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PRODUCTION-INVENTORY AND EMISSION REDUCTION INVESTMENT DECISION UNDER CARBON CAP-AND-TRADE POLICY[☆]

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Abstract. The increasing amount of carbon emissions has caused global warming and challenged the sustainable development of environment. Governments around the world have implemented carbon policies including carbon cap-and-trade policy. In this paper, we focus on how a two-echelon supply chain manages its carbon footprints in production and inventory under carbon cap-and-trade policy. We extend the classical EOQ (economic order quantity) model and study decisions on production-inventory, carbon trading and emission reduction investment in the decentralized and centralized situations. The results show that emission permit sharing can effectively reduce the total cost and total carbon emissions of the supply chain. Moreover, the manufacturer's emission reduction effort rises with the increase of the buying and selling prices of emission permits under centralized decision-making. In addition, a compensation mechanism is proposed for the centralized supply chain with emission permit sharing. It is observed that the buying and selling prices of emission permits have a positive influence on the permit sharing price in the compensation mechanism. Meanwhile, the retailer pays less for using the emission permits if it has a higher carbon cap, while the manufacturer with a higher carbon cap is more capable to provide a high compensation for the retailer.

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1. Introduction

The increasing amount of carbon emissions has caused global warming and challenged the sustainable development of environment and human beings. Governments around the world have implemented regulations and policies to reduce carbon emissions, including carbon tax, carbon cap, carbon cap-and-trade and carbon offset policies [3].

Carbon cap-and-trade policy combines the government's compulsory regulation and the flexible market mechanism. This policy evolved from Coase's Theory of Property Rights, which determined the emission cap according to the region and emission objective. This theory made emission permits become scarce resources and pushed

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forward carbon trading in the market. In recent years, emission trading markets have been founded worldwide, such as European Union Greenhouse Gas Emission Trading Scheme (EU ETS), UK Emissions Trading Group (ETG), Chicago Climate Exchange (CCX) and National Trust of Australia (NSW).

Meanwhile, the research on carbon cap-and-trade policy has been extensively done in the field of supply chain management. Enterprises try to reduce carbon emissions by optimizing decisions in production, inventory, transportation and other activities. It is explained that the reduction of carbon emissions not only helps to cut down costs but also benefits environment and attracts consumers.

In this paper, we assume that a two-echelon supply chain is composed of a manufacturer and a retailer, and the manufacturer is the leader. The purpose is to optimize decisions on the manufacturer's emission reduction investment, the retailer's order quantity and the quantity of traded permits under carbon cap-and-trade policy. The rest of the paper is organized as follows. In Section 2, related literature is reviewed. In Section 3, we develop the decentralized and centralized decision-making models. Section 4 analyzes the model based on numerical results and proposes a compensation mechanism for emission permit sharing. The conclusions are given in Section 5.

2. Literature review

2.1. Carbon cap-and-trade policy

The research concerning cap-and-trade policy has been conducted from both macro and micro perspectives. Macro researches mainly focus on emission permit allocation and the influence of carbon trading on industries; while micro researches mainly analyze the influence of cap-and-trade policy on enterprises' operational decisions.

Rose and Stevens [27] study initial allocation of emission permits under cap-and-trade policy and find that the method of free allocation brings high profits to large monopoly enterprises in carbon trading, while in the long-term it may decrease the production capacity of enterprises. Based on the literature and historical data, Boemare and Quirion [5] evaluate the performances of emission permit allocation methods and auction mechanisms in European countries. Smale *et al.* [28] analyze the influences of carbon trading on the enterprises' production, carbon emissions and profit. Taking electric power industry as an example, Bode [4] focuses on a multi-period carbon trading problem and finds that the free allocation of emission permits is beneficial to enterprises. Demailly and Quirion [8] study the case of steel industry and analyze the influences of European carbon emission trading mechanism on the production and revenue of steel industry. The analysis shows that carbon trading mechanisms hardly contribute to the competitiveness of steel industry.

Some scholars research on operation management under the cap-and-trade policy, including production, transportation, pricing, emission reduction investment and supply chain coordination and so on. Based on the dynamic Arrow–Karlin model, Dobos [9] compares the optimal production quantities with or without emission trading and analyzes the influence of carbon trading on production decisions. Hoen *et al.* [16] consider the constraint of carbon emissions in transportation mode selection and analyze the influence of cap-and-trade policy. Liu *et al.* [23] take customers' environmental awareness into consideration by assuming a positive correlation between demand and carbon emission reduction. Xu *et al.* [32] develop a production and emission reduction decision model in a make-to-order supply chain under cap-and-trade policy, in which they use the wholesale price and cost sharing contracts to coordinate the supply chain.

2.2. Production and inventory management under carbon policies

In recent years, carbon policies are implemented by more and more governments and organizations. Therefore, environmental factors have been taken into account in the operation of firms [13, 25]. Especially, some scholars consider carbon constraints in inventory models. Classical inventory models including EOQ (economic order quantity) model are extended to study how enterprises manage carbon footprints in production and inventory.

Under the carbon cap-and-trade policy, Hua *et al.* [18] extend the traditional EOQ model considering carbon emissions in ordering, storing and producing and analyze the influences of carbon cap and carbon price on the retailer's order quantity, ordering cost and carbon emissions. Based on the research of Hua *et al.* [18], Chen *et al.* [7] study EOQ model under carbon cap, carbon tax, carbon offset, cap-and-trade policies and observe that adjusting order quantity reduces carbon emissions meanwhile doesn't greatly increase cost. Toptal *et al.* [31] study carbon emission reduction investment under different carbon emission regulations. Some researches take transportation cost into account [21]. Battini *et al.* [2] extend the EOQ model by considering transportation cost, suppliers' location and trucks' utilization efficiency. Some scholars consider customers' preference for low-carbon products. Hovelaque and Bironneau [17] assume that there is a negative correlation between demand and carbon emission coefficient and obtain the optimal order quantity which brings the highest profit and lowest carbon emissions.

Some scholars study production and inventory models with constraints on carbon emissions in multi-echelon supply chains [1, 6, 15] and the joint decision problems are also studied. Zeng *et al.* [35] establish a joint replenishment model under different carbon policies which shows that proper replenishment strategy will increase the cost slightly but reduce carbon emissions effectively. Jaber *et al.* [19] assume production rate is the function about carbon emissions and study the joint economic lot-sizing problem in a two-echelon supply chain. Zanoni *et al.* [34] consider that demand is related to price and emission level in the joint economic lot-sizing model.

2.3. Supply chain coordination under carbon policies

Coordination issues are widely discussed in the field of supply chain management [24, 37]. Kanda and Deshmukh [20] make a comprehensive review on supply chain coordination methods. Du *et al.* [11] study the production decision in the supply chain composed of an emission-dependent manufacturer and an emission permit supplier under cap-and-trade policy and propose coordination mechanisms for the supply chain. Yang *et al.* [33] develop the ordering and pricing model under different carbon policies. They compare the profits of the supply chain under four policies and design quantity discount contracts to coordinate supply chain members, respectively. Toptal and Çetinkaya [30] study the production and inventory problem in a two-echelon supply chain under carbon tax and cap-and-trade policies, in which carbon credit sharing is used to coordinate the supply chain. Considering the demand of products is influenced by carbon emission reduction ratio, Swami and Shah [29] study the pricing and emission reduction investment decisions in decentralized and centralized supply chains, and the supply chain is coordinated through a two-part tariff contract; Zhang *et al.* [36] propose the multi-product newsvendor model to make centralized and decentralized decisions, where a revenue sharing contract is adopted to coordinate the supply chain; Du *et al.* [12] design wholesale-price contract, revenue-sharing contract, quantity-discount contract for the decentralized supply chain to achieve coordination. Considering the carbon emission reduction investment of the manufacturer, Dong *et al.* [10] implement revenue sharing contract, buyback contract, two-part tariff contract and find that only revenue sharing contract can effectively coordinate the supply chain. From the above literature review, we conclude that there is limited research on production and inventory problems under carbon cap-and-trade policy. In addition, carbon emission reduction investment is seldom considered in multi-level supply chains. Table 1 indicates the contributions of our research, including optimizing the quantity of traded permits under carbon cap-and-trade regulations, considering the influences of emission reduction investment on supply chain members' carbon trading behaviors, coordinating supply chain members' carbon cap sharing behaviors and so on.

3. The model

3.1. Problem description, assumptions and notations

In this paper, we assume that a two-echelon supply chain consists of a manufacturer and a retailer. The supply chain faces the constant demand of a single product in the infinite horizon. The retailer makes ordering decisions based on the classical EOQ model. Shortages are not allowed and products are delivered without lead time. All activities in the supply chain, including ordering, producing, inventory and procurement cause carbon emissions [7, 18, 30]. The majority of emissions are produced in manufacturing. Thus, the manufacturer needs to make decisions on the efforts in emission reduction.

Table 1. Summary of relevant literature.

	Supply chain structure	Carbon policy	Decision	Emission reduction investment	Supply chain coordination
Hua *et al.* [18]	1 retailer	Cap-and-trade	Order quantity	No	No
Chen *et al.* [7]	1 retailer	Carbon cap, carbon tax, cap-and-offset, cap-and-trade	Order quantity	No	No
Du *et al.* [11]	1 manufacturer, 1 supplier	Cap-and-trade	Production quantity, price of emission permits	No	Yes
Jaber *et al.* [19]	1 manufacturer, 1 retailer	Carbon tax, emissions penalty	Manufacturer's production rate, coordination multiplier	No	Yes
Swami and Shah [29]	1 manufacturer, 1 retailer	–	Wholesale price, retail price, greening efforts	Yes	Yes
Toptal *et al.* [31]	1 retailer	Carbon cap, carbon tax, cap-and-trade	Order quantity, investment amount, traded quantity of emission capacity	Yes	No
Yang *et al.* [33]	1 supplier, 1 retailer	Carbon cap, carbon tax, cap-and-trade	Order quantity, wholesale price	No	Yes
Hammami *et al.* [15]	1 manufacturer, N suppliers	Carbon tax, carbon cap	Manufacturing, ordering, inventory positioning	No	No
Dong *et al.* [10]	1 manufacturer, 1 retailer	Cap-and-trade	Order quantity, sustainability investment	Yes	Yes
Toptal and Çetinkaya [30]	1 manufacturer, 1 retailer	Carbon tax, cap-and-trade	Order quantity, quantity of traded permits	No	Yes
This paper	1 manufacturer, 1 retailer	Cap-and-trade	Order quantity, emission reduction investment, quantity of traded permits	Yes	Yes

We assume that carbon cap-and-trade policy is implemented, which means both the manufacturer and the retailer have carbon caps. In addition, they need to purchase additional carbon emission permits from the carbon market if their emissions exceed the caps. There are three circumstances relative to the carbon caps of supply chain members. (1) The manufacturer (retailer) needs additional emission permits and the retailer (manufacturer) has residual emission permits; (2) both the manufacturer and the retailer are in shortage of emission permits; (3) both of them have residual emission permits. An assumption about carbon trading is that the selling price of emission permits is higher than the buying price due to the existence of transaction costs [26]. Therefore, emission permit sharing benefits supply chain members in the first circumstance, which makes full use of emission permits in the supply chain. We will model emission permit sharing in the supply chain and analyze its influences on the cost, emissions and decisions on production-inventory, carbon trading and emission reduction efforts. The notations used in the model are listed as follows.

