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ON DENSE IDEALS IN SPACES OF ANALYTIC FUNCTIONS

by Mihai PUTINAR(*)

1. Introduction.

Let A be a locally convex space of analytic functions defined in a bounded pseudoconvex domain of $\mathbf{C}^n$ and let I be an ideal of analytic functions defined on the closure of the respective domain. The question treated in the present note is to find conditions which imply the density of the ideal I in A. The motivation for considering this problem comes from some recent works related to topological modules over function algebras (see [5] and the references there). Two factors are important in the previous question : the linear topology of the space A and the relation between the geometries of the domain and the zero set of the ideal I. However, as formulated before the question is too general and much too difficult to approach, even in a single complex variable case. (Think for instance to the deep results of the rational approximation theory.)

Throughout this note we will consider only a very specific case of the above general approximation problem. Namely, given a bounded, strictly pseudoconvex domain Ω of $\mathbf{C}^n$ with real analytic smooth boundary and a positive measure μ supported by the closure of Ω we seek conditions under which an ideal $I \subset \mathcal{O}(\overline{\Omega})$ (of analytic functions defined in neighbourhoods of $\overline{\Omega}$) is dense in the closure $A^p(\mu)$ of $\mathcal{O}(\overline{\Omega})$ into $L^p(\mu)$, for $1 \leq p < \infty$. For instance, assuming that the measure μ is singular with respect to the volume measure of any real smooth (n-1)-dimensional submanifold of $\partial\Omega$ and that the zero set $V(I)$ of the ideal I is contained in a smooth complex

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proper submanifold of $\mathbf{C}^n$ which is disjoint of Ω we prove the density of I in $A^p(\mu)$.

The main result below will be stated in a slightly more general and technically more involved form, for other possible applications. Although the above mentioned approximation statement is not surprising, its proof involves two distinct refined results of complex analysis, namely the flatness and separation of ideals of analytic functions in the space of smooth functions (cf. [11] and [18]) and the structure of the locally peak sets for the algebra $A^\infty(\Omega)$ (cf. [3],[4]). For that reason the proof proposed below cannot apparently be adapted to more general situations, as for instance a polydomain and an ideal with a singular zero locus. It is quite natural to expect that a more general statement with a conceptually simpler proof is true. From that perspective we consider the present note an intermediate step towards a better understanding of similar approximation problems.

A particular case of the main result below was proved with similar methods in the paper [13]. To be more specific we proved there the density of an ideal I into the Bergman space $A^2(\mathrm{dvol}|\Omega)$ under the hypothesis that the set $V(I)$ intersects $\overline{\Omega}$ in finitely many points. As it turned out a posteriori the non-finite intersection case is not a simple consequence of the finite intersection case, as innocently asserted in [13] Corollary 3.3.

The next section contains a review of the necessary terminology and the statement of the main results. The last section is devoted to the technical details of the proofs.

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2. Preliminaries and main results.

Let $\Omega \subset \mathbf{C}^n$ be a bounded strictly pseudoconvex domain. That means that there is a strictly plurisubharmonic function ρ, defined in a neighbourhood U of $\overline{\Omega}$, such that :

$$\Omega = \{z \in U; \rho(z) < 0\}$$

and

$$\partial\Omega = \{z \in U; \rho(z) = 0\}.$$

If grad(ρ) is non-vanishing on $\partial\Omega$ then the boundary of Ω is smooth. If the defining function ρ is real analytic, then we say that the domain Ω has real analytic boundary.

Fix a smooth point x in the boundary of Ω. The gradient of ρ at x defines the exterior normal at x to the real tangent hyperplane $T_x(\partial\Omega)$. The complex structure on $\mathbf{C}^n$ defines an antiinvolution J on $T_x(\partial\Omega) \oplus \mathbf{R}(\mathrm{grad}_x\rho)$. The vector

$$\tau_x = J(\mathrm{grad}_x\rho)$$

lies in the tangent space at $\partial\Omega$ and its orthogonal complement is invariant under J, hence it is a complex vector space :

$$T^c{}_x(\partial\Omega) = T_x(\partial\Omega) \ominus \mathbf{R}(\tau_x).$$

It is the maximal complex subspace of the real tangent hyperplane at $\partial\Omega$, of complex dimension $n-1$.

