overline{z}_n^{\beta_n}}$$

respectively. Let N^k_q be the Neumann operator on $L^{2,k}_{0,q}(D)$ which is defined with respect to $(\cdot,\cdot)_k$ just as N_q is defined with respect to $(\cdot,\cdot)$, and let $\overline{\partial}_q^{*,k}$ be the Hilbert space adjoint of $\overline{\partial}_q$ on $L^{2,k}_{0,q}(D)$.

THEOREM 3.20. — *Let $D \subset\subset \mathbb{C}^n$ be a smoothly bounded domain which satisfies conditions $Z(q)$ and $Z(q+1)$. Then the Neumann operator N^k_q exists and is compact.*

Proof. — The existence of N^k_q follows from the closedness of the image of $\overline{\partial}$ as in Theorem 3.1. If D satisfies conditions $Z(q)$ and $Z(q+1)$, the (usual) canonical solution operators $\overline{\partial}^* N_j$, $j = q, q+1$, satisfy Kohn's subelliptic $\frac{1}{2}$-estimate

$$\|\overline{\partial}^* N_j f\|_{k+\frac{1}{2}} \lesssim \|f\|_k$$

and are therefore compact as operators from $L^{2,k}_{0,j}(D)$ to $L^{2,k}_{0,j-1}(D)$. This implies that the canonical solution operators $\overline{\partial}^{*,k} N^k_j$ are *a fortiori* compact. Just as in the proof of Theorem 3.1, we derive the compactness of N^k_q. □

Note added in proof. — In an interesting survey [6], Fu and Straube have shown the following: Let $U \subseteq \mathbb{C}^n$ be open, smoothly bounded and strictly pseudoconvex, and let $\Omega \subset\subset \mathbb{C}^n$ be a pseudoconvex domain such that the $\overline{\partial}$-Neumann operator on $(0,q)$-forms is compact on Ω and such that $U \cap \Omega$ is a domain. Then the corresponding $\overline{\partial}$-Neumann operator is compact on the intersection $U \cap \Omega$. The proof of this result (with only minor modifications) also applies to the following situation: Let U be as above, and suppose that $\Omega \subset\subset \mathbb{C}^n$ is a smoothly bounded strictly q-convex domain such that $U \cap \Omega$ is a domain (Ω need not be pseudoconvex). Then the $\overline{\partial}$-Neumann operator on $U \cap \Omega$ is compact on $(0,s)$-forms for $s \geqslant q$. For $n > 2$, this is a special case of Theorem 3.16.

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