D	the demand of consumers;
Q	the retailer's order quantity (decision variable);
K_R	the setup cost of the retailer's ordering;
h_R	the retailer's unit holding cost;
c	the retailer's unit procurement cost;
f_R	the retailer's fixed carbon emissions per order;
g_R	the retailer's carbon emission coefficient of inventory;
e_R	the retailer's carbon emission coefficient of procurement;
P	the manufacturer's production rate ($P > D$);
K_M	the setup cost of the manufacturer's manufacturing;
h_M	the manufacturer's unit holding cost;
p_M	the manufacturer's unit production cost;
f_M	the manufacturer's fixed carbon emissions of production setup;
g_M	the manufacturer's carbon emission coefficient of inventory;
C_R	the retailer's carbon cap;
C_M	the manufacturer's carbon cap;
p_s	the selling price of carbon emission permits;
p_b	the buying price of carbon emission permits, $p_b > p_s$;
θ	the carbon emission reduction effort of the manufacturer, $0 < \theta < 1$ (decision variable);
$e_M(\theta)$	the carbon emission coefficient of manufacturing;
$I(\theta)$	the manufacturer's investment in carbon emission reduction;
X_R	the quantity of carbon emission permits traded by the retailer (decision variable);
X_M	the quantity of carbon emission permits traded by the manufacturer (decision variable);
X_s	the quantity of carbon emission permits traded by the supply chain with carbon emission permit sharing (decision variable);
$RC(Q, X_R)$	the retailer's total cost;
$MC(Q, X_M, \theta)$	the manufacturer's total cost;
$TC(Q, X_R, X_M, \theta)$	the total cost of the supply chain in the decentralized model and $TC(Q, X_R, X_M, \theta) = RC(Q, X_R) + MC(Q, X_M, \theta)$;
$SC(Q, X_s, \theta)$	the total cost of the supply chain in the centralized model with carbon emission permit sharing;
$E_R(Q)$	the retailer's carbon emission amount;
$E_M(Q, \theta)$	the manufacturer's carbon emission amount;
$E_T(Q, \theta)$	the total carbon emission amount in the decentralized model;
$E_s(Q, \theta)$	the total carbon emission amount in the centralized model with carbon emission permit sharing;
Q_d^*	the optimal order quantity in the decentralized model;
Q_s^*	the optimal order quantity in the centralized model with carbon emission permit sharing;

Specifically, $e_M = a(1-\theta)$, a denotes the emission amount when no emission reduction efforts are made and $a > 0$. In addition, based on the assumption that $I(\theta)$ is a quadratic function [14, 22, 29], we set $I(\theta) = r\theta^2/2$, where r is the cost coefficient of emission reduction investment and $r > 0$.

Because the manufacturer and the retailer can choose to share carbon caps or not, we establish both the decentralized and centralized decision-making models under the cap-and-trade policy. Precisely, the manufacturer and the retailer share emission permits in the centralized decision-making. The decisions on production-inventory, carbon trading and emission reduction investment in the supply chain are optimized.

3.2. The decentralized decision-making model

In this section, we develop a decentralized decision-making model in the supply chain, in which the manufacturer is the leader. Therefore, the first step is to solve the cost minimization problem of the retailer and obtain the optimal response functions of the order quantity and the quantity of traded permits. The second step is that with the response functions of the order quantity and the quantity of traded permits derived, the manufacturer decides the emission reduction effort and the quantity of traded permits. Then, the costs and emissions of supply chain members are calculated.

3.2.1. The retailer's problem

The retailer's cost minimization problem is given in equation (3.1):

$$\begin{aligned} &\min RC\left(Q, X_R\right) = \begin{cases} RC_1\left(Q, X_R\right) & X_R \leq 0 \\ RC_2\left(Q, X_R\right) & X_R > 0 \end{cases} \\ &\text{s.t. } \tfrac{f_R D}{Q} + \tfrac{g_R Q}{2} + e_R D + X_R = C_R \\ &\qquad Q > 0. \end{aligned} \tag{3.1}$$

Equation (3.1) is interpreted as equations (3.2) and (3.3):

$$RC_1\left(Q, X_R\right) = \frac{K_R D}{Q} + \frac{h_R Q}{2} + cD - p_b X_R. \tag{3.2}$$

$$RC_2\left(Q, X_R\right) = \frac{K_R D}{Q} + \frac{h_R Q}{2} + cD - p_s X_R. \tag{3.3}$$

When $X_R < 0$, the retailer's emissions exceed its cap and it needs to buy emission permits, which is shown in equation (3.2). While when $X_R > 0$, the retailer's emissions don't exceed its cap, therefore, the retailer sells emission permits as equation (3.3) shows. If $X_R = 0$, $RC_1\left(Q, X_R\right) = RC_2\left(Q, X_R\right)$. With Q given, the retailer's emission amount is as follows:

$$E_R\left(Q\right) = \frac{f_R D}{Q} + \frac{g_R Q}{2} + e_R D. \tag{3.4}$$

It is known from equation (3.1) that $X_R\left(Q\right) = C_R - \frac{f_R D}{Q} - \frac{g_R Q}{2} - e_R D$. Substituting $X_R\left(Q\right)$ into equation (3.2), we have:

$$RC_1\left(Q, X_R\left(Q\right)\right) = \frac{\left(K_R + p_b f_R\right) D}{Q} + \frac{\left(h_R + p_b g_R\right) Q}{2} + \left(c + p_b e_R\right) D - p_b C_R. \tag{3.5}$$

It is shown that $RC_1\left(Q, X_R\left(Q\right)\right)$ is a strictly convex function of Q and is minimized when $Q_{d1}^* = \sqrt{\frac{2\left(K_R + p_b f_R\right) D}{h_R + p_b g_R}}$.

Similarly, we have:

$$RC_2\left(Q, X_R\left(Q\right)\right) = \frac{\left(K_R + p_s f_R\right) D}{Q} + \frac{\left(h_R + p_s g_R\right) Q}{2} + \left(c + p_s e_R\right) D - p_s C_R. \tag{3.6}$$

Then, $RC_2\left(Q, X_R\left(Q\right)\right)$ is minimized when $Q^*_{d2} = \sqrt{\frac{2(K_R+p_s f_R)D}{h_R+p_s g_R}}$.

If $\left(C_R - e_R D\right)^2 > 2f_R g_R D$, $X_R\left(Q\right) = 0$, which means that the retailer neither buys nor sells emission permits. Let $Q_1 = \frac{C_R - e_R D - \sqrt{(C_R - e_R D)^2 - 2g_R f_R D}}{g_R}$ and $Q_2 = \frac{C_R - e_R D + \sqrt{(C_R - e_R D)^2 - 2g_R f_R D}}{g_R}$, it is known that $Q_1 < Q_2$. Then, we analyze the relationships among Q^*_{d1}, Q^*_{d2}, Q_1 and Q_2 in different circumstances and decide the optimal order quantity. Theorem 3.1 is provided for the retailer to obtain the optimal decisions in the decentralized model.

Theorem 3.1. *In the decentralized decision-making model, the retailer's optimal order quantity and the quantity of traded permits are as follows:*

I. If $\left(C_R - e_R D\right) \le \sqrt{2g_R f_R D}$, *then* $Q^*_d = Q^*_{d1}$, $X_R\left(Q^*_d\right) = C_R - \frac{f_R D}{Q^*_{d1}} - \frac{g_R Q^*_{d1}}{2} - e_R D$;
II. If $\left(C_R - e_R D\right) > \sqrt{2g_R f_R D}$, *then:*
- *a. If* $\frac{h_R}{g_R} = \frac{K_R}{f_R}$, *then* $Q^*_d = Q^*_{d2}$, $X_R\left(Q^*_d\right) = C_R - \frac{f_R D}{Q^*_{d2}} - \frac{g_R Q^*_{d2}}{2} - e_R D$;
- *b. If* $\frac{h_R}{g_R} < \frac{K_R}{f_R}$,
  - *i. If* $Q_1 < Q_2 \le Q^*_{d1} < Q^*_{d2}$, *then* $Q^*_d = Q^*_{d1}$, $X_R\left(Q^*_d\right) = X_R\left(Q^*_{d1}\right) < 0$;
  - *ii. If* $Q_1 < Q^*_{d1} < Q^*_{d2} \le Q_2$, *then* $Q^*_d = Q^*_{d2}$, $X_R\left(Q^*_d\right) = X_R\left(Q^*_{d2}\right) > 0$;
  - *iii. If* $Q_1 < Q^*_{d1} < Q_2 < Q^*_{d2}$, *then* $Q^*_d = Q_2$, $X_R\left(Q^*_d\right) = X_R\left(Q_2\right) = 0$;
- *c. If* $\frac{h_R}{g_R} > \frac{K_R}{f_R}$,
  - *i. If* $Q_1 \le Q^*_{d2} < Q^*_{d1} < Q_2$, *then* $Q^*_d = Q^*_{d2}$, $X_R\left(Q^*_d\right) = X_R\left(Q^*_{d2}\right) > 0$;
  - *ii. If* $Q^*_{d2} < Q_1 < Q^*_{d1} < Q_2$, *then* $Q^*_d = Q_1$, $X_R\left(Q^*_d\right) = X_R\left(Q_1\right) = 0$;
  - *iii. If* $Q^*_{d2} < Q^*_{d1} < Q_1 < Q_2$, *then* $Q^*_d = Q^*_{d1}$, $X_R\left(Q^*_d\right) = X_R\left(Q^*_{d1}\right) < 0$.

The proof is in Appendix A.

The conditions about $\frac{h_R}{g_R}$ and $\frac{K_R}{f_R}$ are given in Theorem 3.1, and the optimal order quantity and the quantity of traded permits under different conditions are obtained.

$\frac{h_R}{g_R}$ stands for the ratio of holding cost to holding emissions, and $\frac{K_R}{f_R}$ represents the ratio of the setup cost of replenishment to the setup emissions of replenishment. If $\frac{h_R}{g_R} < \frac{K_R}{f_R}$, it indicates that the retailer should increase the order frequency and lower the inventory level, which helps to reduce carbon emissions; while if $\frac{h_R}{g_R} > \frac{K_R}{f_R}$, it is interpreted that the retailer's reduction in order frequencies and the increase of order quantity will help to reduce carbon emissions.

3.2.2. The manufacturer's problem

The manufacturer's cost minimization problem is given by equation (3.7):

$$\begin{aligned}
&\min MC\left(Q, X_M, \theta\right) = \begin{cases} MC_1\left(Q, X_M, \theta\right) & X_M \le 0 \\ MC_2\left(Q, X_M, \theta\right) & X_M > 0 \end{cases} \\
&\text{s.t. } \frac{f_M D}{Q} + \frac{g_M DQ}{2P} + \left(a - a\theta\right)D + X_M = C_M \\
&\quad\ 0 < \theta < 1 \\
&\quad\ Q > 0.
\end{aligned} \tag{3.7}$$

Equation (3.7) is divided into equations (3.8) and (3.9):

$$MC_1\left(Q, X_M, \theta\right) = \frac{K_M D}{Q} + \frac{h_M DQ}{2P} + p_M D + \frac{r\theta^2}{2} - p_b X_M. \tag{3.8}$$

$$MC_2\left(Q, X_M, \theta\right) = \frac{K_M D}{Q} + \frac{h_M DQ}{2P} + p_M D + \frac{r\theta^2}{2} - p_s X_M. \tag{3.9}$$

When $X_M < 0$, the manufacturer needs to buy emission permits, which is shown in equation (3.8). When $X_M > 0$, the manufacturer has residual emission permits to sell, and its cost is expressed as equation (3.9). If $X_M = 0$, $MC_1(Q, X_M, \theta) = MC_2(Q, X_M, \theta)$. Given Q and θ, the manufacturer's emission amount is as follows:

$$E_M(Q,\theta) = \frac{f_M D}{Q} + \frac{g_M DQ}{2P} + (a - a\theta) D. \tag{3.10}$$

It is known from equation (3.7) that $X_M(Q,\theta) = C_M - \frac{f_M D}{Q} - \frac{g_M DQ}{2P} - (a - a\theta) D$. Substituting $X_M(Q,\theta)$ into equation (3.8), we have:

$$MC_1(Q, X_M(Q,\theta), \theta) = \frac{(K_M + p_b f_M) D}{Q} + \frac{(h_M + p_b g_M) DQ}{2P} + (p_M + ap_b) D - p_b C_M + \frac{r\theta^2}{2} - ap_b D\theta. \tag{3.11}$$

Its Hessian Matrix about Q and θ is $\begin{bmatrix} \frac{2(k_M + p_b f_M) D}{Q^3} & 0 \\ 0 & r \end{bmatrix}$. Because the conditions $\frac{2(k_M + p_b f_M) D}{Q^3} > 0$ and $\frac{2r(k_M + p_b f_M) D}{Q^3} > 0$ hold, $MC_1(Q, X_M(Q,\theta), \theta)$ is a strictly convex function of Q and θ. Likewise, $MC_2(Q, X_M(Q,\theta), \theta)$ is proved to be a strictly convex function of Q and θ.

