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REMARKS ON RANDOM WALKS ON SEMI SIMPLE LIE GROUPS

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The purpose of this note is to discuss two problems concerning random walks and harmonic functions on Lie groups, and there-by to complete and enlarge upon the results of A.RAUGI announced in [6] and proved in detail in [7] . Both problems have their origin in the work of H.FURSTENBERG [2,3] as further extended by R.AZENCOTT [1] and A.VIRTSER [8] . For the sake of convenience in notation and bookkeeping, the results of this paper are stated for connected semi simple groups , which are clearly the most important case , but one could easily extend them as in [7] to general almost connected Lie groups (those with only finitely many connected components) .

1. μ-POSITIVE AND μ-NEGATIVE COCYCLES

1.1.- Let G be a connected semisimple Lie group with finite center and with Iwasawa decomposition KAN . Let $\mathfrak{a}$ denote the Lie algebra of A , let $\mathfrak{a}^*$ be its real dual , and let $\mathfrak{a}^*_{\mathbb{C}}$ be the complexification of $\mathfrak{a}^*$. In the discussion that follows , the term <u>root</u> will always refer to an element of $\mathfrak{a}^*$ which is a restricted root of the pair (G , A) . The <u>Weyl group</u> is the finite group W of automorphisms of $\mathfrak{a}$ or $\mathfrak{a}^*$ induced by the conjugation action of the normalizer of A in K ; W permutes the roots . An ordering of the roots is canonically determined by the choice of N . For $x \in G$, write $x = kan$ with $k \in K$, $a \in A$, and $n \in N$, and let $H(x) = \log (a) \in \mathfrak{a}$. Then the functions ϕ on $G\times K$ of the form

$$(1) \qquad \phi(x , k) = \lambda(H(xk)) ,$$

where $\lambda \in \mathfrak{a}^*$, $x \in G$, and $k \in K$, are called <u>A-cocycles</u> (on K) . (See [3 , §6] -the present definition is not quite the same as Furstenberg's but is essentially equivalent) . Every such function is associated with an elementary spherical function Ψ on G defined by

$$(2) \qquad \Psi(x) = \int_K \exp (\phi(x , k)) \, dk .$$

(Here dk denotes normalized Haar measure on the compact group K). In fact , all elementary spherical functions arise this way if we allow λ to be chosen from $\mathfrak{a}^*_{\mathbb{C}}$. For further discussion of spherical functions (about which there is an exten-

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sive literature) , the reader can consult [4] .

Now suppose μ is a probability measure on G . If f is a function on G , $x \in G$, and X_1 , X_2 , ... are independant G-valued random variables each with distribution μ , it is of interest to determine the asymptotic behavior of $f(X_n \dots X_2X_1x)$ as $n \longrightarrow \infty$. This , of course , is the subject of "laws of large numbers" and "central limit theorems" for the left μ-walk on G . (One could analogously consider $f(xX_1X_2 \dots X_n)$ and the right μ-walk on G) . The function f is at our disposal here , and it is convenient to have f related to the group-theoretic structure at hand . Furstenberg showed in [3 , §7] that if f is of the form $\phi(.\,,k)$ with ϕ an A-cocycle as above and $k \in K$, then $n^{-1}f(X_n \dots X_2X_1x)$ converges with probability 1 to a limit $a_\mu(\phi)$ independent of x and k , provided that μ is regular enough . (Furstenberg takes μ of class B_1-absolutely continuous with a first moment-but it is clear from [1] that absolute continuity of μ can be weakened to the condition that μ be "étalé") . What is remarkable is that for certain choices of ϕ , this limit is strictly positive (or strictly negative) for all choices of μ . This was first shown by Furstenberg [3 , Theorem 7.6] under the additional restriction that μ be of class B_∞ (determined by a bounded L^1 function of compact support) and then extended by RAUGI [7 , Cor. (7.15)] to the conclusion that $a_\mu(\phi) > 0$ for all étalé μ with a first moment , whenever the $\lambda \in \mathfrak{a}^*$ that determines ϕ is a positive root . (The apparent change of sign from RAUGI's statement is due to the fact that RAUGI puts K on the <u>right</u> in the Iwasawa decomposition , whereas we are following Furstenberg and putting K on the <u>left</u>) . The purpose of this section is to point out that , with μ restricted to being of class B_∞ , one can deduce this fact directly from [3 , Theorem 7.4] and elementary facts about semisimple groups . This provides another proof of the conjecture on p. 417 of [3] .