A linear subspace $S \subset T_x(\partial\Omega)$ is called *totally real* if $S \cap J.S = 0$. A submanifold of $\partial\Omega$ is totally real when its tangent space is totally real at every point.

Suppose that the boundary of the domain Ω is smooth. Then we denote by $A^\infty(\Omega)$ the algebra of analytic functions in Ω which are indefinitely differentiable on $\overline{\Omega}$. A *peak set* for the algebra $A^\infty(\Omega)$ is a subset E of $\partial\Omega$ with the property that there is a function $f \in A^\infty(\Omega)$ with the properties :

$$f|E = 1 \qquad \textit{and} \qquad f|(\overline{\Omega} \setminus E) < 1.$$

A set $E \subset \partial\Omega$ is called *locally peak* for the algebra $A^\infty(\Omega)$ if for any point $x \in E$ there is an open neighbourhood V of x with the property that $\overline{V} \cap E$ is a peak set. In both definitions we can choose equivalently a function g which is identically zero on E and with $\mathrm{Re}(g) > 0$ elsewhere. (See [4] for details.)

For a domain Ω as before we denote by $\mathcal{O}(\overline{\Omega})$ the algebra of germs of analytic functions defined in neighbourhoods of $\overline{\Omega}$. In other terms, this algebra is an inductive limit :

$$\mathcal{O}(\overline{\Omega}) = \mathrm{ind.lim}_{\mathrm{U}}\mathcal{O}(U)$$

where U runs over a (countable) fundamental system of open neighbourhoods of $\overline{\Omega}$. One knows that for strictly pseudoconvex domains Ω with smooth boundary one can take the neighbourhoods U with the same property. (See for instance [9] Section 1.5.) From this representation the space $\mathcal{O}(\overline{\Omega})$ inherits an inductive limit topology of Fréchet-Schwarz spaces.

Let Ω be a bounded domain in $\mathbf{C}^n$ with real analytic boundary and let I be an ideal of the algebra $\mathcal{O}(\overline{\Omega})$. A deep remark of Frisch [6] asserts that the ideal I is in these conditions finitely generated. An extension of Frisch' Theorem is discussed by Siu in [17].

Let us consider a finitely generated ideal $I = (f_1, \dots, f_p)$ for some elements $f_i \in \mathcal{O}(\overline{\Omega})$. Then each function f_i is defined in a neighbourhood of $\overline{\Omega}$, so that there is an open pseudoconvex domain U which contains $\overline{\Omega}$ and where the functions $f_1, \dots f_p$ are defined. In particular, the ideal $(f_1, \dots, f_p)$ is closed in the Fréchet topology of the space $\mathcal{O}(U)$ (by a classical result of H. Cartan, cf. [9] Theorem 4.1.4). In conclusion any ideal of the algebra $\mathcal{O}(\overline{\Omega})$ is finitely generated and closed in the inductive limit topology. For an ideal I as above we denote by $V(I)$ the set of common zeros of the elements (or equivalently the generators) of I. Thus $V(I)$ is a germ of an analytic set, defined in a neighbourhood of the closure of Ω.

Following Henkin [8] we call a linear functional $l \in \mathcal{O}(\overline{\Omega})'$ an *A-functional* if, for every sequence $f_n \in \mathcal{O}(\overline{\Omega})$ which is uniformly bounded on $\overline{\Omega}$ and satisfies $\lim_{n\to\infty} f_n(z) = 0$ for any point $z \in \Omega$ we have $\lim_{n\to\infty} l(f_n) = 0$. Originally, Henkin has isolated in [8] a class of measures on $\overline{\Omega}$ which interpreted as functionals have this property. (See also [14] Chapter 9 for details.)

The main result of this note can be stated as follows :

THEOREM 1. — *Let $\Omega \subset \mathbf{C}^n$ be a bounded strictly pseudoconvex domain with smooth real analytic boundary and let l be a continuous A-functional on $A^\infty(\Omega)$. Let $I \subset \mathcal{O}(\overline{\Omega})$ be an ideal whose zero set satisfies $V(I) \cap \overline{\Omega} \subset E$, where E is a locally peak sets for the algebra $A^\infty(\Omega)$.*

If $l|I = 0$, then $l = 0$.

In order to state the announced application of Theorem 1 we need more details about locally peak sets with respect to the algebra $A^\infty(\Omega)$. The following remarkable characterization of locally peak sets was obtained by Chaumat and Chollet in [4], as a completion of some prior results due to Hakim and Sibony. (See also [3] for details.)