Theorem 3.2 is provided for the manufacturer to obtain the optimal decisions in the decentralized model. The proof is in Appendix A.

Theorem 3.2. *Given* Q_d^*,

I. *If* $D < \frac{r}{ap_b}$ *and* $C_M < \frac{f_M D}{Q_d^*} + \frac{g_M DQ_d^*}{2P} + \left(a - \frac{a^2 p_b D}{r}\right) D$, *then* $X_M(Q_d^*) < 0$, $\theta = \frac{ap_b D}{r}$;

II. *If* $D < \frac{r}{ap_s}$ *and* $C_M > \frac{f_M D}{Q_d^*} + \frac{g_M DQ_d^*}{2P} + \left(a - \frac{a^2 p_s D}{r}\right) D$, *then* $X_M(Q_d^*) > 0$, $\theta = \frac{ap_s D}{r}$;

III. *If* $\frac{f_M D}{Q_d^*} + \frac{g_M DQ_d^*}{2P} + \left(a - \frac{a^2 p_s D}{r}\right) D \geq C_M \geq \frac{f_M D}{Q_d^*} + \frac{g_M DQ_d^*}{2P} + \left(a - \frac{a^2 p_b D}{r}\right) D$, *then* $X_M(Q_d^*) = 0$, $\theta = \frac{\frac{f_M D}{Q_d^*} + \frac{g_M DQ_d^*}{2P} + aD - C_M}{aD}$.

Theorem 3.2 shows that in the decentralized decision making, if the retailer's optimal order quantity is fixed, the carbon trading behavior of the manufacturer depends on the manufacturer's carbon cap, which also affects the manufacturer's decision on emission reduction investment. When the manufacturer's carbon cap is lower than a threshold, the manufacturer will purchase emission permits from carbon trading market, which also results in the positive correlation between manufacturer's carbon emission reduction effort and the buying price of emission permits. When the manufacturer's carbon cap is higher than a threshold, the manufacturer will sell emission permits, and there is a positive correlation between the manufacturer's emission reduction effort and the selling price.

3.3. The centralized supply chain model

In this section, we consider a centralized supply chain, where the manufacturer and the retailer share carbon caps and make centralized decisions. The objective is to minimize the total cost of the supply chain. In this model, we assume that if a member is short of emission permits while the other member has residual emission permits, they make an internal transfer of emission permits in the supply chain without payment. The total cost in the centralized decision-making model with emission permit sharing is given by equation (3.12) as follows:

$$\begin{aligned}
&\min SC\left(Q, X_s, \theta\right)=\left\{\begin{array}{ll} SC_1\left(Q, X_s, \theta\right) & X_s \leq 0 \\ SC_2\left(Q, X_s, \theta\right) & X_s>0 \end{array}\right. \\
&\text{s.t. } \frac{\left(f_R+f_M\right) D}{Q}+\frac{\left(g_R+\frac{g_M D}{P}\right) Q}{2}+\left(e_R+a-a \theta\right) D+X_s=C_R+C_M \\
&\quad 0<\theta<1 \\
&\quad Q>0.
\end{aligned} \tag{3.12}$$

Equation (3.12) is transformed to equations (3.13) and (3.14):

$$SC_1\left(Q, X_S, \theta\right)=\frac{\left(K_R+K_M\right) D}{Q}+\frac{\left(h_R+\frac{h_M D}{P}\right) Q}{2}+\left(c+p_M\right) D-p_b X_s+\frac{r \theta^2}{2}. \tag{3.13}$$

$$SC_2\left(Q, X_S, \theta\right)=\frac{\left(K_R+K_M\right) D}{Q}+\frac{\left(h_R+\frac{h_M D}{P}\right) Q}{2}+\left(c+p_M\right) D-p_s X_s+\frac{r \theta^2}{2}. \tag{3.14}$$

Given Q and θ, the supply chain's total emission amount is as follows:

$$E_s(Q, \theta)=\frac{\left(f_R+f_M\right) D}{Q}+\frac{\left(g_R+\frac{g_M D}{P}\right) Q}{2}+\left(e_R+a-a \theta\right) D. \tag{3.15}$$

Theorem 3.3 is provided for supply chain members to obtain the optimal decisions in the centralized model. The proof is in Appendix A.

Theorem 3.3. *In the centralized decision-making model,*

I. If $D<\frac{r}{a p_b}$ *and* $C_R+C_M<\left(e_R+a-\frac{a^2 p_b D}{r}\right) D+\frac{\left(f_R+f_M\right) D}{\sqrt{\frac{2\left[K_R+K_M+p_b\left(f_R+f_M\right)\right] D}{h_R+\frac{h_M D}{p}+p_b\left(g_R+\frac{g_M D}{P}\right)}}}$ $+\frac{\left(g_R+\frac{g_M D}{P}\right) \sqrt{\frac{2\left[K_R+K_M+p_b\left(f_R+f_M\right)\right] D}{h_R+\frac{h_M D}{p}+p_b\left(g_R+\frac{g_M D}{P}\right)}}}{2}$, *then* $Q_s^*=\sqrt{\frac{2\left[K_R+K_M+p_b\left(f_R+f_M\right)\right] D}{h_R+\frac{h_M D}{P}+p_b\left(g_R+\frac{g_M D}{P}\right)}}$, $\theta=\frac{a p_b D}{r}$. *The supply chain will buy emission permits from the carbon market that:*

$$X_s=C_R+C_M-\left(e_R+a-\frac{a^2 p_b D}{r}\right) D-\frac{\left(f_R+f_M\right) D}{\sqrt{\frac{2\left[K_R+K_M+p_b\left(f_R+f_M\right)\right] D}{h_R+\frac{h_M D}{p}+p_b\left(g_R+\frac{g_M D}{P}\right)}}}-\frac{\left(g_R+\frac{g_M D}{P}\right) \sqrt{\frac{2\left[K_R+K_M+p_b\left(f_R+f_M\right)\right] D}{h_R+\frac{h_M D}{p}+p_b\left(g_R+\frac{g_M D}{P}\right)}}}{2};$$

II. If $D<\frac{r}{a p_s}$ *and* $C_R+C_M>\left(e_R+a-\frac{a^2 p_s D}{r}\right) D+\frac{\left(f_R+f_M\right) D}{\sqrt{\frac{2\left[K_R+K_M+p_s\left(f_R+f_M\right)\right] D}{h_R+\frac{h_M D}{p}+p_s\left(g_R+\frac{g_M D}{P}\right)}}}$ $+\frac{\left(g_R+\frac{g_M D}{P}\right) \sqrt{\frac{2\left[K_R+K_M+p_s\left(f_R+f_M\right)\right] D}{h_R+\frac{h_M D}{p}+p_s\left(g_R+\frac{g_M D}{P}\right)}}}{2}$, *then* $Q_s^*=\sqrt{\frac{2\left[K_R+K_M+p_s\left(f_R+f_M\right)\right] D}{h_R+\frac{h_M D}{P}+p_s\left(g_R+\frac{g_M D}{P}\right)}}$, $\theta=\frac{a p_s D}{r}$. *The supply chain can sell extra emission permits to gain profits that:*

$$X_s=C_R+C_M-\left(e_R+a-\frac{a^2 p_s D}{r}\right) D-\frac{\left(f_R+f_M\right) D}{\sqrt{\frac{2\left[K_R+K_M+p_s\left(f_R+f_M\right)\right] D}{h_R+\frac{h_M D}{p}+p_s\left(g_R+\frac{g_M D}{P}\right)}}}-\frac{\left(g_R+\frac{g_M D}{P}\right) \sqrt{\frac{2\left[K_R+K_M+p_s\left(f_R+f_M\right)\right] D}{h_R+\frac{h_M D}{p}+p_s\left(g_R+\frac{g_M D}{P}\right)}}}{2}.$$

In the centralized decision-making with emission permit sharing, whether the supply chain buys or sells emission permits depends on the carbon cap of the supply chain. When the carbon cap of the supply chain is below a threshold, the supply chain will buy emission permits from the market, and the retailer's optimal order quantity and the quantity of traded permits are affected by the buying price of emission permits. When the cap is higher than a threshold, the supply chain sells emission permits. Accordingly, the retailer's optimal order quantity and the quantity of traded permits are influenced by the selling price of permits. The influences of carbon trading prices on the emission reduction effort are illustrated in Corollary 3.4.

Corollary 3.4. *In the case of emission permit sharing, the emission reduction effort of the manufacturer is positively related to the prices of carbon trading. When the supply chain buys emission permits from the market, the manufacturer's emission reduction effort becomes higher with the increase of the buying price of permits. When the supply chain sells emission permits, the manufacturer's emission reduction effort increases with the increase of the selling price.*

The results show that if the supply chain needs to buy emission permits from the market, the higher the buying price is, it is more likely that the manufacturer increases its emission reduction investment. If the supply chain sells permits, the higher selling price pushes the manufacturer to increase emission reduction investment because selling the residual permits brings more profits to the supply chain. Meanwhile, because the selling price is higher than the buying price, in the case that the supply chain purchases emission permits, the manufacturer's investment effort in emission reduction is greater than the investment effort in the case that the supply chain sells emission permits.

4. Numerical examples and model analysis

4.1. Comparative analysis of decentralized and centralized decisions

In this section, numerical examples are given to provide supply chain members with strategies on production-inventory and emission reduction in decentralized and centralized decision-making models. Moreover, the total cost and emissions of the supply chain along with the quantity of traded permits are compared and analyzed.

Based on the parameters used by Toptal and Çetinkaya [30], we set $D = 50$, $P = 150$, $c = 12$, $e_R = 5$, $p_M = 8$, $a = 7$, $r = 10\,000$, $p_b = 7.5$, $p_s = 6$ and the other parameters as shown in Table 1. The optimal solutions in decentralized and centralized models are shown in Tables 2 and 3, respectively.

A significant observation is that the total cost of supply chain with emission permit sharing is lower than that without emission permit sharing. However, the influence of emission permit sharing on the total carbon emissions of the supply chain depends on parameters.

Let $R = \frac{E_s(Q_s^*)}{E_T\left(Q_d^*\right)} = \frac{\frac{(f_R+f_M)D}{Q_s^*}+\frac{\left(g_R+\frac{g_M D}{P}\right)Q_s^*}{2}+(e_R+e_M)D}{\frac{(f_R+f_M)D}{Q_d^*}+\frac{\left(g_R+\frac{g_M D}{P}\right)Q_d^*}{2}+(e_R+e_M)D}$ denote the ratio of the total emissions in the centralized model to the total emissions in the decentralized model, which is used to measure the emission reduction performance of emission permit sharing. If $R \geq 1$, emission permit sharing cannot reduce carbon emissions and curb global warming; while if $R < 1$, emission permit sharing effectively reduces the carbon emissions of the

Table 2. Table of parameters used in the models.

Set of parameters	K_R	h_R	f_R	g_R	C_R	K_M	h_M	f_M	g_M	C_M
1	900	1	40	0.5	300	1000	0.5	135	0.25	450
2	900	1	20	0.5	300	1000	0.5	135	0.25	450
3	900	1	10	0.5	300	1000	0.5	135	0.25	450
4	900	1	40	0.8	300	1000	0.5	135	0.25	450
5	900	1	40	1	300	1000	0.5	135	0.25	450
6	900	1	40	0.5	300	1000	0.5	100	0.25	450
7	900	1	40	0.5	300	1000	0.5	60	0.25	450
8	900	1	40	0.5	300	1000	0.5	135	0.5	450
9	900	1	40	0.5	300	1000	0.5	135	1	450

TABLE 3. The optimal solutions in the decentralized model.