1.2. <u>Proposition</u>. - *With notation as above , if* $\lambda \in \mathfrak{a}^*$ *is a positive root and* ϕ *is the* A-*cocycle defined by (1) , then* $a_\mu(\phi) > 0$ *for all probability measures* μ *of class* B_∞ .

<u>Proof</u>. - By [3 , Theorem 7.4] , we will have $a_\mu(\phi) < 0$ whenever the elementary spherical function Ψ defined by (2) satisfies $\|\Psi\|_{L^\infty(G)} \leq 1$. By the easy direction of the Helgason-Johnson Theorem [4 , p. 66] , this will be the case if $\lambda = i\nu - \rho$ with ρ half the sum of the positive roots and $\nu \in \mathfrak{a}^* + iC_\rho$, where C_ρ is the (closed) convex hull of the orbit of ρ under the Weyl group W . This is equivalent (since λ is real-valued) to having $\lambda \in -(\rho + C_\rho)$. But if $s_0 \in W$ is the element taking every positive root to a negative root , if α is a

simple root ("with multiplicity"), and if $s_\alpha \in W$ is the reflection in the hyperplane normal to α, then $s_\alpha s_0 \rho = s_\alpha(-\rho) = \alpha - \rho$, and thus $\alpha = \rho + (\alpha - \rho) \in \rho + C_\rho$. This shows that each A-cocycle associated to the negative of a simple root is μ-negative for all μ of class B_∞. Since the μ-positive and μ-negative cocycles constitute convex cones which are negatives of one another, and since each positive root is a sum of simple roots (possibly with repetitions), the conclusion of the proposition follows.

1.3. <u>Remark</u>. - It is interesting to observe that the convex cone generated by $\rho + C_\rho$ is exactly the same as that generated by the positive roots (since for $s \in W$, $\rho + s\rho = \rho + (ss_0)(s_0\rho) = \rho - (ss_0)\rho$ is a sum of positive roots by dominance of ρ). Furthermore, the Helgason-Johnson Theorem also shows that $\|\Psi\|_{L^\infty(G)} \leqslant 1$ <u>only</u> when $\lambda \in i\mathfrak{a}^* - (\rho + C_\rho)$. Thus the present method produces no more cocycles positive for all μ than does RAUGI's. If G is of real rank 1, every non zero A-cocyle is either μ-positive for all μ or μ-negative for all μ, since every element of $\mathfrak{a}^*$ is a scalar multiple of a root. But when the real rank of G is > 1, it is possible for a $\lambda \in \mathfrak{a}^*$ to be neither in the cone C^+ generated by the positive roots nor in the cone C^- generated by the negative roots. It seems likely that in this case the corresponding A-cocycle ϕ would be μ-positive for certain μ and μ-negative for certain other μ.

When μ is the class B_∞ probability measure defined by a K-biinvariant positive continuous function g of total mass 1, it is easy to chech that

$$a_\mu(\phi) = \int_G g(x) \int_K \phi(x, k)\,dk\,dx$$

$$= \int_{\mathfrak{a}^+} g(\exp X)\,\lambda(T(X))\,\Delta(X)\,dx ,$$

where $\mathfrak{a}^+ = \{X \in \mathfrak{a} : \alpha(X) > 0$ for all positive roots $\alpha\}$, Δ is a certain positive continuous function on $\mathfrak{a}^+$, dx is Lebesgue measure, and $T : \mathfrak{a} \longrightarrow \mathfrak{a}$ is given by

$$\text{(3)} \qquad T(X) = \int_K H((\exp X)k)\,dk.$$

Since $g|_{\mathfrak{a}^+}$ can have support in an arbitrarily small neighborhood of a given point $X \in \mathfrak{a}^+$, Proposition 1.2. implies that $\alpha(T(X)) > 0$ for all positive roots α. In other words, we have the following (purely group-theoretic) result :

1.4. <u>Corollary.</u> - *With* T *defined as in (3),* $T(\mathfrak{a}^+) \subseteq \mathfrak{a}^+$. *If one could show that in fact* $T(\mathfrak{a}^+) = \mathfrak{a}^+$, *then for* $\lambda \notin (C^+ \cup C^-)$, $\lambda \circ T$ *would change sign in* $\mathfrak{a}^+$, *and one could choose* g *so that* $a_\mu(\phi)$ *was either positive or negative.*

2. SEMISIMPLE GROUPS WITH INFINITE CENTER

2.1. The purpose of this section is to examine random walks on semisimple Lie groups with infinite center, and in particular to extend some of the results of [8]. Our principal result, Theorem 2.10, is exactly what is needed to make it possible to drop the condition on the center of G_o/R in the proof of theorem (8.4) of [7]. Although semisimple groups with infinite center may not often arise in practice,, it is nice to have a unified treatment of random walks and harmonic functions valid for all connected Lie groups (without side conditions on the center).

