G. V. CHUDNOVSKY

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CONSTRUCTIVE AND NON-CONSTRUCTIVE METHODS OF PROOFS OF IRRATIONALITY AND TRANSCENDENCY AND ALGEBRAIC INDEPENDENCE OF PERIODS OF ABELIAN VARIETIES.

by G. V. Chudnovsky

The methods announced in the title are based on the deformation theory for Fuchsian linear differential equations. They include effective constructions of the Padé approximations to generalized hypergeometric functions giving the periods of certain algebraic varieties, of the Padé approximations to systems of Abelian integrals of the first and second kind, and of the Padé approximations to systems of Abelian functions on certain Abelian varieties. We shall describe the arithmetic structure of the coefficients of the polynomials in the Padé approximations, their sizes and the remainder term.

Example 1 : Let k be an algebraic number such that $0<|k|<1$, and consider the equation : $y^2=(1-x^2)(1-k^2x^2)$. If $|k|^2<(H(k^2))^{-\delta}$ for some positive δ, then

$$|x\pi+y\omega+z\eta|>H^{-C}$$

where $C=C(\delta)$.

If now $k^2=1/q$ and $q\geq q_0(\varepsilon)$, then

$$|x\pi+y\omega+z\eta|>H^{-3-\varepsilon}.$$

In these formulae, x, y, z are integers, $0<\max(|x|,|y|,|z|)=H$ and ω and η are respectively the period and quasi period of the corresponding elliptic curve.

The same type of result can be proved for n elliptic curves.

Example 2 : If $E: y^2 = 4x^3 - g_2x - g_3$ is defined over $\mathbb{Q}$ and $\wp(u) = 1/q$ for $q \geq q_0(\varepsilon)$, then

$$|u\wp'(u) - r/s| > |s|^{-2-\varepsilon}.$$

We obtain similar results, under much milder restrictions, for $\zeta(u) + \alpha u$, $\alpha \in \mathbb{Q}$, as well.

These are the most elementary examples. It should be noted that we can compute all the constants explicitly.

We shall also present results on algebraic independence for sets of numbers connected with the exponential function and Abelian functions. We propose new bounds for the types of transcendence of two numbers, as η/ω and $\zeta(u) - (\eta/\omega)u$, the algebraic independence of which was earlier proved by the author . Finally, we consider an elliptic function $\wp(z)$ with complex multiplication over K, and obtain the bound

$$|P(\wp(\alpha),\wp(\beta))| > H(P)^{-C_2 d(P)^2}$$

where $\log\log H(P) \geq d(P)^4$ and $C_2 = C_2([K(\alpha,\beta):\mathbb{Q}]) > 0$.

Institut des Hautes Etudes Scientifiques
35, route de Chartres
91440 Bures-sur-Yvette
(France)