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SLOPE FILTRATIONS: *ERRATUM*

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Abstract. Erratum for the paper "Slope Filtrations", Confluentes Mathematici, Vol. 1, No. 1 (2009) 1–85

(1) Example 1.2.2.(2): actually, the usual orthogonal sum is neither a coproduct (nor a product), and in fact, the category of hermitian spaces does not admit finite coproducts (this is not used elsewhere).

(2) Discard Lemma 1.2.8 and the undefined notion of "refinement", which are used only in Proposition 1.4.18. Replace the given proof of Proposition 1.4.18 by the following:

Proof. — It suffices to show (by induction on rk N) that for any strict subobject N of M, the point (rk $N, \deg N$) lies below $NP(M)$. If i denotes the minimal index such that N is contained in the notch M_i of the flag $\mathcal{F}(M)$, this amounts to: $\deg N \leqslant \deg M_{i-1} + \lambda_i(\mathrm{rk}\, N - \mathrm{rk}\, M_{i-1})$.

Let $p : N \hookrightarrow M_i \twoheadrightarrow M_i/M_{i-1}$ be the composed morphism. Since M_i/M_{i-1} is semistable of slope λ_i and $N/(N\cap M_{i-1}) \to Im\, p$ is epi-monic, one has $\mu(N/(N\cap M_{i-1})) \leqslant \lambda_i$. By additivity of rk and deg in the sequence $0 \to (N \cap M_{i-1}) \to N \to N/(N \cap M_{i-1}) \to 0$, one gets

$$\deg N \leqslant \lambda_i(\mathrm{rk}\, N - \mathrm{rk}(N \cap M_{i-1})) + \deg(N \cap M_{i-1}).$$

On the other hand, by induction, the point $(\mathrm{rk}\,(N\cap M_{i-1}), \deg\,(N\cap M_{i-1}))$ lies below $NP(M)$, which implies that

$$\deg(N \cap M_{i-1}) \leqslant \deg M_{i-1} - \lambda_i(\mathrm{rk}\, M_{i-1} - \mathrm{rk}(N \cap M_{i-1})).$$

By combining the last two inequalities, one gets $\deg N \leqslant \deg M_{i-1} + \lambda_i(\mathrm{rk}\, N - \mathrm{rk}\, M_{i-1})$ as wanted. □

(3) In Lemma 1.2.18, replace the sum $N+P$ in the sense of Section 1.2.3, which is the image of $N \oplus P \to Q$, by the coimage of this morphism (which is the usual sum of N and P in the abelian envelop $\mathcal{A}$). This lemma is not used elsewhere in the paper.

References

[1] Yves André, *Slope filtrations*, Confluentes Math. 1(1):1–85, 2009.

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