Let G be a connected semisimple Lie group with center Z, let μ be an étalé probability measure on G, and let π_μ be the Poisson space of μ. Then if Z is infinite, it may happen [5] that G is not transitive on π_μ. More precisely, let T_μ be the closed semigroup generated by the support of μ and let S_μ be the open semigroup of [1, Définition IV.2]. Then by [1, Proposition IV.5], G is transitive on π_μ if and only if $[Z : Z \cap S_\mu S_\mu^{-1}] < \infty$. Since π_μ depends only on T_μ by [1, Théorème II.4], one expects to be able to phrase this condition in terms of T_μ, and in fact, $Z \cap S_\mu S_\mu^{-1} = Z \cap T_\mu T_\mu^{-1}$:

2.2. Lemma. - *Let* G *be a group with center* Z, *and let* S *and* T *be non-empty sub-semigroups of* G *satisfying* $S \subseteq T$ *and* $TS \subseteq S$. *Then* $Z \cap SS^{-1} = Z \cap TT^{-1}$

Proof. - Since $S \subseteq T$, $Z \cap SS^{-1} \subseteq Z \cap TT^{-1}$. But if $z \in Z \cap TT^{-1}$, we can write $z = t_1 t_2^{-1}$ with $t_1, t_2 \in T$. Let $s \in S$. Then $z = (t_1 s)^{-1} z (t_1 s)$ (since z is central) $= s^{-1} t_1^{-1} t_1 t_2^{-1} t_1 s = (t_2 s)^{-1} (t_1 s) \in (TS)^{-1}(TS) \subseteq S^{-1}S$, so $Z \cap TT^{-1} \subseteq Z \cap S^{-1}S = Z \cap SS^{-1}$.

Corollary. - *In the setting of* 2.1 *above*, $Z \cap S_\mu S_\mu^{-1} = Z \cap T_\mu T_\mu^{-1}$.

Proof. - By [1, p. 76], we can take $S = S_\mu$ and $T = T_\mu$ in the lemma.

2.3.- Let KAN be an Iwasawa decomposition of G and let M be the centralizer of A in K. We begin by analyzing the asymptotic behavior of the left μ-walk on $G/AN \cong K$ (equivalently, we could deal with the right μ-walk on $AN\backslash G$) in the case where G is transitive on π_μ. The results are basically as in the finite center case, except for the fact that since K is no longer assumed compact, it is necessary to consider infinite measures.

2.4. Lemma. - *Let* $G = KAN$ *and* M *be as in* 2.3 , *and let* M_1 *be an open subgroup of finite index in M. Then MAN leaves fixed on* $G/AN \cong K$ *exactly one Radon measure* m *(up to scalar multiples) , namely , the image in* K *of a Haar measure on* M . M_1AN *leaves fixed on* $G/AN \cong K$ *exactly (up to scalar multiples) the convex combinations of the restrictions of this measure to the right* M_1*-cosets in* M . *In particular , the cone of* M_1AN*-invariant (positive) Radon measures on* G/AN *is generated by its extremal rays , and there are exactly* $[M : M_1]$ *of these* .

Proof. - $G/ZAN = K/Z$ is compact , so every Radon measure on it is a multiple of a probability measure . By [3 , Theorem 2.6] , there is exactly one probability measure λ on G/ZAN which is MAN/Z-invariant , and λ is obviously normalized Haar measure on the image of M/Z .

Let m be an MAN-invariant Radon measure on $G/AN \cong K$. Since $Z \subseteq M$, m is Z-invariant and projects to an MAN/Z-invariant measure on K/M . Hence m is of the form

$$\int_K f\, dm = \int_K \int_Z f(zk)dz\, d\lambda(\dot{k})$$

for some Haar measure dz on Z , and this proves the first assertion .