Let Ω be a bounded strictly pseudoconvex domain of $\mathbf{C}^n$ with smooth boundary and let E be a subset of $\partial\Omega$. Then E is locally peak with respect to the algebra $A^\infty(\Omega)$ if and only if for each point $x \in E$ there is an open neighbourhood V of x in $\mathbf{C}^n$ and a smooth totally real submanifold N of $V \cap \partial\Omega$ containing $E \cap V$ and with the property that

$$T_z(N) \subset T^c{}_z(\partial\Omega),$$

for every point $z \in E \cap V$.

A second part of the main result in [4] asserts that one can impose the condition $T_z(N) \subset T^c{}_z(\partial\Omega)$ at any point of N, and then the condition on N to be totally real is automatically satisfied.

In fact one proves in [4] that one can choose the manifold N of maximal real dimension $n-1$ and with the above property.

Let Ω be as before a bounded strictly pseudoconvex domain with real analytic smooth boundary in $\mathbf{C}^n$ and let μ be a positive Borel measure supported by the closure of Ω. If μ is singular with respect to the $(n-1)$-dimensional integration measure along any $(n-1)$-submanifold of $\partial\Omega$ then, in view of the preceding result, $\mu(E) = 0$ for any locally peak set $E \subset \partial\Omega$. We denote by $A^p(\mu)$ the closure of the space of analytic functions defined in neighbourhoods of $\overline{\Omega}$ into $L^p(\mu)$, for any fixed $p \in [1,\infty)$.

These spaces of analytic functions contain as particular cases the Bergman and one of the possible Hardy spaces of the domain Ω, for the following choices $\mu = \mathrm{dvol}_\Omega$ and $\mu = \mathrm{dvol}_{\partial\Omega}$, respectively. For these two choices it is evident that any point of Ω is a bounded point evaluation for the space $A^p(\mu)$. Thus any ideal of analytic functions $I \subset \mathcal{O}(\overline{\Omega})$ which has at least a common zero inside Ω is not dense in $A^p(\mu)$. For that reason we exclude this possibility from the very beginning.

The principal application of Theorem 1 can be stated as follows :

COROLLARY 1. — *Let Ω be a bounded strictly pseudoconvex domain of $\mathbf{C}^n$ with real analytic smooth boundary, let $I \subset \mathcal{O}(\overline{\Omega})$ be an ideal and let μ be a positive measure supported by $\overline{\Omega}$. Assume that $\mu(N) = 0$ for every smooth (n-1)-dimensional submanifold N of $\partial\Omega$ and that $V(I) \cap \overline{\Omega} \subset E$, where $E \subset \partial\Omega$ is a locally peak set for the algebra $A^\infty(\Omega)$.*

Then the ideal I is dense in $A^p(\mu)$ for every $p \in [1,\infty)$.

As we will see in the proof below we can assume in the preceding statement only that $\mu(F) = 0$ for any locally peak set F (which is a slightly weaker condition). In the usual applications to the Bergman or Hardy spaces the condition imposed to the measure μ is automatically satisfied. The following consequence of Corollary 1 gives a geometric criterion for the second condition in the statement involving the zero set $V(I)$. We do not know how much this latter condition can be relaxed in general.

COROLLARY 2. — *Let Ω and μ be as in Theorem 1 and let $I \subset \mathcal{O}(\overline{\Omega})$ be an ideal with the zero set included in a finite union of smooth complex submanifolds of a neighbourhood of $\overline{\Omega}$, each disjoint of Ω. Then I is dense in all spaces $A^p(\mu)$,$p \in [1, \infty)$.*

In particular this corollary shows that the ideal of a linear complex variety which is tangent to Ω is dense in the Bergman and Hardy spaces of the domain Ω.

3. Proofs.

Let K be a compact subset of $\mathbf{C}^n$. We denote by $\mathcal{E}(K)$ the space of Whitney C^∞ jets on K. This is a Fréchet space in the natural topology of uniform convergence on K of all partial derivatives of a jet. If K is a smooth manifold with boundary, then $\mathcal{E}(K)$ is the space of smooth functions on K. We denote by $m(K)$ the closed subspace of $\mathcal{E}(\mathbf{C}^n)$ consisting of all functions ψ which are flat on K, that is such that any partial derivative of ψ, of any order, vanishes on K. In that case we have an exact sequence of Fréchet spaces :

$$0 \longrightarrow m(K) \longrightarrow \mathcal{E}(\mathbf{C}^n) \longrightarrow \mathcal{E}(K) \longrightarrow 0.$$

The monographs [11] Chapter 1 and [18] Chapter IV treat these topics in detail.

