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SOME CHARACTERIZATIONS OF ULTRABORNLOGICAL SPACES *

by Manuel VALDIVIA

In this paper we show that an ultrabornological space E is the inductive limit of a family of nuclear Fréchet spaces and we prove also that E is the inductive limit of a family of nuclear (DF)-spaces.

The vector spaces that we use here are defined over the field K of the real or complex numbers. With the word "space" we shall mean "separated locally convex spaces". Given the space E , then E' is its topological dual. We denote by $\sigma(E, E')$ and $\mu(E, E')$ the weak and Mackey topologies, respectively, on E . If A is a bounded closed absolutely convex set in the space E , then E_A is the normed space on the linear hull of A , with A as closed unit ball. If C is a compact set, with non-empty interior, in the n -dimensional euclidean space \mathbb{R}^n , \mathcal{O}_C is the space of all the real or complex valued functions, infinitely differentiable, with compact support contained in C , provided with the topology of the uniform convergence on all the derivatives of order q , $q = 0, 1, 2, \dots$. \mathcal{O}'_C is the topological dual of \mathcal{O}_C , with the strong topology. $\mathcal{O}(\Omega)$ and $\mathcal{O}'(\Omega)$ are the well-known spaces of L. Schwartz, with the strong topologies, being Ω an open set of \mathbb{R}^n . If $x = (x_1, x_2, \dots, x_n)$ is a point of \mathbb{R}^n and $p = (p_1, p_2, \dots, p_n)$, being p_j a non-negative integer, $j = 1, 2, \dots, n$, then x^p denotes $x_1^{p_1} x_2^{p_2} \dots x_n^{p_n}$, $|p| = p_1 + p_2 + \dots + p_n$ and $dx = dx_1 dx_2 \dots dx_n$.

In [4] we have shown the following result : a) *Let E be a Banach space. If $\{x_n\}$ is a sequence in E , such that, for every positive integer p , the sequence $\{2^{pn} x_n\}$ converges to the origin then there is in E a compact absolutely convex set B , so that E_B is a Hilbert space and in $E_B \{x_n\}$ is a sequence such that $\{2^{pn} x_n\}$ converges to the origin.*

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We follow the same way, as we did in [4] for result a), to prove Lemma 1.

LEMMA 1. — *Let E be a Banach space. Let $\{\lambda_n\}$ be a strictly increasing sequence of positive integers. If $\{x_n\}$ is a sequence in E, such that, for every positive integer p, the sequence $\{\lambda_{pn}^p x_n\}$ converges to the origin, then there is in E a weakly compact absolutely convex set B, so that E_B is a Hilbert space and in $E_B\{x_n\}$ is a sequence such that $\{\lambda_{pn}^p x_n\}$ converges to the origin.*

Proof. — Clearly, it is sufficient to carry out the proof when $x_n \neq 0$, $n = 1, 2, \dots$, which we are going to suppose. Let f be the linear mapping of l^2 into E such that

$$f(\{a_n\}) = \sum_{n=1}^{\infty} a_n (x_n / \|x_n\|^{1/2}).$$

Since $\{\lambda_{2n}^2 x_n\}$ converges to the origin and $\sum_{n=1}^{\infty} \lambda_{2n}^{-2} < \infty$, then $\sum_{n=1}^{\infty} \|x_n\|$ is convergent. On the other hand

$$\left\| \sum_{n=q}^{\infty} a_n (x_n / \|x_n\|^{1/2}) \right\| \leq \left(\sum_{n=q}^{\infty} |a_n|^2 \right)^{1/2} \left(\sum_{n=q}^{\infty} \|x_n\| \right)^{1/2}.$$