Now if $\gamma \in M$, $(M_1AN)(M_1\gamma AN) = AN(M_1\gamma AN)$ (since M_1 normalizes AN) = $M_1\gamma AN$ (since $M_1\gamma$ normalizes AN) , so the restrictions of m to right M_1-cosets in M (or rather , their images in G/AN) are M_1AN-invariant . Conversely , suppose σ is an M_1AN-invariant measure on G/AN . Let $r = [M : M_1]$ and choose representatives $\gamma_1 , \dots , \gamma_r$ for M/M_1 in M. Then $\Sigma\gamma_i \cdot \sigma$ is MAN-invariant , hence is a multiple of m , and is therefore supported on MAN . It follows that σ is supported on MAN , or on M if we identify G/AN with K . But an M_1-invariant measure on M is a sum of translates of Haar measure on M_1 . This proves the lemma .

2.5. Lemma . - *Let* $G = KAN$ *and* M *be as in* 2.3 . *Let* μ *be an étalé probability measure on* G *such that* G *is transitive on* π_μ . *We take* $\pi_\mu = G/M_\mu AN$ *with* M_μ *an open subgroup of* M ; *since* π_μ *is compact* , $[M : M_\mu] < \infty$. *Let* ν *denote the Poisson kernel for* μ *on* π_μ , *and let* X *be a locally compact (left) G-space* . *Then if* λ *is an* $M_\mu AN$*-invariant Radon measure on* X , *we can define a* μ*-stationary measure* $\nu * \lambda$ *on* X *by*

$$\int_X f\, d(\nu * \lambda) = \int_X \int_{\pi_\mu} f(xy)\, d\nu(\dot{x})\, d\lambda(y)$$

(the integral converges since ν *has compact support). Conversely , every* μ*-stationary Radon measure on* X *is of the form* $\nu * \lambda$ *with* λ *as above* .

Proof. - This follows from trivial modifications in the proof of [3 , Lemma 2.1].

2.6. Theorem. - *Assume the hypotheses of* 2.5.. *Then* $G/AN \cong K$ *admits (up to scalar multiples) exactly* $[M : M_\mu]$ *ergodic* μ*-stationary Radon measures , and every* μ*-stationary measure on* K *is (up to scalar multiples) a convex combination of these . With probability* 1 *, any element of* K *enters the support of one of the ergodic measures after finitely many steps of the left* μ*-walk on* G/AN *, and then stays there thereafter .*

Proof. - Note that ergodic μ-stationary measures on K , considered up to scalar multiples , are the same as extremal rays in the cone of μ-stationary measures on K . By Lemmas 2.4 and 2.5 , this cone is generated by its extremal rays , and there are exactly $[M : M_\mu]$ of these . Also , we see that the ergodic measures have disjoint supports whose union E is the inverse image in K (under the natural map $p : K \longrightarrow K/M_\mu$) of the support of ν . Hence , if $k \in K$ and $X_1 , \ldots , X_n , \ldots$ are independent G-valued random variables with distribution μ , $X_n \ldots X_1 kAN$ eventually lands in E if and only if $X_n \ldots X_1 p_1(k)AN$ eventually lands in $p_1(\mathrm{supp}\ \nu)$, where p_1 denotes projection onto K/M . But we know that this last assertion is true either by the ergodic theorem (cf . [3 , p. 397]) or else by the Doeblin condition [8 , Lemma 1] applied to the unique stationary μ-process on the maximal boundary $G/MAN \cong K/M$ of G .

Now we go on to the general case (in which G is not necessarily transitive on π_μ) .

2.7. Lemma. - *Let* G *be a connected semisimple Lie group with (not necessarily finite) center* Z *, and let* S *be a non-empty open sub-semigroup of* G *. Then one can choose an Iwasawa decomposition* KAN *of* G *so that* S *intersects* ZA *and so that the projection of* $S \cap ZA$ *onto* A *contains a set of the form* $C \cap B^c$ *, where* C *is an open subcone of the (open) positive Weyl chamber in* A *and* B^c *is the complement of some relatively compact neighborhood* B *of the identity element* e *.*

Proof. - Since the set of regular elements in G is dense , and since S is non-empty and open , S meets some Cartan subgroup H of G . Choose a Cartan decomposition $\mathfrak{k} + \mathfrak{p}$ of the Lie algebra of G so that $H = H_K \cdot H_{\mathfrak{p}}$, where H_K lies in the connected subgroup K of G with Lie algebra $\mathfrak{k}$, and where $H_{\mathfrak{p}}$ is a vector group contained in $\exp(\mathfrak{p})$. We have $Z \subseteq H_K$, and H_K/Z is compact . (For all this , see [9 , §1.4]) . Hence the projection onto H_K/Z of the image in H/Z of $S \cap H$ is a non-empty sub-semigroup of a compact group , and is therefore [10] a group . In particular , the identity element of H_K/Z lies in the image of $S \cap H$, which means that S meets $ZH_{\mathfrak{p}}$. Now we can choose an Iwasawa decomposition KAN of G with K as above and with $H_{\mathfrak{g}} \subseteq A$, so S meets ZA . Since $S \cap ZA$ is open , it must contain an element of the form za with $z \in Z$ and with a regular (that is , with a contained in one of the open Weyl chambers of A) , and assuming a suitable ordering of the roots (and thus a suitable choice of N) , a **will** be in the positive Weyl chamber . The last statement now follows from the structure of open sub-semigroups of vector groups .

