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Superdiffusive bounds on self-repellent precesses in $d = 2$ — extended abstract

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Abstract

We prove superdiffusivity with multiplicative logarithmic corrections for a class of models of random walks and diffusions with long memory. The family of models includes the "true" (or "myopic") self-avoiding random walk, self-repelling Durrett-Rogers polymer model and diffusion in the curl-field of (mollified) massless free Gaussian field in 2D. We adapt methods developed in the context of bulk diffusion of ASEP by Landim-Quastel-Salmhofer-Yau (2004).

We study the long time asymptotics of the *self repelling Brownian polymer process (SRBP)* in $\mathbb{R}^d$ defined by the SDE

$$dX_t = dB_t\big(\eta_0 - \operatorname{grad} V * l_t\big)(X(t))dt, \tag{0.1}$$

where B_t is standard Brownian motion in $\mathbb{R}^d$, $\eta_0 : \mathbb{R}^d \to \mathbb{R}^d$ is a gradient vector field with sufficient regularity,

$$l_t(A) := |\,\{s \in [0,t] : X_s \in A\}\,|$$

is the occupation time measure of the process X_t and $V : \mathbb{R}^d \to [0,\infty)$ is a C^∞, spherically symmetric approximate identity with sufficiently fast decay at infinity. It is assumed that $V(\cdot)$ is positive definite:

$$\hat{V}(p) = \int_{\mathbb{R}^2} e^{ip\cdot x} V(x) dx \geq 0.$$

The process is pushed by the negative gradient of its own local time, mollified by convoluting with V. The process was introduced by Durrett and Rogers in [2] and further investigated in a series of probability papers. For a survey and complete list of references see [6]. It is phenomenologically similar to the so-called *true self-avoiding random walk (TSAW)* which arose in the physics literature initiated by Amit, Parisi and Peliti in [1] and further investigated in a series of physics papers. For a survey and complete list of references see [10].

Conjectures based on scaling and renormalization group arguments regarding this family of models are the following (see e.g. [1]):

– In $d = 1$: $X(t) \sim t^{2/3}$ with intricate, non-Gausssian scaling limit.
– In $d = 2$: $X(t) \sim t^{1/2}(\log t)^{1/4}$ and Gaussian (that is Wiener) scaling limit expected.
– In $d \geq 3$: $X(t) \sim t^{1/2}$ with Gaussian (i.e. Wiener) scaling limit expected.

Some if these conjectures had been proven or at least partially settled. For results in $d = 1$ see [9], [13], [12], [8] and the survey [10]. For results in $d \geq 3$ see [3]. The present lecture concentrates on recent results on the $d = 2$ case. The complete results and proofs will appear in [11].

The natural framework of formulation of the problem and results is *the environment seen by the random walker.* Let

$$\eta_t(x) := \big(\eta_0 - \operatorname{grad} V * l_t\big)(X(t)).$$

Then $t \mapsto \eta_t$ is a Markov process with continuous sample paths in the function space

$$\Omega = \big\{\omega \in C^\infty(\mathbb{R}^2 \to \mathbb{R}^2) \,:\, \operatorname{rot}\omega \equiv 0, \|\,\omega\,\|_{k,m,r} < \infty\big\}$$

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where $\| \omega \|_{k,m,r}$ are the seminorms

$$\| \omega \|_{k,m,r} = \sup_{x\in\mathbb{R}^d} \left(1+|\,x\,|\right)^{-1/r} \left|\partial^{|\,m\,|}_{m_1,\dots,m_d}\omega_k(x)\right|$$

defined for $k = 1, 2$, multiindices $m = (m_1, \dots, m_d)$, $m_j \geq 0$, and $r \geq 1$.

It was proved in [8] (for $d = 1$) and [3] (for arbitrary d) that the Gaussian measure π on Ω defined by the covariances

$$\int_\Omega \omega_k(x)d\pi(\omega) = 0, \qquad K_{kl}(x-y), \qquad \text{with} \qquad \hat{K}_{kl}(p) = \frac{p_k p_l}{|\,p\,|^2}\hat{V}(p)$$

is stationary and ergodic for the Markov process η_t.

The random vector field ω distributed according to π the gradient of the massless free Gaussian field smeared out by convolution with U, where $U * U = V$.

Let

$$\hat{E}(\lambda) := \int_0^\infty e^{-\lambda t}\mathbf{E}\left(|\,X_t\,|^2\right)dt.$$

The main result of [11], reported in this lecture is the following theorem.

Theorem 1. *Consider the process defined by the stochastic differential equation* (0.1) *in* $\mathbb{R}^2$ *and let the initial vector field* η_0 *be sampled from the stationary distribution* π*. Then there exist constants* $0 < C_1, C_2 < \infty$ *such that for* $0 < \lambda < 1$ *the following bounds hold*

$$C_1 \log|\log\lambda\,| \leq \lambda^2\hat{E}(\lambda) \leq C_2\,|\log\lambda\,|$$

Remarks: (1) Modulo Tauberian inversion, these bounds mean in real time

$$C_3 t\log\log t \leq \mathbf{E}\left(|\,X(t)\,|^2\right) \leq C_4 t\log t,$$

with $0 < C_3, C_4 < \infty$ and for t sufficiently large.
(2) The upper bound is straightforward, it follows directly form estimates on diffusion in random scenery. The superdiffusive lower bound is the main result.

The proof of Theorem 1 follows the main lines of [5]. See also [4]. However on the computational level there are notable differences. For similar recent results referring to second class particle motion in various asymmetric exclusion processes see also [7].

Full proofs are available in [11].

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