Set of parameters	Q_d^*	θ	$X_R\,(Q_d^*)$	$X_M\,(Q_d^*)$	$TC\,(Q_d^*)$	$E_R\,(Q_d^*)$	$E_M\,(Q_d^*)$	$E_T\,(Q_d^*)$
1	158.94	0.21	−2.32	61.41	1559.87	302.32	388.59	690.91
2	159.69	0.21	11.32	124.58	1093.15	288.68	325.42	615.10
3	154.92	0.21	8.04	123.47	1135.03	291.96	326.53	618.49
4	130.93	0.21	−17.65	116.49	1455.89	317.65	333.51	651.16
5	118.82	0.21	−26.24	111.74	1615.70	326.24	338.26	664.5
6	158.94	0.21	−2.32	135.42	1084.35	302.32	314.58	616.90
7	158.94	0.21	−2.32	148.00	1040.33	302.32	302.00	604.32
8	158.94	0.21	−2.32	117.19	1111.34	302.32	332.21	634.53
9	158.94	0.21	−2.32	104.54	1300.49	302.32	345.46	647.78

TABLE 4. The optimal solutions in the centralized model.

Set of parameters	Q_s^*	θ	$X_s\,(Q_s^*)$	$SC\,(Q_s^*)$	$E_s\,(Q_s^*)$
1	251.33	0.21	115.38	1052.86	634.62
2	246.78	0.21	120.53	1026.23	629.47
3	244.15	0.21	123.00	1013.98	627.00
4	213.91	0.21	88.47	1258.57	661.53
5	196.41	0.21	72.89	1381.41	677.11
6	242.82	0.21	124.25	1007.88	625.75
7	231.94	0.21	134.68	957.31	615.32
8	239.87	0.21	107.86	1109.31	642.14
9	219.07	0.21	92.65	1226.04	657.35

supply chain. Next, we'll discuss how will the optimal solutions and R change with the changes in selected parameters including f_R, g_R, f_M and g_M.

From the results with Sets 1, 2 and 3 of parameters, it is found that when the two members share carbon caps, if the retailer's fixed amount of carbon emissions per order decreases, the optimal order quantity of the retailer declines, and the total cost and carbon emissions of the supply chain also decrease, while the quantity of traded permits of the supply chain increases. It is explained that if carbon caps are fixed, when the emissions of the supply chain decline, the emission permits for sale increase and bring extra revenue to the supply chain. However, in the decentralized decision-making, results become different. Because $R_1 = 0.918$, $R_2 = 1.023$ and $R_3 = 1.014$, it is shown that with the Set 1 of parameters, emission permit sharing reduces both the total cost and the total carbon emissions of the supply chain. However, the comparison among the results with Sets 1, 2 and 3 of parameters indicates that with the decrease of f_R, the value of R exceeds one meaning that emission permit sharing cannot reduce carbon emissions effectively.

Comparing the results of Sets 1, 4 and 5 of parameters, we see that when the two members share carbon caps, with the increase of the retailer's carbon emission coefficient of inventory, the optimal order quantity of the retailer and the quantity of traded permits of the supply chain decrease, while the total cost and carbon emissions of the supply chain increase. However, the quantity of traded permits, the cost and carbon emissions of the retailer increase with the increase of g_R in the decentralized decision-making model. It indicates that the retailer's emission coefficient of inventory has a significant impact on its carbon emissions and then affects the total emissions of the supply chain. Meanwhile, the results that $R_4 = 1.016$ and $R_5 = 1.019$ show that R increases with the increase of g_R, which implies that with the increase of the retailer's emission coefficient of inventory, emission permit sharing becomes a barrier for the emission reduction of the supply chain.

Comparing the results of Sets 1, 6 and 7 of parameters, it is shown that in the circumstance of emission permit sharing, the reduction of the manufacturer's carbon emissions of production setup causes the decrease of the optimal order quantity and the decline in the total cost and emissions of the supply chain, while the quantity of traded permits increases. In addition, we find that the impacts of f_M and f_R on the decision variables and objectives are the same when the two members share carbon caps. In the decentralized decision-making, the quantity of traded permits, the cost and emissions of the retailer don't change with the change in f_M because the optimal order quantity in the decentralized model is not related to f_M, while the cost and emissions of the manufacturer decline with the decrease of f_M. The results that $R_6 = 1.014$ and $R_7 = 1.018$ indicate that R increases with the increase of f_M. It is interpreted that with the reduction of the manufacturer's carbon emissions of production setup, the carbon emissions of supply chain in the circumstance of emission permit sharing are more than in the circumstance without sharing. However, it has caused a decline in the total cost of supply chain.

Based on the results with Sets 1, 8 and 9 of parameters, we analyze the impact of manufacturer's emission coefficient of inventory. It shows that in the circumstance of emission permit sharing, the optimal order quantity of the retailer and the quantity of traded permits of the supply chain decrease with the increase of g_M, while the total cost and carbon emissions of the supply chain increase. However, the quantity of traded permits, the cost and emissions of the retailer don't change with the change of g_M in the decentralized model because the optimal order quantity in the decentralized model is not related to g_M. Moreover, the results that $R_8 = 1.012$ and $R_9 = 1.015$ imply that with the increase of the manufacturer's emission coefficient of inventory, emission permit sharing doesn't help to reduce carbon emissions compared with the circumstance without emission permit sharing.

4.2. Compensation mechanism in the centralized model

In this section, we provide a compensation mechanism to coordinate the manufacturer and the retailer. Then, the surplus is allocated which comes from emission permit sharing in the centralized model. There are two scenarios of emission permit sharing. In scenario 1, the retailer is short of emission permits and the manufacturer has residual emission permits. In scenario 2, the retailer has extra emission permits and the manufacturer is lack of permits. The coordination mechanism in which one member compensates another member to achieve a win-win situation is discussed, which may in the form of the fixed payment or the fixed price of shared permits.

4.2.1. Scenario 1: the retailer compensates the manufacturer

If $X_R(Q_s^*) < 0$, $X_M(Q_s^*) > 0$ and $-X_R(Q_s^*) < X_M(Q_s^*)$, it means that the manufacturer gives its residual emission permits to the retailer at no charge in the centralized model. Thus, the retailer should offer compensation for the manufacturer to maintain the cooperation relationship. Let C_1 denote the fixed compensation that the retailer offers to the manufacturer. Let ΔRC denote the cost difference of the retailer between the centralized model with the compensation mechanism and the decentralized model. Adopting an effective compensation mechanism, the cost difference should satisfy the condition in equation (4.1) as follows:

$$\Delta RC = \frac{K_R D}{Q_s^*} + \frac{h_R Q_s^*}{2} + cD + C_1 - \left(\frac{K_R D}{Q_d^*} + \frac{h_R Q_d^*}{2} + cD - p_b X_R(Q_d^*)\right) \leq 0. \tag{4.1}$$

From equation (4.1), we derive the scope of the fixed compensation that:

$$\begin{aligned} C_1 &\leq \left(\frac{K_R D}{Q_d^*} + \frac{h_R Q_d^*}{2} + cD - p_b X_R(Q_d^*)\right) - \left(\frac{K_R D}{Q_s^*} + \frac{h_R Q_s^*}{2} + cD\right) \\ &= \left(\frac{K_R D}{Q_d^*} + \frac{h_R Q_d^*}{2} + cD - p_b X_R(Q_d^*)\right) - \left(RC(Q_s^*, X_R(Q_s^*)) + p_b X_R(Q_s^*)\right) \\ &= RC(Q_d^*, X_R(Q_d^*)) - RC(Q_s^*, X_R(Q_s^*)) - p_b X_R(Q_s^*)\ . \end{aligned}$$

Let p_c^1 denote the permit sharing price in scenario 1. Then, the compensation mechanism is effective in coordinating the supply chain if p_c^1 satisfies the following condition:

$$p_c^1 \le \frac{\left(\frac{K_R D}{Q_d^*}+\frac{h_R Q_d^*}{2}+cD-p_b X_R(Q_d^*)\right)-\left(\frac{K_R D}{Q_s^*}+\frac{h_R Q_s^*}{2}+cD\right)}{-X_R(Q_s^*)}.$$

4.2.2. Scenario 2: the manufacturer compensates the retailer

If $X_R\left(Q_s^*\right)>0$, $X_M\left(Q_s^*\right)<0$ and $X_R\left(Q_s^*\right)>-X_M\left(Q_s^*\right)$, it means that the retailer gives its residual emission permits to the manufacturer free of charge in the centralized model. Therefore, the manufacturer should compensate the retailer. Let C_2 denote the fixed compensation that the manufacturer offers to the retailer. Let ΔMC denote the cost difference of the manufacturer between the centralized model with the compensation mechanism and the decentralized model. The mechanism contributes to a win-win situation if ΔMC satisfies the condition in equation (4.2) as follows:

$$\Delta MC=\frac{K_M D}{Q_s^*}+\frac{h_M D Q_s^*}{2P}+p_M D+\frac{r\theta^2}{2}+C_2-\left(\frac{K_M D}{Q_d^*}+\frac{h_M D Q_d^*}{2P}+p_M D+\frac{r\theta^2}{2}-p_b X_M\left(Q_d^*\right)\right)\le 0. \quad (4.2)$$

From equation (4.2), it is derived that:

$$\begin{aligned}
C_2 &\le \left(\frac{K_M D}{Q_d^*}+\frac{h_M D Q_d^*}{2P}+p_M D+\frac{r\theta^2}{2}-p_b X_M\left(Q_d^*\right)\right)-\left(\frac{K_M D}{Q_s^*}+\frac{h_M D Q_s^*}{2P}+p_M D+\frac{r\theta^2}{2}\right)\\
&=\left(\frac{K_M D}{Q_d^*}+\frac{h_M D Q_d^*}{2P}+p_M D+\frac{r\theta^2}{2}-p_b X_M\left(Q_d^*\right)\right)-\left(MC\left(Q_s^*, X_M\left(Q_s^*\right)\right)+p_b X_M\left(Q_s^*\right)\right)\\
&=MC\left(Q_d^*, X_M\left(Q_d^*\right)\right)-MC\left(Q_s^*, X_M\left(Q_s^*\right)\right)-p_b X_M\left(Q_s^*\right).
\end{aligned}$$

Let p_c^2 denote the permit sharing price in scenario 2. Then, the compensation mechanism is effective in coordinating the supply chain if p_c^2 is below the upper bound that:

$$p_c^2 \le \frac{\left(\frac{K_M D}{Q_d^*}+\frac{h_M D Q_d^*}{2P}+p_M D+\frac{r\theta^2}{2}-p_b X_M\left(Q_d^*\right)\right)-\left(\frac{K_M D}{Q_s^*}+\frac{h_M D Q_s^*}{2P}+p_M D+\frac{r\theta^2}{2}\right)}{-X_M\left(Q_s^*\right)}$$

.

4.3. Sensitivity analysis

4.3.1. Sensitivity analysis of the carbon trading prices

The comparative analysis above indicates that emission permit sharing can reduce the total cost and carbon emissions of the supply chain. In this section, we will further discuss the impacts of carbon trading prices on decision variables and objectives.

First, we analyze how the buying price of emission permits (p_b) affects the optimal solutions when the supply chain buys emission permits from the carbon market. Let $D=100$, $P=110$, $c=12$, $e_R=5$, $p_M=8$, $a=6$, $r=10\,000$, $p_s=6$, $K_R=500$, $h_R=2$, $f_R=80$, $g_R=0.5$, $C_R=300$, $K_M=500$, $h_M=2$, $f_M=150$, $g_M=0.25$ and $C_M=350$.