2.8. Lemma. - *Let* G , Z , *and* S *be as in the hypotheses of* 2.7 , *and let* K , A , *and* N *be chosen as in that lemma . Let* M *be the centralizer of* A *in* K , *and let* Z' *be a subgroup of* Z *such that* $Z' \cap S^{-1}S = \{e\}$. *Let* $p : G \longrightarrow G/Z'$ *be the canonical map . Then for any* $m \in M$, p *is injective on* SmAN .

Proof. - Assume this is false . Then for some $m \in M$, $z \neq e$, in Z' , a_1 and a_2 in A , n_1 and n_2 in N , and s_1 and s_2 in S , we have $zs_1ma_1n_1 = s_2ma_2n_2$; or $s_1^{-1}s_2 = (zma_1n_1)(ma_2n_2)^{-1} = zm(a_1n_1n_2^{-1}a_2^{-1})m^{-1}$, and $zan \in S^{-1}S$ for suitable $a \in A$, $n \in N$ (recall M normalizes AN) . Let C and B^c be as in Lemma 2.7 , and choose $h_1 \in C \cap B^c$. Also choose $z_1 \in Z$ with $z_1h_1 \in S$ (this is possible by the conclusion of 2.7) . Then for any integer q , we have $z_1^{-q}h_1^{-q} \in S^{-1}$ and $z_1^{a}h_1^{q} \in S$, so that

$$(z_1^{-q}h_1^{-q})(zan)(z_1^{q}h_1^{q}) = zah_1^{-q}nh_1^{q} \in S^{-1}S .$$

(Note that z, z_1 , and a commute with h_1) . But as $q \longrightarrow \infty$, $h_1^{-q}nh_1^{q} \longrightarrow e$ (cf. [1 , Lemme III.1]) , so $za \in S^{-1}S$. One can now choose $h_2 \in C \cap B^c$ so that $h_3 = ah_2 \in C \cap B^c$, and if z_2 , $z_3 \in Z$ are such that z_2h_2 , $z_3h_3 \in S$, we get $(z_3h_3)^{-1}(za)(z_2h_2) = z_3^{-1}z_2z \in S^{-1}S$. In fact , it is easy to see that we can take $z_2 = z_3$ if h_2 is chosen far enough away from the identity element of A , so $z \in S^{-1}S$. This actually implies that $z \in S^{-1}S$, for we can choose a neighborhood U of e in G and an element $s \in S$ with $Us \subseteq S$ (since S is open) and with $zu \in S^{-1}S$ for some $u \in U^{-1}$, and then $z \in z(s^{-1}U^{-1}uUs) = (Us)^{-1}(zu)(Us) \subseteq S^{-1}S$. But this is a contradiction , since we assumed $z \neq e$ and $Z' \cap S^{-1}S = \{e\}$.

2.9. Theorem. - *Let* G *be a connected semisimple Lie group with possibly infinite center* Z , *and let* μ *be an étalé probability measure on* G . *Then we can choose an Iwasawa decomposition* KAN *of* G *such that with respect to the left* μ*-walk on* $G/AN \cong K$, K *decomposes into a transient set and a countable disjoint union of ergodic invariant sets . With probability* 1 , *any element of* K *enters one of the ergodic sets after finitely many steps of the left* μ *-walk on* G/AN .