Therefore, f is well defined and maps every bounded set of l^2 in a bounded set of E and, thus, f is continuous. Let g be the canonical mapping of l^2 onto $l^2/f^{-1}(0)$. If $f = h \circ g$, then h is a continuous injective linear mapping of the Hilbert space $l^2/f^{-1}(0)$ in E. If U is the closed unit ball of l^2 , then $g(U)$ is the closed unity ball of $l^2/f^{-1}(0)$ and, therefore,

$$B = h(g(U)) = f(U) = \left\{ \sum_{n=1}^{\infty} a_n (x_n / \|x_n\|^{1/2}) : \sum_{n=1}^{\infty} |a_n|^2 \leq 1, \right. \\ \left. a_n \in K, n = 1, 2, \dots \right\},$$

hence E_B can be identified with $l^2/f^{-1}(0)$. Given two positive integers p and r , there exists a positive integer n_1 such that $r\lambda_{pn}^p \|x_n\|^{1/2} < 1$, $n \geq n_1$. Since $x_n / \|x_n\|^{1/2} \in B$, $n = 1, 2, \dots$, we have that for $n \geq n_1$

$$\begin{aligned} \lambda_{pn}^p x_n &= \lambda_{pn}^p (x_n / \|x_n\|^{1/2}) \|x_n\|^{1/2} \in \lambda_{pn}^p \|x_n\|^{1/2} B = \\ &= r \lambda_{pn}^p \|x_n\|^{1/2} (1/r) B \subset (1/r) B, \end{aligned}$$

and, therefore, $\{\lambda_{pn}^p x_n\}$ converges to the origin in E_B . Finally, B is weakly compact in E , since U is weakly compact in l^2 and $B = f(U)$.
 Q.E.D.

In [4] we have proved the following result : b) *Let E be a Hilbert space of infinite dimension. If $\{x_n\}$ is a sequence in E such that, for every positive integer p , the sequence $\{2^{pn} x_n\}$ converges to the origin, then there is in E an orthogonal sequence $\{y_n\}$, so that its closed absolutely convex hull contains $\{x_n\}$ and $\{2^{pn} y_n\}$ converges to the origin.*

We follow the same way, as we did in [4] for result b), to prove Lemma 2.

LEMMA 2. — *Let $\{\lambda_n\}$ be a strictly increasing sequence of positive integers. Let E be a Hilbert space of infinite dimension. If $\{x_n\}$ is a sequence in E , such that, for every positive integer p , the sequence $\{\lambda_{pn}^p x_n\}$ converges to the origin, then there is in E an orthogonal sequence $\{u_n\}$ so that its closed absolutely convex hull contains $\{x_n\}$ and $\{\lambda_{pn}^p u_n\}$ converges to the origin.*

Proof. — Since E has infinite dimension, we can choose a sequence $\{y_n\}$ in E , $y_n \neq 0$, $n = 1, 2, \dots$, with infinite dimensional linear hull, such that the closed absolutely convex hull of the sequence $\{y_n\}$ contains $\{x_n\}$, and so that, for every positive integer p , the sequence $\{\lambda_{pn}^p y_n\}$ converges to the origin. By induction we select an increasing sequence of positive numbers $\{n_q\}$ setting

$$\|y_{n_1}\| = \sup\{\|y_n\| : n = 1, 2, \dots\}$$

$$\|y_{n_q}\| = \sup\{\|y_n\| : n = n_{q-1} + 1, n_{q-1} + 2, \dots\}, q > 1.$$

Let $z_n = (y_n / \|y_{n_q}\|) \|y_{n_q}\|$, $n = n_{q-1} + 1, n_{q-1} + 2, \dots, n_q$, $q = 1, 2, \dots$, $n_0 = 0$. The closed absolutely convex hull of the sequence $\{z_n\}$ contains $\{x_n\}$, $\|z_n\| \geq \|z_{n+1}\|$, $n = 1, 2, \dots$, and, for every positive integer p , the sequence $\{\lambda_{pn}^p z_n\}$ converges to the origin. We construct, by induction, a family of sequences in E , $\{z_{qn}\}$, $q = 1, 2, \dots$

We set $z_{1n} = z_n$, $n = 1, 2, \dots$. We suppose the sequence $\{z_{qn}\}$ already constructed. Let $z_{qn(q)}$ be the first non-zero element of this sequence. If H_q is a hyperplane in E , orthogonal to $z_{qn(q)}$, passing through the origin, we represent by $z_{(q+1)n}$ the orthogonal projection of z_{qn} onto H_q . Clearly, we can choose a positive integer p_0 such that $\sum_{n=1}^{\infty} \lambda_{p_0 n}^{-p_0} < 1$.