Figure 1 shows that in the centralized model with emission permit sharing, the optimal order quantity of the retailer increases with the increasing buying price, which causes the increase of its own carbon emissions. Meanwhile, the manufacturer improves its emission reduction effort to reduce the total carbon emissions of the supply chain. As the emissions of the supply chain decrease, the emission permits bought from the market also decrease. As illustrated in Figure 2, although the cost of carbon trading has decreased, the overall cost of the supply chain has risen due to the increase in order quantity and the investment in emission reduction by the

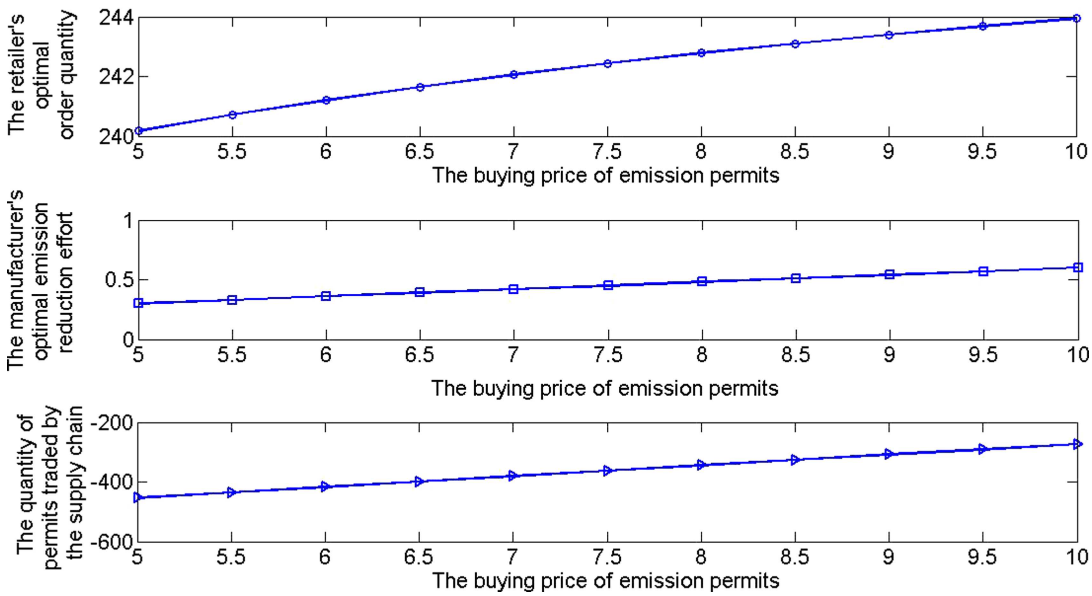

FIGURE 1. Impacts of the buying price of emission permits on decision variables.

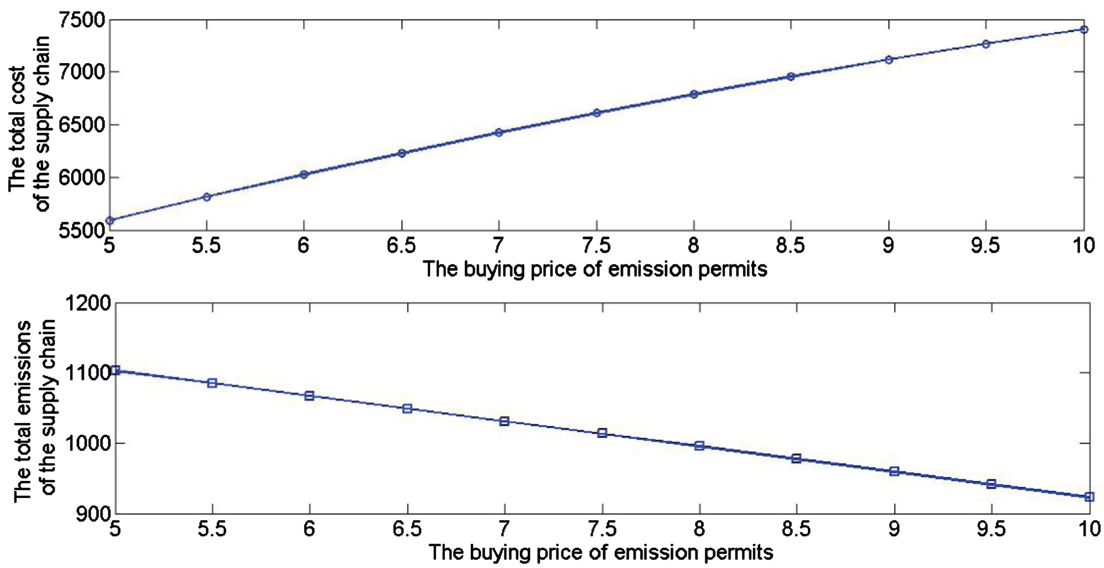

FIGURE 2. Impacts of the buying price of emission permits on the total cost and emissions of the supply chain.

manufacturer. This shows that when the supply chain needs to buy emission permits, the higher the buying price is, the higher cost the supply chain bears.

Next, we examine how the selling price of emission permits (p_s) affects the optimal solutions when the supply chain has emission permits to sell. We use the data set that $D = 50$, $P = 150$, $c = 12$, $e_R = 5$, $p_M = 8$, $a = 7$, $r = 10\,000$, $K_R = 900$, $h_R = 1$, $f_R = 40$, $g_R = 0.5$, $K_M = 1000$, $h_M = 0.5$, $f_M = 135$, $g_M = 0.25$, $p_b = 7.5$, $C_R = 300$ and $C_M = 450$.

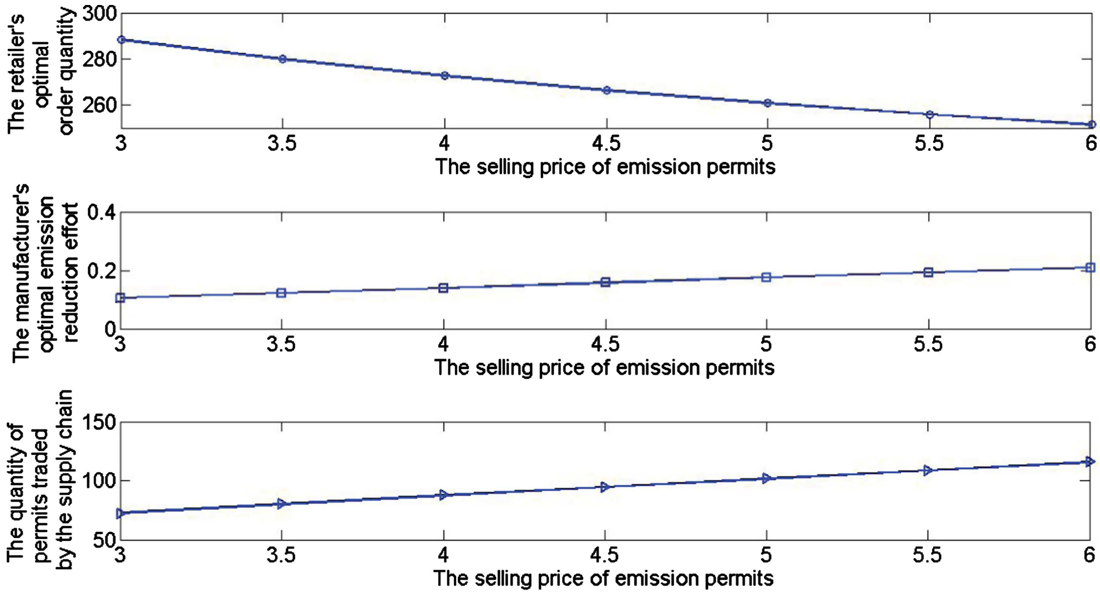

FIGURE 3. Impacts of the selling price of emission permits on decision variables.

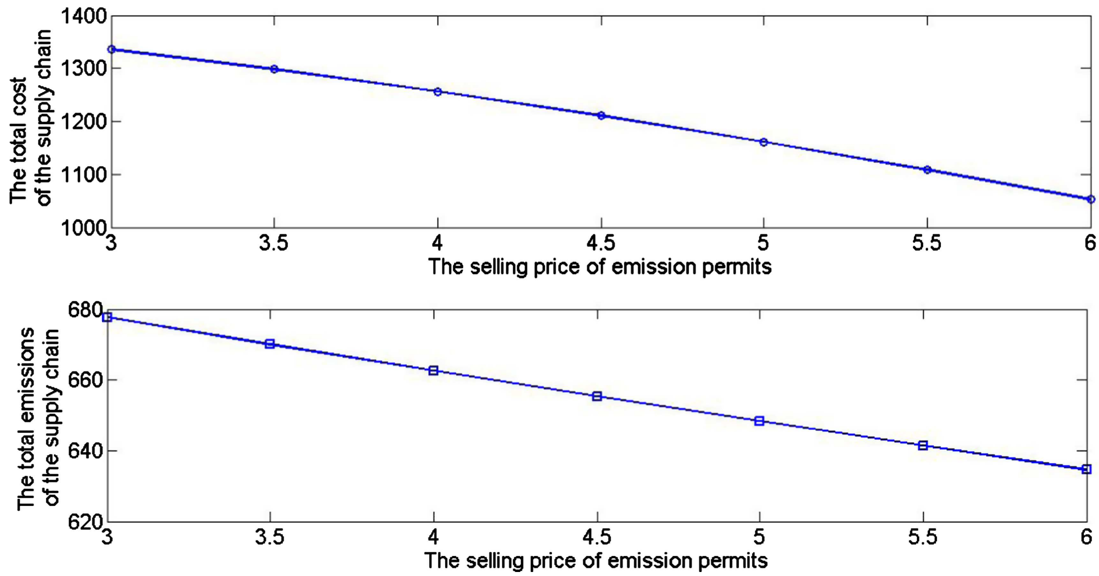

FIGURE 4. Impacts of the selling price of emission permits on the total cost and emissions of the supply chain.

Figure 3 shows that with the increase of selling price, the optimal order quantity of the retailer decreases, which leads to the reduction in its own emissions. In addition, the manufacturer is also improving its emission reduction effort, so the total carbon emissions of the supply chain decline (Fig. 4). What's more, there are extra emission permits for the supply chain to sell to the carbon market. Although the manufacturer's higher emission reduction effort brings higher investment cost, the supply chain gains profits from selling emission

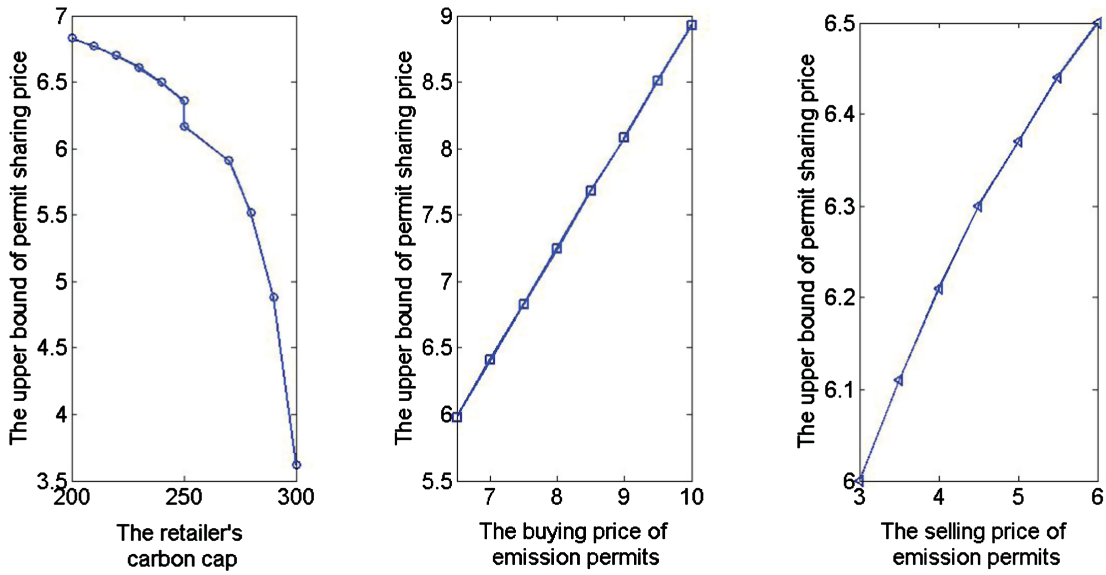

FIGURE 5. Impacts of carbon parameters on the upper bound of permit sharing price (the retailer compensates the manufacturer).