Proof. - If $[Z : Z \cap S_\mu S_\mu^{-1}] < \infty$, Theorem 2.6 applies . If not , it follows from the structure theorem for finitely generated abelian groups (as applied to Z) that we can choose a subgroup Z' of Z with $Z' \cap S_\mu S_\mu^{-1} = \{e\}$ and $[Z : Z'.(Z \cap S_\mu S_\mu^{-1})] < \infty$. Apply 2.7 with $S = S_\mu$ to get an Iwasawa decomposition of G , and let M be the centralizer of A in K . The set $S_\mu MAN$ is open and invariant for the left μ-walk on $G/AN \cong K$, and with probability 1 , any element of K enters $S_\mu MAN$ after finitely many steps of the μ-walk on G/AN because of the ergodicity of the μ-walk on G/MAN (the maximal boundary of G). (The μ-walk on G/MAN may be viewed as a walk for the image of μ in G/Z on the same space , so [8 , p.671] and [3] apply .) Hence it is enough to decompose $S_\mu MAN$ into a transient set and countably many ergodic sets . Let $m_1, m_2, \dots$ be a sequence dense in M. Then $S_\mu MAN$ is the (not necessarily disjoint) union of the invariant sets $S_\mu m_i AN$. Let $p : G \longrightarrow G/Z'$ be the canonical map . Then p is clearly equivariant for the μ-walk on G/AN and the $p(\mu)$-walk on $G/Z'AN$, and by 2.8 , p is injective on each $S_\mu m_i AN$. But by construction of Z' , the $p(\mu)$-walk on $G/Z'AN$ satisfies the hypotheses of Theorem 2.6 . Therefore each $S_\mu m_i AN$ decomposes into a transient set and finitely many ergodic sets . Since ergodic sets (up to sets of measure zero) either coincide or are disjoint , we conclude that $S_\mu MAN$ can be decomposed into a transient set (the union of the transient parts of the $S_\mu m_i AN$) and countably many ergodic sets. This proves the theorem .

2.10. Theorem. - *Let* G *be a connected semisimple Lie group with Iwasawa decomposition* KAN *and with possibly infinite center* Z *, and let* μ *be an étalé probability measure on* G *. Then with respect to the left* μ*-walk on* $G/AN = K$, K *decomposes into a transient set and a countable disjoint union of ergodic invariant sets . With probability* 1 *, any element of* K *enters one of the ergodic sets after finitely many steps of the left* μ*-walk on* G/AN .

Proof. - (kindly suggested by A.RAUGI) . The only difference between this theorem and 2.9 is that here our Iwasawa decomposition is specified in advance . Let $G = K_1A_1N_1$ be a decomposition as provided by 2.9 . Then for some $k \in K$, $A_1N_1 = kANk^{-1}$. Let $G/A_1N_1 = T \cup E_i$ be a decomposition of G/A_1N_1 into a transient set T and countably many ergodic sets E_i . Then Tk and the E_ik are evidently invariant for the left μ-walk and give a partition of G/AN ; they are also clearly transient and ergodic , respectively . Finally , if $x \in K$, and if $X_1, X_2, \dots$ are independent G-valued random variables each with distribution μ , then by 2.9 we know that with probability 1 , $X_n \dots X_1(xk^{-1})A_1N_1 \in E_i$ for some i and some n . This says that $X_n \dots X_1 xAN \in E_ik$, so that x enters one of the ergodic sets for the μ-walk on G/AN after finitely many steps .

REFERENCES

1. AZENCOTT (R.) . Espaces de Poisson des groupes localement compacts , Lecture Notes in Math. , no. 148.. Berlin and New York , Springer-Verlag , 1970 .

2. FURSTENBERG (H.) . A Poisson formula for semi-simple Lie groups , Ann. of Math. ((2) , t. 77 , 1963 , p. 335-386 .

3. FURSTENBERG (H.) . Noncommuting random products , Trans. Amer. Math. Soc. , t. 108 , 1963 , 377-428 .

4. GANGOLLI (R.) . Spherical functions on semisimple Lie groups , in Symmetric Spaces , ed. by W. M. Boothby and G. L. Weiss . New York , Marcel Dekker , 1972 .

5. MOORE (C. C.) and ROSENBERG (J.) . Comments on a paper of Brown and Guivarc'h , Ann. scient. Ec. Norm. Sup. (4) , t. 8 , 1975 , p. 379-381 .

6. RAUGI (A.) . Fonctions harmoniques sur les groupes localement compacts à base dénombrable , G. R. Acad. Sci. Paris , t. 282, 1976 , Série A , p. 653-656 .

7. RAUGI (A.) . Fonctions harmoniques sur les groupes localement compacts à base dénombrable , to appear .

8. VIRTSER (A.D.) . Central limit theorem for semisimple Lie groups , Theory of Prob. and Appl. , t. 15 , 1970 , p. 667-687 .

9. WARNER (G.) . Harmonic Analysis on Semi-Simple Lie Groups I . Berlin and New York , Springer-Verlag , 1972 .

10. WRIGHT (F. B.) . Semigroups in compact groups , Proc. Amer. Math. Soc. , t. 7 , 1956 , p. 309-311 .

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