If

$$u_q = \lambda_{p_0 q}^{p_0} (\|z_{n(q)}\| / \|z_{qn(q)}\|) z_{qn(q)},$$

then the sequence $\{u_n\}$ is orthogonal. Given any positive number r we obtain a positive integer r_0 such that $n(r_0) \leq r < n(r_0 + 1)$. We can set

$$z_r = \sum_{q=1}^{r_0} a_q z_{qn(q)} = \sum_{q=1}^{r_0} b_q u_q.$$

If (x, y) is the inner product of any two elements $x, y \in E$, then $(z_r, u_q) = b_q (u_q, u_q)$, and so

$$\begin{aligned} |b_q| &\leq \|z_r\| / \|u_q\| = \|z_r\| / (\lambda_{p_0 q}^{p_0} \|z_{n(q)}\|) \leq \\ &\leq \|z_{n(r_0)}\| / (\lambda_{p_0 q}^{p_0} \|z_{n(q)}\|) \leq \lambda_{p_0 q}^{-p_0}, \end{aligned}$$

and, therefore, if $v \in E'$ and $|\langle v, u_n \rangle| \leq 1$, $n = 1, 2, \dots$, then

$$|\langle v, z_r \rangle| \leq \sum_{q=1}^{r_0} |b_q \langle v, u_q \rangle| \leq \sum_{q=1}^{\infty} \lambda_{p_0 q}^{-p_0} \leq 1,$$

hence the closed absolutely convex hull of $\{u_n\}$ contains z_r and, therefore, it contains $\{x_n\}$. Finally, for every pair of positive integers p and q , we get

$$\begin{aligned} \|\lambda_{pq}^p u_q\| &= \lambda_{pq}^p \lambda_{p_0 q}^{p_0} \|z_{n(q)}\| \leq \lambda_{(p+p_0)q}^p \lambda_{(p+p_0)q}^{p_0} \|z_q\| = \\ &= \lambda_{(p+p_0)q}^{p+p_0} \|z_q\| \end{aligned}$$

and, therefore, the sequence $\{\lambda_{pn}^p u_n\}$ converges to the origin.

Q.E.D.

Markushevich proves in [1] that every separable infinite-dimensional Banach space has a basis in the wide sense, (see also [2] p. 116). In Lemma 3 we shall give a more general result than the one given above. We shall need it later.

LEMMA 3. — *Let E be an infinite-dimensional space, with $E'[\sigma(E', E)]$ separable. Suppose that there is in E a bounded countable total set. If E is sequentially complete there is a Markushevich basis $\{x_n, u_n\}$ for E, such that the sequence $\{x_n\}$ is bounded.*

Proof. — Let $\{y_n\}$ be a bounded total sequence in E. Let f be the linear mapping from l^2 into E such that, if $\{a_n\} \in l^2$,

$$f(\{a_n\}) = \sum_{n=1}^{\infty} n^{-1} a_n y_n$$

If q is a continuous seminorm in E, then $q(y_n) < c, n = 1, 2, \dots$, and, therefore,

$$\begin{aligned} q\left(\sum_{n=m}^{\infty} n^{-1} a_n y_n\right) &\leq \sum_{n=m}^{\infty} n^{-1} |a_n| q(y_n) \leq \\ &\leq c \left(\sum_{n=m}^{\infty} n^{-2}\right)^{1/2} \left(\sum_{n=m}^{\infty} |a_n|^2\right)^{1/2}; \end{aligned}$$

from here, and being E sequentially complete, it follows that f is well-defined and it is bounded and so it is continuous. If B is the closed unit ball of l^2 and $A = f(B)$, then E_A can be identified with the Hilbert space $l^2/f^{-1}(0)$. If $\{v_n\}$ is a total sequence in $E'[\sigma(E', E)]$ whose elements are linearly independent, then $\{v_n\}$ is total in

$$(E_A)' [\sigma((E_A)', E_A)],$$

and applying the Gram-Schmidt process we obtain an orthonormal sequence $\{u_n\}$ in $(E_A)' [\mu((E_A)', E_A)]$. If x_n is a continuous linear form on $(E_A)' [\sigma((E_A)', E_A)]$, such that $\langle x_n, u_n \rangle = 1, \langle x_n, u_m \rangle = 0, n \neq m, n, m = 1, 2, \dots$, then $\{x_n\}$ is total in E_A and, therefore, $\{x_n, u_n\}$ is a Markushevich basis for E, such that $\{x_n\}$ is a bounded sequence in E.