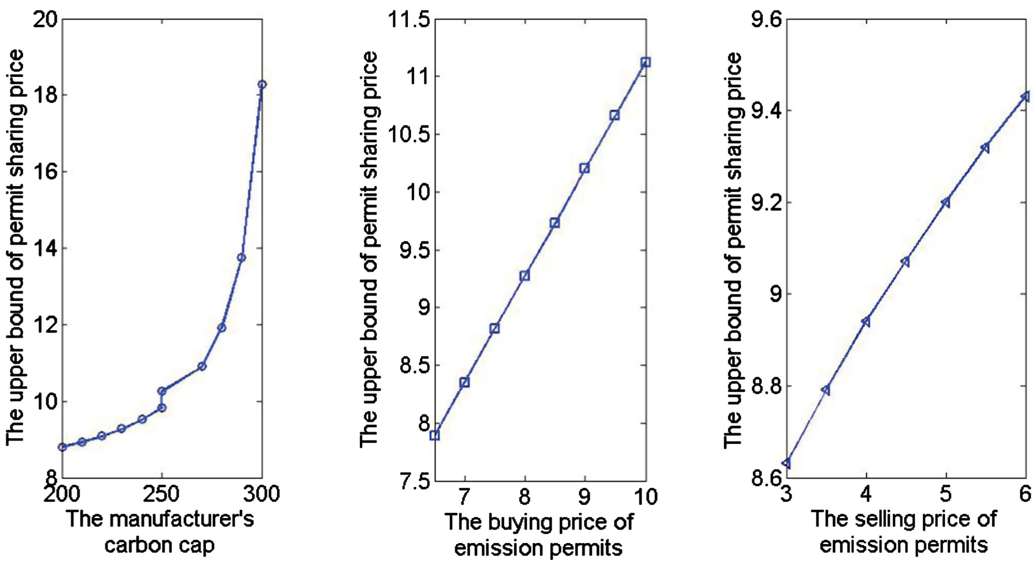

FIGURE 6. Impacts of carbon parameters on the upper bound of permit sharing price (the manufacturer compensates the retailer).

permits. Thus, the total cost of the supply chain has declined. This indicates that when the supply chain has residual emission permits to sell, the higher the selling price is, the more profit the supply chain gains (Fig. 4).

4.3.2. Sensitivity analysis of the permit sharing price in the compensation mechanism

In scenario 1, the manufacturer is the provider of emission permits. Using the basic data set that $D = 50$, $P = 150$, $c = 12$, $e_R = 5$, $p_M = 8$, $a = 7$, $r = 10000$, $K_R = 900$, $h_R = 1$, $f_R = 40$, $g_R = 0.5$, $K_M = 1000$,

$h_M = 0.5$, $f_M = 135$, $g_M = 0.25$, we analyze the influences of carbon parameters on the permit sharing price. The first subplot of Figure 5 reflects the influence of the retailer's carbon cap using the other carbon parameters that $p_b = 7.5$, $p_s = 6$ and $C_M = 450$; the second subplot of Figure 5 shows the influence of the buying price of emission permits using the data that $p_s = 6$, $C_M = 450$ and $C_R = 200$; the third subplot depicts the influence of the selling price of emission permits using the data that $p_b = 7.5$, $C_M = 450$ and $C_R = 250$.

As Figure 5 shows, if the retailer has a higher carbon cap, the upper bound of permit sharing price decreases, which means that the retailer pays less for using the emission permits. While when the buying and selling prices of emission permits increase, the upper bound of permit sharing price also increases. This indicates that carbon trading prices have a positive influence on the permit sharing price.

In scenario 2, the retailer is the provider of emission permits. Using the basic data set that $D = 50$, $P = 150$, $c = 12$, $e_R = 5$, $p_M = 8$, $a = 7$, $r = 10000$, $K_R = 900$, $h_R = 1$, $f_R = 40$, $g_R = 0.5$, $K_M = 1000$, $h_M = 0.5$, $f_M = 135$, $g_M = 0.25$, we analyze the influences of carbon parameters on the permit sharing price. The first subplot of Figure 6 reflects the influence of the manufacturer's carbon cap using the other carbon parameters that $p_b = 7.5$, $p_s = 6$ and $C_R = 450$; the second subplot of Figure 6 shows the influence of the buying price of emission permits using the data that $p_s = 6$, $C_M = 200$ and $C_R = 450$; the third subplot depicts the influence of the selling price of emission permits using the data that $p_b = 7.5$, $C_M = 250$ and $C_R = 450$.

As Figure 6 shows, the buying and selling prices of emission permits also have a positive influence on the upper bound of permit sharing price. However, with the increase of the manufacturer's carbon cap, the permit sharing price has a higher upper bound, which indicates that the manufacturer with a higher carbon cap is more capable to provide a satisfying compensation for the retailer.

5. Conclusions

In this paper, we focus on how a two-echelon supply chain manages its carbon footprints in production and inventory under carbon cap-and-trade policy. We extend the classical EOQ model and study decisions on production-inventory, carbon trading and emission reduction investment in the decentralized and centralized situations. In addition, the compensation mechanism is proposed for the centralized supply chain with emission permit sharing. A series of numerical examples are analyzed to show the characteristics of analytical solutions and provide significant observations.

The results show that in the decentralized model, the manufacturer's optimal emission reduction effort is affected by its carbon cap. When the carbon cap of the manufacturer declines, the investment in emission reduction becomes less. While in the centralized model with emission permit sharing, the carbon cap of the supply chain affects the manufacturer's decision on emission reduction investment, the supply chain's carbon trading amount and the total cost of supply chain. In addition, the comparison between solutions in decentralized and centralized models shows that emission permit sharing can effectively reduce the total cost and total carbon emissions of the supply chain. From the sensitivity analysis, we see that under centralized decision-making, when the supply chain purchases emission permits from the trading market, with the increase in the buying price of carbon emissions, the manufacturer's emission reduction effort rises, while the cost of the supply chain doesn't decrease; when the supply chain has residual emission permits to sell, the manufacturer's emission reduction effort increases with the increase of the selling price, while the total cost of the supply chain declines. Meanwhile, it is also observed that the buying and selling prices of emission permits have a positive influence on the permit sharing price in the compensation mechanism. The carbon caps of the manufacturer and the retailer have opposite influences on the permit sharing price that the retailer pays less for using the emission permits if it has a higher carbon cap, while the manufacturer with a higher carbon cap is more capable to provide a high compensation for the retailer.

In the future research, extensions will be made in the following directions. We will consider the demand which is stochastic or affected by consumers' preference for environment-friendly products. We will also assume a dynamic carbon trading price which depends on the supply-demand relationship in carbon market.

Appendix A

A.1 Proof of Theorem 3.1

For the proof of Theorem 3.1, we present the following lemmas and corollaries.

Lemma A.1. *When $(C_R - e_R D) \leq \sqrt{2g_R f_R D}$, the retailer doesn't sell emission permits, that is $X_R \leq 0$, $Q_d^* = Q_{d1}^* = \sqrt{\frac{2(K_R+p_b f_R)D}{h_R+p_b g_R}}$ and $X_R(Q_d^*) = C_R - e_R D - \sqrt{\frac{D(h_R f_R + k_R g_R + 2p_b f_R g_R)^2}{2(k_R+p_b f_R)(h_R+p_b g_R)}}$.*

Proof. For any Q, we have $\frac{f_R D}{Q} + \frac{g_R Q}{2} + e_R D \geq \sqrt{2f_R g_R D}$, it turns out that $\frac{f_R D}{Q} + \frac{g_R Q}{2} \geq \sqrt{2f_R g_R D}$. Because $X_R(Q) = C_R - \frac{f_R D}{Q} - \frac{g_R Q}{2} - e_R D$, there is $X_R(Q) \leq C_R - e_R D - \sqrt{2f_R g_R D}$. Hence, if $C_R - e_R D \leq \sqrt{2f_R g_R D}$, it implies that $X_R \leq 0$ and the retailer doesn't sell emission permits. Then we derive that $Q_d^* = Q_{d1}^* = \sqrt{\frac{2(K_R+p_b f_R)D}{h_R+p_b g_R}}$, $X_R(Q_d^*) = C_R - e_R D - \sqrt{\frac{D(h_R f_R + k_R g_R + 2p_b f_R g_R)^2}{2(k_R+p_b f_R)(h_R+p_b g_R)}}$. □

Lemma A.2. *If $f_R h_R < K_R g_R$, then $Q_{d1}^* < Q_{d2}^*$; If $f_R h_R = K_R g_R$, then $Q_{d1}^* = Q_{d2}^*$; If $f_R h_R > K_R g_R$, then $Q_{d1}^* > Q_{d2}^*$.*

Proof. We'll prove the first part of Lemma A.2. The proofs of other two parts are similar.

Because $p_b > p_s$, if $f_R h_R < K_R g_R$, then $(p_b - p_s) f_R h_R < (p_b - p_s) K_R g_R$.

Add $K_R h_R + p_b p_s f_R g_R$ to the both sides of the inequality, we have:
$(K_R + p_b f_R)(h_R + p_s g_R) < (K_R + p_s f_R)(h_R + p_b g_R)$.

Rearrange the above inequality, it is derived that $\frac{(K_R+p_b f_R)}{(h_R+p_b g_R)} < \frac{(K_R+p_s f_R)}{(h_R+p_s g_R)}$.

Furthermore, we get $\sqrt{\frac{2(K_R+p_b f_R)D}{(h_R+p_b g_R)}} < \sqrt{\frac{2(K_R+p_s f_R)D}{(h_R+p_s g_R)}}$, which implies that $Q_{d1}^* < Q_{d2}^*$. □

Lemma A.3. *When $(C_R - e_R D) > \sqrt{2g_R f_R D}$ and $f_R h_R < K_R g_R$, the following cases don't exist.*

I. $Q_{d1}^* \leq Q_1 < Q_2 \leq Q_{d2}^*$;
II. $Q_{d1}^* \leq Q_1 < Q_{d2}^* < Q_2$;
III. $Q_{d1}^* < Q_{d2}^* \leq Q_1 < Q_2$.

Proof.

I. Because $RC_2(Q, X_R(Q))$ is a strictly convex function of Q and is minimized at Q_{d2}^*, which implies that: if $Q_1 < Q_2 \leq Q_{d2}^*$, then $RC_2(Q_1, X_R(Q_1)) > RC_2(Q_2, X_R(Q_2))$.
When $Q = Q_1$ or $Q = Q_2$, we know that $X_R(Q) = 0$ and $RC_1(Q, X_R(Q)) = RC_2(Q, X_R(Q))$. Thus, we have:

$$RC_1(Q_1, X_R(Q_1)) > RC_1(Q_2, X_R(Q_2)). \tag{A.1}$$

While, for $RC_1(Q, X_R(Q))$, if $Q_{d1}^* \leq Q_1 < Q_2$, we have:

$$RC_1(Q_1, X_R(Q_1)) < RC_1(Q_2, X_R(Q_2)). \tag{A.2}$$

Equation (A.1) is contradictory with equation (A.2), thus, $Q_{d1}^* \leq Q_1 < Q_2 \leq Q_{d2}^*$ doesn't hold.

II. If $Q_1 < Q_{d2}^* < Q_2$, we have $RC_1(Q_{d2}^*, X_R(Q_{d2}^*)) < RC_2(Q_{d2}^*, X_R(Q_{d2}^*))$ and $RC_2(Q_{d2}^*, X_R(Q_{d2}^*)) < RC_2(Q_1, X_R(Q_1))$, therefore, it is derived that $RC_1(Q_{d2}^*, X_R(Q_{d2}^*)) < RC_2(Q_1, X_R(Q_1))$.
When $Q = Q_1$, we obtain that $RC_1(Q_1, X_R(Q_1)) = RC_2(Q_1, X_R(Q_1))$, therefore, it is derived that:

$$RC_1(Q_{d2}^*, X_R(Q_{d2}^*)) < RC_1(Q_1, X_R(Q_1)). \tag{A.3}$$

While, for $RC_1\left(Q, X_R\left(Q\right)\right)$, if $Q^*_{d1} \leq Q_1 < Q^*_{d2}$, we have:

$$RC_1\left(Q^*_{d2}, X_R\left(Q^*_{d2}\right)\right) > RC_1\left(Q_1, X_R\left(Q_1\right)\right). \tag{A.4}$$

Equation (A.3) is contradictory with equation (A.4). Thus, $Q^*_{d1} \leq Q_1 < Q^*_{d2} < Q_2$ doesn't hold.

