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A CORRECTION TO “COUNTING RATIONAL POINTS ON A CERTAIN EXPONENTIAL-ALGEBRAIC SURFACE”

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by Jonathan PILA (*)

Abstract. — We correct a mistake in the paper. All the theorems remain correct.

Résumé. — Nous corrigeons une erreur dans la papier. Tous les théorèmes restent vrais.

This note is to correct a part of the paper [1]. All the theorems remain correct (though some of the implied constants will be adversely affected). I am very grateful to John Armitage for pointing out my error.

The mistake occurs in the early lines of the proof of Theorem 6.1, p. 507. The exhibited parameterization of the surface

$$\mathcal{X} = \{(x, y, z) \in (0,1)^3 : \log x \log y = -\log z\}$$

is not *mild* (as defined in the paper), as claimed there, because the function

$$z = \exp\left(\left(1 - \frac{1}{s^g}\right)\left(1 - \frac{1}{t^g}\right)\right), \quad (s,t) \in (0,1)^2$$

is not mild. The seemingly innocuous change from $-1/s^g$ to $1 - 1/s^g$ (and the same for t) to cover all of $\mathcal{X}$ spoils the mildness, and indeed the surface needs to be non-trivially reparameterized near the $x = 1$ boundary for small y (and near the $y = 1$ boundary for small x), as well as near the origin.

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We should therefore use instead the following parameterization of $\mathcal{X}$ by four maps $\theta_i : (0,1)^2 \to (0,1)^3, i = 1, \ldots, 4$ (here $g \in \mathbb{Z}$, $g > 0$):

$$\theta_1(s,t) = \left(\exp\left(-\frac{1}{s^g}\right), \exp\left(-\frac{1}{t^g}\right), \exp\left(-\frac{1}{(st)^g}\right)\right),$$

$$\theta_2(s,t) = \left(\exp\left(-s^g\right), \exp\left(-\frac{1}{(st)^g}\right), \exp\left(-\frac{1}{t^g}\right)\right),$$

$$\theta_3(s,t) = \left(\exp\left(-\frac{1}{(st)^g}\right), \exp\left(-t^g\right), \exp\left(-\frac{1}{s^g}\right)\right),$$

$$\theta_4(s,t) = \left(\exp\left(-s^g\right), \exp\left(-t^g\right), \exp\left(-(st)^g\right)\right).$$

These maps cover, respectively, the (x,y)-regions: $(0,\frac{1}{e}) \times (0,\frac{1}{e})$, $(\frac{1}{e},1) \times (0,\frac{1}{e})$, $(0,\frac{1}{e}) \times (\frac{1}{e},1)$, and $(\frac{1}{e},1) \times (\frac{1}{e},1)$. The first three maps are $(A, 1+1/g)$-mild for some suitable $A = A(g) > 0$, by Proposition 2.8, and the last is $(A(g), 0)$-mild, being analytic on the closure of $(0,1)^2$. The omitted lines are in the "algebraic part" of $\mathcal{X}$ and play no role.

BIBLIOGRAPHY

[1] J. Pila, "Counting rational points on a certain exponential-algebraic surface", *Ann. Inst. Fourier* **60** (2010), no. 2, p. 489-514.

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