Q.E.D.

THEOREM 1. — *Let F be a sequentially complete infinite-dimensional space with the following properties :*

- 1) *There is in F a bounded countable total set.*
- 2) *There is in $F'[\sigma(F', F)]$ a countable total set which is equicontinuous in F.*

3) If u is an injective linear mapping from F into F , with closed graph, then u is continuous.

If E is an infinite-dimensional Banach space then E is the inductive limit of a family of spaces equal to F , spanning E .

Proof. — According to Lemma 3 we construct a Markushevich basis $\{x_n, u_n\}$ for E so that the sequence $\{x_n\}$ is bounded. Since in the proof of Lemma 3 we chose the sequence $\{v_n\}$ with the unique conditions that it is total in $F'[\sigma(F', F)]$ and linearly independent, we can suppose, according to property 2), that $\{u_n\}$ is in the linear hull of an equicontinuous set in F . We determine a sequence $\{\lambda_n\}$ of positive integers, strictly increasing, such that the sequence $\{\lambda_n^{-1} u_n\}$ be equicontinuous in F . Let M be the $\sigma(F', F)$ -closed absolutely convex hull of $\{\lambda_n^{-1} u_n\}$. Let \mathfrak{S} be the family of all the sequences of E holding the two following properties : α) If $\{y_n\} \in \mathfrak{S}$ then, for every positive integer p , the sequence $\{\lambda_{pn}^p y_n\}$ converges to the origin in E . β) If $\{y_n\} \in \mathfrak{S}$ and $\{a_n\}, \{b_n\}$ are two different bounded sequences of K then $\sum_{n=1}^{\infty} a_n y_n$ and $\sum_{n=1}^{\infty} b_n y_n$ are different points of E .

Given an element $\{y_n\} = s \in \mathfrak{S}$ we define a linear mapping f_s from F into E so that, for every $x \in F$,

$$f_s(x) = \sum_{n=1}^{\infty} \langle x, u_n \rangle y_n$$

Let M^0 be the polar set of M in F . We have that $\sum_{n=1}^{\infty} \lambda_n^{-2} < \infty$ and

$\|\lambda_n^3 y_n\| < c, n = 1, 2, \dots$, being $\|y\|$ the norm of every $y \in E$. Thus, if $x \in M^0$, it results that

$$\left\| \sum_{n=1}^{\infty} \langle x, u_n \rangle y_n \right\| \leq \sum_{n=1}^{\infty} \lambda_n^{-2} |\langle x, \lambda_n^{-1} u_n \rangle| \cdot \|\lambda_n^3 y_n\| \leq c \sum_{n=1}^{\infty} \lambda_n^{-2},$$

from here, f_s is well-defined and transforms M^0 , which is a neighbourhood of the origin in F , in a bounded set of E , hence f_s is continuous. According to property β) the mapping f_s is injective. Then, if $E_s = f_s(F)$ we can give to E_s a topology \mathfrak{C}_s , finer than the induced topology by E , such that $E_s[\mathfrak{C}_s]$ can be identified with the