The first result of this section is a technical lemma derived from Malgrange flatness and separation theorem ([11] Theorem 1.1, p. 82) and from J.J. Kohn regularity theorem for the $\overline{\partial}$-operator on weakly pseudoconvex domains ([8]).

LEMMA 1. — *Let Ω be a bounded, strictly pseudoconvex domain with real analytic smooth boundary in $\mathbf{C}^n$, and let $(f_1, \dots f_m)$ be a system of analytic functions defined in a neighbourhood of $\overline{\Omega}$.*

Then the ideal $(f_1, \ldots, f_m)\mathcal{E}(\overline{\Omega})$ *is closed in the Fréchet topology of* $\mathcal{E}(\overline{\Omega})$ *and*

$$A^\infty(\Omega) \cap (f_1, \ldots, f_m)\mathcal{E}(\overline{\Omega}) = (f_1, \ldots, f_m)A^\infty(\Omega). \tag{1}$$

A proof of Lemma 1 can be deduced from Nagel's note [12]. More precisely, the closeness of the ideal $(f_1, \ldots, f_m)\mathcal{E}(\overline{\Omega})$ is a consequence of [12]-Theorem 4.8, while the algebraic relation (1) is implied by [12]-Theorem 3.2.

Since $A^\infty(\Omega)$ is a closed subspace of $\mathcal{E}(\overline{\Omega})$, Lemma 1 shows in particular that the ideal $(f_1, \ldots, f_m)A^\infty(\Omega)$ is closed in $A^\infty(\Omega)$.

Proof of Theorem 1. — Let Ω be a bounded, strictly pseudoconvex domain with smooth real analytic boundary in $\mathbf{C}^n$ and let l be a continuous A-functional on $A^\infty(\Omega)$ which vanishes on the ideal $I \subset \mathcal{O}(\overline{\Omega})$, and suppose that $V(I) \cap \overline{\Omega}$ is contained in a locally peak set $E \subset \partial\Omega$ for the algebra $A^\infty(\Omega)$.

Then by the Theorem of Frisch [6] the ideal I is finitely generated. Let $f = (f_1, \ldots, f_m)$ be a system of generators of I. Since the boundary of Ω is smooth and strictly pseudoconvex, the functions $f_1, \ldots, f_m$ are defined in a neighbourhood of $\overline{\Omega}$, as in Lemma 1.

Lemma 1 implies that the two spaces in the following diagram are Fréchet and the map i induced by the natural inclusion is a strict morphism of Fréchet spaces (i.e. one to one with closed range in this case) :

$$i : A^\infty(\Omega)/(f_1, \ldots, f_m)A^\infty(\Omega) \longrightarrow \mathcal{E}(\overline{\Omega})/(f_1, \ldots, f_m)\mathcal{E}(\overline{\Omega}).$$

By Hahn-Banach Theorem we can therefore extend the functional l to a continuous linear functional on the codomain of i. By lifting, this functional is represented by a distribution $u \in \mathcal{E}(\overline{\Omega})'$. By Whitney extension theorem ([16] Théorème IV.2.2) u is a distribution in $\mathbf{C}^n$ with support in $\overline{\Omega}$. Since u annihilates the space $(f_1, \ldots, f_m)\mathcal{E}(\overline{\Omega})$, the support of u is in fact included in $V(I) \cap \overline{\Omega} \subset E$. Let d denote the order of the distribution u.

Let $h_1, \ldots, h_p$ be a finite system of peak functions of the algebra $A^\infty(\Omega)$, with the property that, for any point $x \in E$ there is at least one function $h_j, 1 \leq j \leq p$, which is equal to one at x. Let us denote by g_r the following sequence of elements of $A^\infty(\Omega)$:

$$g_r = ((1 - {h_1}^r) \ldots (1 - {h_p}^r))^{d+1}.$$

The set $V(I) \cap \partial\Omega$ is real analytic hence regular in the sense of Whitney ([18] Corollaire VI.1.7). Thus the distribution $g_r u$ vanishes for any positive integer r. (See [15] Section III.9 Théorème XXXIV.)

