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Dynamical Systems / *Systèmes dynamiques*

Appendix to the paper "On the Billingsley dimension of Birkhoff average in the countable symbolic space"

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Abstract. This appendix gives a lower bound of the Billingsley-Hausdorff dimension of a level set related to Birkhoff average in the *"non-compact"* symbolic space $\mathbb{N}^{\mathbb{N}}$, defined by Gibbs measure.

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The authors in [1] estimate the upper bound of the Billingsley dimension of the levels sets $\widehat{E}_f(\alpha)$, defined by Gibbs measures. In the following, we give the lower bound.

Theorem 1. *Let φ be a potential function of summable variations. We assume that φ admits a unique Gibbs measure ν, then*

$$\dim_\nu \widehat{E}_f(\alpha) = \sup\left\{\gamma(\nu,\mu);\ \int_{\mathscr{X}} f \,\mathrm{d}\mu = \alpha\right\}.$$

Proof. For any $\mu \in \mathscr{P}_\sigma(\mathscr{X})$, define the set of μ-generic points by

$$G_\mu = \left\{x \in \mathscr{X};\ \lim_{n\to+\infty} \frac{1}{n} S_n f(x) = \int_{\mathscr{X}} f \mathrm{d}\mu \quad \text{for all} \quad C_b(\mathscr{X})\right\}.$$

Remark that the sets G_μ for which $\int_{\mathscr{X}} f \mathrm{d}\mu = \alpha$ are all included in the set $\widehat{E}_f(\alpha)$. Thus by using Theorem 1.1 in [2], we obtain

$$\sup\left\{\gamma(\nu,\mu);\ \int_{\mathscr{X}} f \,\mathrm{d}\mu = \alpha\right\} \le \dim_\nu \widehat{E}_f(\alpha).$$

□

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