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Superdiffusivity for directed polymer in corelated random environment

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Abstract

The directed polymer in random environment models the behavior of a polymer chain in a solution with impurities. It is a particular case of random walk in random environment. In $1+1$ dimensional environment is has been shown by Petermann that this random walk is superdiffusive. We show superdiffusivity properties are reinforced were there are long ranged correlation in the environment and that super diffusivity also occurs in higher dimensions.

The Directed Polymer in Random Environment has been introduced by Henley and Huse [7] (in dimention $1 + 1$) as an effective model for the interface of the 2 dimentional Ising model with impurities. In higher dimension this model can be used to describe wave diffusion in a time dependent random environment or the spatial configuration of a polymer in a solution with impurities. It gives a natural framework to study so-called Anderson localization [1], and for this reason, has been extensively studied by physicists and mathematicians.

We give now the definition of a discrete time, continous space version of the model, introduced by Petermann in his PhD Thesis [12]. Let $(S_n)_{n\geqslant O} \in \mathbb{N}^{\mathbb{R}^d}$ be a discrete time random walk in $\mathbb{R}^d$ with i.i.d. standard gaussian increments ($S_n := \sum_{i=1}^n X_i$ where X_i i.i.d. and $X_1 \sim \mathcal{N}(0, I_d)$), P denotes its probaility law. Let $(\omega_{i,x})_{i,x\in\mathbb{N}\times\mathbb{R}^d}$ be a fixed realization of a translation invariant Gaussian field (law denoted by $\mathbf{P}$) with covariance function $\mathbf{E}(\omega_{i,x}, \omega_{i',x'}) = \mathbf{1}_{i=i'}Q(x - x')$, where $Q(0) = 1$, Q non-negative and $Q(x) \to 0$ when $|x| \to \infty$.

We define $\mu_N^{\beta,\omega}$ the polymer measure for the system of size N, for inverse temperature β and environment

$$\frac{\mathrm{d}\mu_N^{\beta,\omega}}{\mathrm{d}P}(S) := \frac{1}{Z_N^{\beta,\omega}} e^{\beta\sum_{n=1}^N \omega_{n,S_n}}, \tag{0.1}$$

where

$$Z_N^{\beta,\omega} := E\left[e^{\beta\sum_{n=1}^N \omega_{n,S_n}}\right]. \tag{0.2}$$

The aim of the sudy of the model is to describe the properties of $(S_n)_{n\in[1,N]}$ under measure $\mu_N^{\beta,\omega}$ for large N. In particular one wants to know the scaling of S_N under μ_N. One has several options

(i) The influence of disorder is too weak to change the trajectorial property of the random walk S, it has diffusive scaling and satisfies invariance principle (convergence in law to Brownian Motion after rescaling).

(ii) The presence of disorder modifies the properies of S crucially: in order to find more favorable environment, under $\mu_N^{\beta,\omega}$, S_N tends to move away from zero at superdiffusive rate ($|S_N| \approx N^\xi$ with $\xi \geqslant 0$).

Whether one find oneself in case (i) or (ii) may depend on β, d and Q. This issue has first been studied in a discrete-space version of this model, with environment given by i.i.d. gaussians but results proved in this setup can be proved to hold in the case of compactly suported Q. The

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conclusion of a several works (among which [8, 2, 6]) is that for low β and $d \geqslant 3$ we have diffusivity. On the other-hand it was proved that for $d = 1, 2$ and any β or for $d \geqslant 3$ and β sufficiently large, significant localization phenomena occur [3, 4, 5, 9].

The model we present here was introduced by Petermann to study superdiffusivity issues [12]. He succeded to prove that the typical value of $\max_{n\in[0,N]} |S_n|$ under the polymer measure $\mu_N^{\beta,\omega}$ is of order at least $N^{3/5-\varepsilon}$ for $d = 1$ and compactly supported Q. Méjane proved later that $\max_{n\in[0,N]} |S_n|$ was at most of order $N^{3/4}$ in any dimention d for any Q. Results of a similar type had been proved earlier by Wütrich in the case of Brownian Motion in Poissonian obstacles. Physicists predicts that for $d = 1$, $\max_{n\in[0,N]} |S_n| \approx N^{2/3}$.

We have been investigating how this picture breaks when there is non-vanishing correlation in the environment. We focused on the case $Q(x) \approx |x|^{-\theta}$ where $\theta > 0$ is a fixed parameter. The conclusions of this work [10] is that when $d \geqslant 3$, the high temperature (i.e. small β) behavior is drastically modified, when $\theta < 2$ whereas invariance principle still holds for $\theta > 2$. In dimension 2 we also get superdiffusivity when $\theta < 2$ and we get it for all θ in dimension 1.

The result we get does not depend on the dimension but only on θ

Theorem 0.1. *When $d \geqslant 2$ and $\theta < 2$ or $d = 1$ and $\theta < 1$, we have*

$$\lim_{\varepsilon\to 0} \liminf_{t\to\infty} \mathbf{P}\mu_t^{\beta,\omega}\left\{ \sup_{0\leqslant s\leqslant t} \|B_s\| \geqslant \varepsilon t^{\frac{3}{4+\theta}} \right\} = 1.$$

For $d = 1$, $Q \in \mathbb{L}_1(\mathbb{R})$ we have

$$\lim_{\varepsilon\to 0} \liminf_{t\to\infty} \mathbf{P}\mu_t^{\beta,\omega}\left\{ \sup_{0\leqslant s\leqslant t} \|B_s\| \geqslant \varepsilon t^{\frac{3}{5}} \right\} = 1.$$

In particular, for small θ, we get that $\max_{n\in[0,N]} |S_n| >> N^{2/3}$ showing that presence of correlation can change the universal behavior of the model.

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