On the other hand :

$$\sup_{z \in \Omega} |g_r(z)| \leq 2^{p(d+1)},$$

whence the sequence g_r is uniformly bounded on $\overline{\Omega}$. Moreover,

$$\lim_{r \to \infty} g_r(z) = 1,$$

for any point $z \in \Omega$.

Let ϕ be an arbitrary element of $A^\infty(\Omega)$. Then the sequence $g_r\phi$ is uniformly bounded on $\overline{\Omega}$ and it converges pointwisely in Ω to ϕ. Because l is an A-functional we obtain :

$$l(\phi) = \lim_{r \to \infty} l(g_r \phi).$$

But for each positive r we have :

$$l(g_r \phi) = u(g_r \phi) = 0,$$

therefore the functional l is equal to zero on the whole space $A^\infty(\Omega)$. This finishes the proof of Theorem 1.

Proof of Corollary 1. — Let Ω be a domain and μ a measure as in the statement of Corollary 1. Fix the real number p in the interval $[1, \infty)$. Let q denote the real number which satisfies $\dfrac{1}{p} + \dfrac{1}{q} = 1$.

By keeping the notations of Corollary 1, let $k \in L^q(\mu)$ be a function which represents a continuous functional on $A^p(\mu)$ which vanishes on the ideal I. We will adapt the proof of Theorem 1 to this particular situation.

First we remark that the multiplication with k is a linear continuous operator between $A^\infty(\Omega)$ and $L^q(\mu)$. Then with the notations of the previous proof we have to compute the following limit

$$L = \lim_{r \to \infty} \int_{\overline{\Omega}} g_r \phi k \, d\mu.$$

The sequence of functions $g_r\phi k$ is pointwisely bounded by a multiple of k and it converges pointwisely to $\chi_A \phi k$, where $A = \{x \in \overline{\Omega};\ g_1(x) = 0\}$ is a locally peak set of $A^\infty(\Omega)$. By Chaumat-Chollet theorem and the assumption of Corollary 1 we obtain $\mu(A) = 0$. Thus by Lebesgue dominated convergence theorem the limit L is zero and the proof of Corollary 1 is finished.

Proof of Corollary 2. — First we assume that the ideal I is reduced and it defines a smooth complex submanifold $V(I)$ of an open neighbourhood U of $\overline{\Omega}$. By assumption the set $V(I)$ is disjoint of Ω. We will prove that in that case $V(I) \cap \partial\Omega$ is a locally peak set for the algebra $A^\infty(\Omega)$, so Theorem 1 applies.

Fix a point $x \in V(I) \cap \partial\Omega$. After a linear change of coordinates we can assume locally that $x = 0$, $V(I)$ is a linear variety and the defining function ρ of the domain Ω satisfies :

$$\partial\rho/\partial x_1(0) = 1, \qquad \partial\rho/\partial y_1(0) = \partial\rho/\partial x_j(0) = \partial\rho/\partial y_j(0) = 0.$$

Above $z_k = x_k + iy_k, k \in [1, n]$, are the new complex coordinates in a neighbourhood of zero.

We denote for simplicity $w = z_1, u = x_1, v = y_1, \zeta_{j-1} = z_j, j \in [2, n]$ and $\zeta = (\zeta_1, \dots, \zeta_{n-1})$. We put $s_k = \mathrm{Re}(\zeta_k)$ and $t_k = \mathrm{Im}(\zeta_k), k \in [1, n-1]$. We work in a domain $D \times \Delta$, where $w \in D \subset \mathbf{C}$ and $\zeta \in \Delta \subset \mathbf{C}^{n-1}$ are balls centered at zero in the respective spaces. Since the linear variety $V(I) \cap (D \times \Delta)$ does not intersect Ω, it is included in the real tangent hyperplane $T_{(0,0)}(\partial\Omega)$, hence it is included in the complex hyperplane $W = \{(w, \zeta) \in D \times \Delta; w = 0\}$.