III. If $Q^*_{d2} \leq Q_1$, it implies that $\sqrt{\frac{2(K_R+p_s f_R)D}{h_R+p_s g_R}} \leq \frac{C_R-e_R D-\sqrt{(C_R-e_R D)^2-2g_R f_R D}}{g_R}$.
Taking the square of both sides, it turns out that:
$\frac{(K_R+p_s f_R)D}{h_R+p_s g_R} \leq \frac{(C_R-e_R D)^2-(C_R-e_R D)\sqrt{(C_R-e_R D)^2-2g_R f_R D}-g_R f_R D}{(g_R)^2}$.
Furthermore, it is proved that:
$\frac{(K_R g_R+p_s f_R g_R)D}{h_R+p_s g_R} \leq \frac{(C_R-e_R D)^2-(C_R-e_R D)\sqrt{(C_R-e_R D)^2-2g_R f_R D}-g_R f_R D}{g_R}$.
When $Q^*_{d1} < Q^*_{d2}$, $f_R h_R < K_R g_R$ holds, therefore, we have:
$\frac{(f_R h_R+p_s f_R g_R)D}{h_R+p_s g_R} < \frac{(C_R-e_R D)^2-(C_R-e_R D)\sqrt{(C_R-e_R D)^2-2g_R f_R D}-g_R f_R D}{g_R}$.
Rearrange the above inequality, we have:

$$\left(C_R - e_R D\right)^2 - 2g_R f_R D > \left(C_R - e_R D\right)\sqrt{\left(C_R - e_R D\right)^2 - 2g_R f_R D}. \tag{A.5}$$

Since $(C_R - e_R D) > \sqrt{2g_R f_R D}$, equation (A.5) doesn't hold. Therefore, $Q^*_{d1} < Q^*_{d2} \leq Q_1 < Q_2$ doesn't hold.

□

On the basis of Lemma A.3, we present Corollary A.4.

Corollary A.4. *If $(C_R - e_R D) > \sqrt{2g_R f_R D}$ and $f_R h_R < K_R g_R$, the following cases may exist.*

I. $Q_1 < Q_2 \leq Q^*_{d1} < Q^*_{d2}$;
II. $Q_1 < Q^*_{d1} < Q^*_{d2} \leq Q_2$;
III. $Q_1 < Q^*_{d1} < Q_2 < Q^*_{d2}$.

Lemma A.5. *If $(C_R - e_R D) > \sqrt{2g_R f_R D}$ and $f_R h_R > K_R g_R$, the following cases don't exist.*

I. $Q^*_{d2} < Q_1 < Q_2 \leq Q^*_{d1}$;
II. $Q_1 \leq Q^*_{d2} < Q_2 \leq Q^*_{d1}$;
III. $Q_1 < Q_2 \leq Q^*_{d2} < Q^*_{d1}$.

Proof.

I. If $Q^*_{d2} < Q_1 < Q_2 \leq Q^*_{d1}$, $RC_1\left(Q^*_{d1}, X_R\left(Q^*_{d1}\right)\right) > RC_2\left(Q^*_{d1}, X_R\left(Q^*_{d1}\right)\right)$.
Moreover, $RC_2\left(Q^*_{d1}, X_R\left(Q^*_{d1}\right)\right) \geq RC_2\left(Q_2, X_R\left(Q_2\right)\right) > RC_2\left(Q_1, X_R\left(Q_1\right)\right)$,
which implies that $RC_1\left(Q^*_{d1}, X_R\left(Q^*_{d1}\right)\right) > RC_2\left(Q_1, X_R\left(Q_1\right)\right)$.
When $Q = Q_1$, we know that $RC_1\left(Q_1, X_R\left(Q_1\right)\right) = RC_2\left(Q_1, X_R\left(Q_1\right)\right)$.
Thus, we have:

$$RC_1\left(Q^*_{d1}, X_R\left(Q^*_{d1}\right)\right) > RC_1\left(Q_1, X_R\left(Q_1\right)\right). \tag{A.6}$$

While $RC_1\left(Q, X_R\left(Q\right)\right)$ is minimized at Q^*_{d1}, therefore, equation (A.6) doesn't hold and $Q^*_{d2} < Q_1 < Q_2 \leq Q^*_{d1}$ doesn't hold.

II. If $Q_1 \leq Q^*_{d2} < Q_2$, we have $RC_2\left(Q^*_{d2}, X_R\left(Q^*_{d2}\right)\right) > RC_1\left(Q^*_{d2}, X_R\left(Q^*_{d2}\right)\right)$ and $RC_1\left(Q^*_{d2}, X_R\left(Q^*_{d2}\right)\right) > RC_1\left(Q_2, X_R\left(Q_2\right)\right) \geq RC_2\left(Q^*_{d1}, X_R\left(Q^*_{d1}\right)\right)$. Thus, we have $RC_2\left(Q^*_{d2}, X_R\left(Q^*_{d2}\right)\right) > RC_1\left(Q^*_{d1}, X_R\left(Q^*_{d1}\right)\right)$.

While if $Q_2 \leq Q_{d1}^*$, it implies that $RC_1\left(Q_{d1}^*, X_R\left(Q_{d1}^*\right)\right) > RC_2\left(Q_{d1}^*, X_R\left(Q_{d1}^*\right)\right)$, furthermore, we derive that:

$$RC_2\left(Q_{d2}^*, X_R\left(Q_{d2}^*\right)\right) > RC_2\left(Q_{d1}^*, X_R\left(Q_{d1}^*\right)\right) \tag{A.7}$$

While $RC_2\left(Q, X_R\left(Q\right)\right)$ is minimized at Q_{d2}^*, therefore, equation (A.7) doesn't hold and $Q_1 \leq Q_{d2}^* < Q_2 \leq Q_{d1}^*$ doesn't hold.

III. If $Q_2 \leq Q_{d2}^*$, it implies that $\frac{C_R - e_R D + \sqrt{(C_R - e_R D)^2 - 2g_R f_R D}}{g_R} \leq \sqrt{\frac{2(K_R + p_s f_R)D}{h_R + p_s g_R}}$. Taking the square of both sides, it turns out that:
$\frac{(C_R - e_R D)^2 + (C_R - e_R D)\sqrt{(C_R - e_R D)^2 - 2g_R f_R D} - g_R f_R D}{g_R} \leq \frac{(K_R g_R + p_s f_R g_R)D}{h_R + p_s g_R}$.
Because when $Q_{d1}^* > Q_{d2}^*$, $f_R h_R > K_R g_R$ holds, then we have:
$\frac{(C_R - e_R D)^2 + (C_R - e_R D)\sqrt{(C_R - e_R D)^2 - 2g_R f_R D} - g_R f_R D}{g_R} < \frac{(f_R h_R + p_s f_R g_R)D}{h_R + p_s g_R}$.
Rearrange the above inequality, we have:

$$\left(C_R - e_R D\right)^2 - 2g_R f_R D < -\left(C_R - e_R D\right)\sqrt{\left(C_R - e_R D\right)^2 - 2g_R f_R D}. \tag{A.8}$$

Since $\left(C_R - e_R D\right) > \sqrt{2g_R f_R D}$, equation (A.8) doesn't hold. Therefore, $Q_1 < Q_2 \leq Q_{d2}^* < Q_{d1}^*$ doesn't take place.

□

On the basis of Lemma A.5, we present Corollary A.6.

Corollary A.6. *When $\left(C_R - e_R D\right) > \sqrt{2g_R f_R D}$ and $f_R h_R > K_R g_R$, the following cases may exist.*

I. $Q_1 \leq Q_{d2}^* < Q_{d1}^* < Q_2$;
II. $Q_{d2}^* < Q_1 < Q_{d1}^* < Q_2$;
III. $Q_{d2}^* < Q_{d1}^* \leq Q_1 < Q_2$.

Synthesizing the above lemmas and corollaries, we can compare Q_1, Q_2, Q_{d1}^* and Q_{d2}^*, and obtain retailer's optimal decisions in the decentralized model. Next, we'll discuss the condition of $Q_1 < Q_2 \leq Q_{d1}^* < Q_{d2}^*$, the other proofs are similar.

Proof. When $Q_1 < Q_2 \leq Q_{d1}^* < Q_{d2}^*$, there are three circumstances.

I. When $Q > Q_2$, then $X_R\left(Q\right) < 0$, which implies that $RC\left(Q, X_R\left(Q\right)\right) = RC_1\left(Q, X_R\left(Q\right)\right)$. Since $RC_1\left(Q, X_R\left(Q\right)\right)$ is minimized at Q_{d1}^*, and $Q_2 \leq Q_{d1}^*$, we have $RC\left(Q, X_R\left(Q\right)\right) \geq RC_1\left(Q_{d1}^*, X_R\left(Q_{d1}^*\right)\right)$.
II. When $Q_1 \leq Q \leq Q_2$, then $X_R\left(Q\right) \geq 0$, which implies that $RC\left(Q, X_R\left(Q\right)\right) = RC_2\left(Q, X_R\left(Q\right)\right)$. Since $RC_2\left(Q, X_R\left(Q\right)\right)$ is minimized at Q_{d2}^* and $Q \leq Q_2 < Q_{d2}^*$, we have $RC\left(Q, X_R\left(Q\right)\right) \geq RC\left(Q_2, X_R\left(Q_2\right)\right)$. Because $X_R\left(Q_1\right) = X_R\left(Q_2\right) = 0$, we can obtain that $RC\left(Q_2, X_R\left(Q_2\right)\right) = RC_2\left(Q_2, X_R\left(Q_2\right)\right) = RC_1\left(Q_1, X_R\left(Q_1\right)\right)$. Because $RC_1\left(Q_1, X_R\left(Q_1\right)\right) \geq RC_1\left(Q_{d1}^*, X_R\left(Q_{d1}^*\right)\right)$, then we have $RC\left(Q, X_R\left(Q\right)\right) \geq RC_1\left(Q_{d1}^*, X_R\left(Q_{d1}^*\right)\right)$.
III. When $Q < Q_1$, then $X_R\left(Q\right) < 0$, which implies that $RC\left(Q, X_R\left(Q\right)\right) = RC_1\left(Q, X_R\left(Q\right)\right)$. Since $RC_1\left(Q, X_R\left(Q\right)\right)$ is minimized at Q_{d1}^*, and $Q_1 \leq Q_{d1}^*$, we have $RC\left(Q, X_R\left(Q\right)\right) \geq RC\left(Q_1, X_R\left(Q_1\right)\right)$. In addition, the relationship that $RC\left(Q_1, X_R\left(Q_1\right)\right) \geq RC\left(Q_2, X_R\left(Q_2\right)\right)$ implies $RC\left(Q, X_R\left(Q\right)\right) \geq RC\left(Q_2, X_R\left(Q_2\right)\right)$. Because $RC\left(Q_2, X_R\left(Q_2\right)\right) \geq RC\left(Q_{d1}^*, X_R\left(Q_{d1}^*\right)\right)$, we have $RC\left(Q, X_R\left(Q\right)\right) \geq RC_1\left(Q_{d1}^*, X_R\left(Q_{d1}^*\right)\right)$.