space F . Let us see now that E is the locally convex hull of the family of spaces $\{E_s[\mathfrak{C}_s] : s \in \mathfrak{S}\}$. Let U be an absorbing absolutely convex set in E such that, for every $s \in \mathfrak{S}$, $U \cap E_s$ is a neighbourhood of the origin in $E_s[\mathfrak{C}_s]$. Let us suppose that U is not a neighbourhood of the origin in E . We take $z_1 \in E$, $\|z_1\| = \lambda_1^{-1}$. Let w_1 be an element of E' such that $\langle w_1, z_1 \rangle = 1$. Supposing constructed $\{w_m, z_m\}_{m=1}^n$ so that $\langle w_m, z_m \rangle = 1$, $\langle w_m, z_p \rangle = 0$, $m \neq p$, $m, p = 1, 2, \dots, n$, let $H_n = \bigcap_{m=1}^n w_m^{-1}(0)$. Since $U \cap H_n$ is not a neighbourhood of the origin in H_n , for the induced topology by E , we choose $z_{n+1} \in H_n$, $z_{n+1} \notin U$, $\|z_{n+1}\| = \lambda_{(n+1)}^{-(n+1)2}$, and in E' w_{n+1} such $\langle w_{n+1}, z_{n+1} \rangle = 1$ and $\langle w_{n+1}, z_m \rangle = 0$, $m = 1, 2, \dots, n$. If $y_n = \lambda_n z_n$ the sequence $r = \{y_n\}$ belongs to \mathfrak{S} and, therefore, $U \cap E_r$ is a neighbourhood of the origin in $E_r[\mathfrak{C}_r]$. The sequence $\{\lambda_n^{-1} x_n\}$ converges to the origin in F , from here the sequence $\{f_r(\lambda_n^{-1} x_n)\} = \{\lambda_n^{-1} y_n\} = \{z_n\}$ converges to the origin in $E_r[\mathfrak{C}_r]$ and, therefore, there is a positive integer n_1 such that $z_n \in U \cap E_r$, for $n \geq n_1$, which is a contradiction. Given a point $z \in E$, $z \neq 0$, the sequence $\{z_n\}$ can be constructed so that $z_1 = (\lambda_1/\|z\|)z$ and, therefore, $E = \cup \{E_s : s \in \mathfrak{S}\}$. Let us see now that the family $\{E_s : s \in \mathfrak{S}\}$ is directed by inclusion. Let s_1 and s_2 be two elements of \mathfrak{S} so that $s_1 = \{y_n\}$, $s_2 = \{y'_n\}$. We put $t_{2n-1} = \lambda_n^2 y_n$, $t_{2n} = \lambda_n^2 y'_n$, $n = 1, 2, \dots$. For every positive integer p , the sequence $\{\lambda_{pn}^p t_n\}$ converges to the origin in E . Let A be the closed absolutely convex hull of the sequence $\{t_n\}$. If $y \in E_{s_1}$ there is a element x of F such that

$$y = \sum_{n=1}^{\infty} \langle x, u_n \rangle y_n$$

and, therefore,

$$y = \sum_{n=1}^{\infty} \lambda_n^{-2} \langle x, u_n \rangle t_{2n-1}$$

On the other hand $\sum_{n=1}^{\infty} \lambda_n^{-2} < \infty$ and $|\langle x, u_n \rangle| < a$, $n = 1, 2, \dots$,

hence it follows that y is in the linear hull of A . Reasoning in an analogous way for the case when y is in E_{s_2} , we have that the linear hull of A contains $E_{s_1} \cup E_{s_2}$. By Lemma 1 there is in E a weakly compact absolutely convex set B such that E_B is a Hilbert space and in $E_B \{t_n\}$ is a sequence so that, for every positive integer p ,

$\{\lambda_{p_n}^p t_n\}$ converges to the origin. By Lemma 2 we find an orthogonal sequence $\{q_n\}$ in E_B which has the property α) and its closed absolutely convex hull contains $\{t_n\}$. Let $s_3 = \{q_n\}$. Since $\{q_n\}$ is orthogonal in E_B we have that $\{q_n\}$ has the property β), from here $s_3 \in \mathfrak{S}$. Let us see now that $E_{s_3} \supset E_{s_1} \cup E_{s_2}$; it is sufficient to prove that E_{s_3} contains the closed absolutely convex hull P of the sequence $\{q_n\}$. If $z \in P$ it is obvious that

$$z = \sum_{n=1}^{\infty} c_n q_n, \quad \sum_{n=1}^{\infty} |c_n| \leq 1$$

Since $\{x_n\}$ is a bounded sequence in the sequentially complete space F , then $\sum_{n=1}^{\infty} c_n x_n$ belongs to F and, therefore,

$$f_{s_3} \left(\sum_{n=1}^{\infty} c_n x_n \right) = \sum_{n=1}^{\infty} \left\langle \sum_{p=1}^{\infty} c_p x_p, u_n \right\rangle q_n = \sum_{n=1}^{\infty} c_n q_n = z \in E_{s_3}$$

Finally, if $s, r \in \mathfrak{S}$ and $E_s \subset E_r$, let u_s be the canonical injection from $E_s[\mathfrak{C}_s]$ into E . The mapping u_s is continuous and so its graph is closed in $E_s[\mathfrak{C}_s] \times E_s[\mathfrak{C}_r]$. According to property 3) it results that \mathfrak{C}_r induces in E_s a topology coarser than \mathfrak{C}_s .