From now on we closely follow a (standard) construction presented in the proof of Proposition 9 in [4]. The Jacobian of the pair of functions (ρ, v) is obviously non-degenerated at (0,0), therefore by the Implicit Function Theorem, after possibly shrinking the balls D and Δ there is a smooth function $h : \Delta \longrightarrow D$ such that :

$$M = \{(w, \zeta) \in D \times \Delta; \rho(w, \zeta) = v = 0\} = \{(h(\zeta), \zeta); \zeta \in \Delta\}$$

is a smooth real submanifold of $\partial\Omega \cap (D \times \Delta)$ which contains $V(I) \cap \partial\Omega \cap (D \times \Delta)$. We have to find a smooth totally real submanifold of M, which still contains $V(I) \cap \partial\Omega$ and whose tangent plane is contained in the complex tangent plane of $\partial\Omega$ at the points of intersection $V(I) \cap \partial\Omega$.

The function $\theta(\zeta) = \rho(0, \zeta)$ is strictly plurisubharmonic and non-negative in the ball Δ, whence by the main result of [7], there is, after possibly shrinking Δ to a smaller neighbourhood of zero, a totally real smooth submanifold N' of Δ which contains the zeros of $\theta(\zeta)$. Moreover it is known from [7] that after a complex linear change of coordinates in the ζ-space one has :

$$N' = \{\zeta \in \Delta; (\partial\theta/\partial s_k)(\zeta) = 0, 1 \leq k \leq n-1\}.$$

It remains to remark that the smooth submanifold of $\partial\Omega$ defined by :

$$N = \{(h(\zeta), \zeta); \zeta \in N'\}$$

is totally real and at the points z of $W \cap \partial\Omega$ one has $T_z \subset T^c{}_z(\partial\Omega)$ because W is a complex linear variety. This finishes the first part of the proof of Corollary 2.

Assume now that $I \subset \mathcal{O}(\overline{\Omega})$ is an ideal of the form $I = I_1 \ldots I_m$, where each factor is the reduced ideal of a smooth complex manifold which is disjoint of Ω. Then a simple recurrence based on the first part of the proof shows that I is dense in $A^p(\mu)$. Finally if $J \subset \mathcal{O}(\overline{\Omega})$ is an ideal whose zero locus is contained in the zero locus of the ideal I above, since we are working on a Stein compact set, there is by the Nullstellensatz Theorem an integer N such that $I^N \subset J$. In conclusion the ideal J is dense in the space $A^p(\mu)$. This finishes the proof of Corollary 2.

Remarks. — a) We have reproduced from [4] the quite involved local construction in the proof of Corollary 2 in order to guarantee the smoothness of the totally real variety which contains locally $V(I) \cap \partial\Omega$. Equivalently, one can argue as follows : by [7], the zeroes of the function $\rho|V(I)$ are locally contained in a totally real submanifold N' of $V(I)$. Then one projects N' onto $\partial\Omega$ in the direction given by grad(ρ) and we have to check that this projection is a smooth, totally real submanifold N of $\partial\Omega$. However, the analysis of this latter assertion in local coordinates is similar to the first part of the above proof of Corollary 2.

b) The real analyticity of the boundary of the domain Ω was needed twice in the proof of Theorem 1. First in Lemma 1 and second for the fact that the set $V(I) \cap \partial\Omega$ is regular in the sense of Whitney. By means of the more general results of Bierstone and Milman [2] (see also [1] and the references in [2]) one can relax a part of the conditions in Theorem 1. More precisely, let Ω be a bounded strictly pseudoconvex domain in $\mathbf{C}^n$ with smooth boundary and let I be a finitely generated ideal of the algebra $\mathcal{O}(\overline{\Omega})$. It is known by the general theory developed in [2] that the ideal I induces a stratification $\Sigma_n \subset \Sigma_{n-1} \subset \ldots \subset \Sigma_0$ of a neighbourhood Σ_0 of $\overline{\Omega}$ with complex analytic spaces. (More exactly, the stratification is induced by the invariant diagram of initial exponents of the localizations of the analytic module I).

If all strata $\Sigma_k, 0 \leq k \leq n$, are regularly situated with respect to $\overline{\Omega}$, then Theorems 1.2 and 1.5 of [2] show that Lemma 1 and Theorem 1 are still valid. (In particular Theorem 1 preserves exactly the same proofs.)

c) We remark at the end that the characterization of (locally) peak sets on more general classes of domains, such as the weakly pseudoconvex domains, is more subtle. (See [16] for references.)

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