Q.E.D.

THEOREM 2. — *If E is an ultrabornological space, then E is the inductive limit of a family of nuclear Fréchet spaces, spanning E .*

Proof. — If the topology of E is the finest locally convex topology, then E is the inductive limit of the finite-dimensional subspaces of E . In the other case, E is the inductive limit of a family of infinite-dimensional Banach spaces spanning E , and, therefore, it is enough to make the proof for the case that E is an infinite-dimensional Banach space. We take F , in Theorem 1, equal to \mathcal{O}_C , which is nuclear and separable, and its topology is defined by a countable family of norms, and so properties 1), 2) and 3) of Theorem 1 hold, hence E is the inductive limit of a family of spaces equal to \mathcal{O}_C , spanning E .

Q.E.D.

THEOREM 3. — *If E is an ultrabornological space, then E is the inductive limit of a family of nuclear (DF)-spaces, spanning E.*

Proof. — It is analogous to the proof of Theorem 2, changing \mathcal{O}_C to its strong dual \mathcal{O}'_C .

Q.E.D.

THEOREM 4. — *If E an infinite-dimensional Banach space, then E is the inductive limit of a family of spaces equal to $\mathcal{O}(\Omega)$, spanning E.*

Proof. — Let $\{w_m\}$ be a linearly independent sequence in $\mathcal{O}'(\Omega)$, whose elements are the monomial functions x^p with p_j any non-negative integer, $j = 1, 2, \dots, n$. The sequence $\{w_m\}$ is total in $\mathcal{O}'(\Omega)$. If $w_m = x^{p(m)}$, let $v_m = w_m \cdot m^{-|p(m)|-n-1}$. If $\varphi \in \mathcal{O}(\Omega)$ there exists a positive integer m_0 such that the support A of φ is contained in the ball with center 0 and radius m_0 . If $m \geq m_0$ we have

$$\begin{aligned} |\langle v_m, \varphi \rangle| &= m^{-|p(m)|-n-1} \left| \int_A \varphi(x) x^{p(m)} dx \right| \\ &\leq m^{-|p(m)|-n-1} m_0^{|p(m)|+n} \sup_{x \in A} |\varphi(x)| \leq m^{-1} \sup_{x \in A} |\varphi(x)| \end{aligned}$$

hence $\{v_n\}$ converges weakly to the origin in $\mathcal{O}'(\Omega)$ and, therefore, $\{v_n\}$ is equicontinuous in $\mathcal{O}(\Omega)$.

Let C be a compact set in Ω with non-empty interior. Then \mathcal{O}_C is a subspace of $\mathcal{O}(\Omega)$ and $\{v_n\}$ is total and linearly independent in \mathcal{O}'_C , being also equicontinuous in \mathcal{O}_C . We apply now Lemma 3 and we obtain from $\{v_m\}$ a Markushevich basis $\{x_m, u_m\}$ for \mathcal{O}_C , such that $\{x_m\}$ is bounded in \mathcal{O}_C , and also in $\mathcal{O}(\Omega)$. Obviously $\{u_n\}$ is total in $\mathcal{O}'(\Omega)$. On the other hand if u is a linear mapping from $\mathcal{O}(\Omega)$ into $\mathcal{O}(\Omega)$, with closed graph, then u is continuous, (see [3], p. 17). Following now the same method as in the proof of Theorem 1, the conclusion of the theorem is obtained.

Q.E.D.

COROLLARY 1.4. — *If E is an ultrabornological space such that $\sigma(E', E) \neq \mu(E', E)$, then E is the inductive limit of a family of spaces equal to $\mathcal{O}(\Omega)$, spanning E.*

COROLLARY 2.4. — *The space $\mathcal{O}'(\Omega)$ is the inductive limit of a family of spaces equal to $\mathcal{O}(\Omega)$, spanning $\mathcal{O}'(\Omega)$.*

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