□

A.2 Proof of Theorem 3.2

I. When $X_M \leq 0$, the manufacturer's model is as follows:

$\min MC_1\left(Q_d^*, X_M, \theta\right) = \frac{K_M D}{Q_d^*} + \frac{h_M D Q_d^*}{2P} + p_M D - p_b X_M + \frac{r\theta^2}{2}$

s.t. $\frac{f_M D}{Q_d^*} + \frac{g_M D Q_d^*}{2P} + (a - a\theta) D + X_M = C_M$

$0 < \theta < 1$

$X_M \leq 0.$

To solve this problem, we set up the Lagrange function with Lagrange multipliers μ_1, μ_2, μ_3, μ_4:

$$L_1\left(X_M, \theta, \mu_1, \mu_2, \mu_3, \mu_4\right) = \frac{K_M D}{Q_d^*} + \frac{h_M D Q_d^*}{2P} + p_M D - p_b X_M + \frac{r\theta^2}{2} + \mu_1\left[\frac{f_M D}{Q_d^*} + \frac{g_M D Q_d^*}{2P} + (a - a\theta) D + X_M - C_M\right] + \mu_2\theta + \mu_3(1-\theta) - \mu_4 X_M. \tag{A.9}$$

The Kuhn-Tucker conditions for equation (A.9) are:

$$\begin{cases} \frac{\partial L_1}{\partial \theta} = r\theta - \mu_1 a D + \mu_2 - \mu_3 = 0 \\ \frac{\partial L_1}{\partial X_M} = -p_b + \mu_1 - \mu_4 = 0 \\ \mu_1\left[\frac{f_M D}{Q_d^*} + \frac{g_M D Q_d^*}{2P} + (a - a\theta) D + X_M - C_M\right] = 0 \\ \mu_2\theta = 0 \\ \mu_3(1-\theta) = 0 \\ \mu_4 X_M = 0 \\ \mu_i \geq 0, \quad i = 1,2,3,4 \end{cases}$$

Since $\theta \neq 0$, $\theta \neq 1$, then $\mu_2 = \mu_3 = 0$, $\mu_1 = \frac{r\theta}{aD}$, $\mu_4 = \mu_1 - p_b$.

i. If $\mu_1 - p_b = 0$ and $X_M \neq 0$, then $\mu_1 = p_b$, $\theta = \frac{ap_b D}{r}$, $X_M = C_M - \frac{f_M D}{Q_d^*} - \frac{g_M D Q_d^*}{2P} - \left(a - \frac{a^2 p_b D}{r}\right) D$ at the condition of $0 < \theta < 1$ and $X_M < 0$. Therefore, the optimal solutions exist only when $D < \frac{r}{ap_b}$ and $C_M < \frac{f_M D}{Q_d^*} + \frac{g_M D Q_d^*}{2P} + \left(a - \frac{a^2 p_b D}{r}\right) D$.

ii. If $\mu_1 - p_b \neq 0$ and $X_M = 0$, then $\theta = \frac{\frac{f_M D}{Q_d^*} + \frac{g_M D Q_d^*}{2P} + aD - C_M}{aD}$.

II. When $X_M > 0$, the manufacturer's model is as follows:

$$\begin{aligned} &\min MC_2\left(Q_d^*, X_M, \theta\right) = \frac{K_M D}{Q_d^*} + \frac{h_M D Q_d^*}{2P} + p_M D - p_s X_M + \frac{r\theta^2}{2} \\ &\text{s.t. } \frac{f_M D}{Q_d^*} + \frac{g_M D Q_d^*}{2P} + (a - a\theta) D + X_M = C_M \\ &\quad 0 < \theta < 1 \\ &\quad X_M > 0. \end{aligned}$$

To solve this problem, we set up the Lagrange function with Lagrange multipliers $\hat{\mu}_1$, $\hat{\mu}_2$, $\hat{\mu}_3$, $\hat{\mu}_4$:

$$L_2\left(X_M, \theta, \hat{\mu}_1, \hat{\mu}_2, \hat{\mu}_3, \hat{\mu}_4\right) = \frac{K_M D}{Q_d^*} + \frac{h_M D Q_d^*}{2P} + p_M D - p_s X_M + \frac{r\theta^2}{2} + \hat{\mu}_1\left[\frac{f_M D}{Q_d^*} + \frac{g_M D Q_d^*}{2P} + (a - a\theta) D + X_M - C_M\right] + \hat{\mu}_2\theta + \hat{\mu}_3(1-\theta) + \hat{\mu}_4 X_M. \tag{A.10}$$

The Kuhn-Tucker conditions for equation (A.10) are:

$$\begin{cases} \frac{\partial L_2}{\partial \theta} = r\theta - \hat{\mu}_1 aD + \hat{\mu}_2 - \hat{\mu}_3 = 0 \\ \frac{\partial L_2}{\partial X_M} = -p_s + \hat{\mu}_1 + \hat{\mu}_4 = 0 \\ \hat{\mu}_1 \left[\frac{f_M D}{Q_d^*} + \frac{g_M D Q_d^*}{2P} + (a - a\theta) D + X_M - C_M \right] = 0 \\ \hat{\mu}_2 \theta = 0 \\ \hat{\mu}_3 (1 - \theta) = 0 \\ \hat{\mu}_4 X_M = 0 \\ \hat{\mu}_i \geq 0, \quad i = 1, 2, 3, 4 \end{cases}$$

Since $\theta \neq 0$, $\theta \neq 1$, $X_M \neq 0$, we have $\hat{\mu}_2 = \hat{\mu}_3 = \hat{\mu}_4 = 0$, $\hat{\mu}_1 = p_s$, $\theta = \frac{ap_s D}{r}$, $X_M = C_M - \frac{f_M D}{Q_d^*} - \frac{g_M D Q_d^*}{2P} - \left(a - \frac{a^2 p_s D}{r}\right) D$ at the condition of $0 < \theta < 1$ and $X_M > 0$. Therefore, the optimal solutions exist only when $D < \frac{r}{ap_s}$, $C_M > \frac{f_M D}{Q_d^*} + \frac{g_M D Q_d^*}{2P} + \left(a - \frac{a^2 p_s D}{r}\right) D$.
Because $p_b > p_s$, we have $\frac{f_M D}{Q_d^*} + \frac{g_M D Q_d^*}{2P} + \left(a - \frac{a^2 p_s D}{r}\right) D > \frac{f_M D}{Q_d^*} + \frac{g_M D Q_d^*}{2P} + \left(a - \frac{a^2 p_b D}{r}\right) D$. Therefore, when $\frac{f_M D}{Q_d^*} + \frac{g_M D Q_d^*}{2P} + \left(a - \frac{a^2 p_s D}{r}\right) D \geq C_M \geq \frac{f_M D}{Q_d^*} + \frac{g_M D Q_d^*}{2P} + \left(a - \frac{a^2 p_b D}{r}\right) D$, $X_M = 0$, $\theta = \frac{\frac{f_M D}{Q_d^*} + \frac{g_M D Q_d^*}{2P} + aD - C_M}{aD}$.

A.3 Proof of Theorem 3.3

I. When $X_s \leq 0$, the optimal model of supply chain is as follows:

$\min SC_1 (Q, X_S, \theta) = \frac{(K_R + K_M) D}{Q} + \frac{\left(h_R + \frac{h_M D}{P}\right) Q}{2} + (c + p_M) D - p_b X_s + \frac{r\theta^2}{2}$

s.t. $\frac{(f_R + f_M) D}{Q} + \frac{\left(g_R + \frac{g_M D}{P}\right) Q}{2} + (e_R + a - a\theta) D + X_s = C_R + C_M$

$0 < \theta < 1$

$Q > 0$

$X_s \leq 0.$

Then, we set up the Lagrange function with Lagrange multipliers μ_1, μ_2, μ_3, μ_4, μ_5:

$$\begin{aligned} L_1 (Q, X_S, \theta, \mu_1, \mu_2, \mu_3, \mu_4, \mu_5) = {} & \frac{(K_R + K_M) D}{Q} + \frac{\left(h_R + \frac{h_M D}{P}\right) Q}{2} + (c + p_M) D - p_b X_s + \frac{r\theta^2}{2} + \mu_1 \\ & \times \left[\frac{(f_R + f_M) D}{Q} + \frac{\left(g_R + \frac{g_M D}{P}\right) Q}{2} \right. \\ & \left. + (e_R + a - a\theta) D + X_s - C_R - C_M \right] \\ & + \mu_2 \theta + \mu_3 (1 - \theta) + \mu_4 Q - \mu_5 X_s. \end{aligned} \tag{A.11}$$

The Kuhn-Tucker conditions for equation (A.11) are:

$$\begin{cases} \frac{\partial L_1}{\partial Q} = -\frac{(K_R+K_M)D}{Q^2} + \frac{h_R+\frac{h_M D}{P}}{2} + \mu_1\left[-\frac{(f_R+f_M)D}{Q^2} + \frac{g_R+\frac{g_M D}{P}}{2}\right] + \mu_4 = 0 \\ \frac{\partial L_1}{\partial \theta} = r\theta - \mu_1 aD + \mu_2 - \mu_3 = 0 \\ \frac{\partial L_1}{\partial X_s} = -p_b + \mu_1 - \mu_5 = 0 \\ \mu_1\left[\frac{(f_R+f_M)D}{Q} + \frac{\left(g_R+\frac{g_M D}{P}\right)Q}{2} + (e_R + a - a\theta)D + X_s - C_R - C_M\right] = 0 \\ \mu_2\theta = 0 \\ \mu_3(1-\theta) = 0 \\ \mu_4 Q = 0 \\ \mu_5 X_s = 0 \\ \mu_i \geq 0, \quad i = 1,2,3,4,5 \end{cases} \tag{A.12}$$

Since $\theta \neq 0$, $\theta \neq 1$, $Q \neq 0$, we have $\mu_2 = \mu_3 = \mu_4 = 0$. From equation (A.12), we derive that $\mu_1 = \frac{r\theta}{aD}$. Because $r \neq 0$, $\theta \neq 0$, then $\mu_1 \neq 0$. It is also derived from equation (A.12) that $\mu_5 = \mu_1 - p_b$. Furthermore, we have:

$$\begin{cases} -\frac{(K_R+K_M)D}{Q^2} + \frac{h_R+\frac{h_M D}{P}}{2} + \mu_1\left[-\frac{(f_R+f_M)D}{Q^2} + \frac{g_R+\frac{g_M D}{P}}{2}\right] = 0 \\ (\mu_1 - p_b)X_s = 0 \\ \frac{(f_R+f_M)D}{Q} + \frac{\left(g_R+\frac{g_M D}{P}\right)Q}{2} + (e_R + a - a\theta)D + X_s - C_R - C_M = 0 \end{cases}$$

i. If $\mu_1 - p_b = 0$ and $X_s \neq 0$, then

$\mu_1 = p_b\theta = \frac{ap_bD}{r}$, $Q = \sqrt{\frac{2[K_R+K_M+p_b(f_R+f_M)]D}{h_R+\frac{h_M D}{P}+p_b\left(g_R+\frac{g_M D}{P}\right)}}$, and

$$X_s = C_R + C_M - \left(e_R + a - \frac{a^2 p_b D}{r}\right)D - \frac{(f_R+f_M)D}{\sqrt{\frac{2[K_R+K_M+p_b(f_R+f_M)]D}{h_R+\frac{h_M D}{p}+p_b\left(g_R+\frac{g_M D}{P}\right)}}} - \frac{\left(g_R+\frac{g_M D}{P}\right)\sqrt{\frac{2[K_R+K_M+p_b(f_R+f_M)]D}{h_R+\frac{h_M D}{p}+p_b\left(g_R+\frac{g_M D}{P}\right)}}}{2}$$ at the condition of $0 < \theta < 1$ and $X_s < 0$.

Therefore, the optimal solutions exist only when $D < \frac{r}{ap_b}$,

$$C_R + C_M < \left(e_R + a - \frac{a^2 p_b D}{r}\right)D + \frac{(f_R+f_M)D}{\sqrt{\frac{2[K_R+K_M+p_b(f_R+f_M)]D}{h_R+\frac{h_M D}{p}+p_b\left(g_R+\frac{g_M D}{P}\right)}}} + \frac{\left(g_R+\frac{g_M D}{P}\right)\sqrt{\frac{2[K_R+K_M+p_b(f_R+f_M)]D}{h_R+\frac{h_M D}{p}+p_b\left(g_R+\frac{g_M D}{P}\right)}}}{2}$$ hold.

ii. If $\mu_1 - p_b \neq 0$ and $X_s = 0$, then $\mu_1 = \frac{r\theta}{aD}$ at the condition of

$$\begin{cases} \frac{-(K_R+K_M)D}{Q^2} + \frac{h_R+\frac{h_M D}{P}}{2} = \frac{r\theta}{aD}\left[\frac{(f_R+f_M)D}{Q^2} - \frac{g_R+\frac{g_M D}{P}}{2}\right] \\ \frac{(f_R+f_M)D}{Q} + \frac{\left(g_R+\frac{g_M D}{P}\right)Q}{2} + (e_R + a - a\theta)D = C_R + C_M \end{cases}.$$

II. When $X_s > 0$, the optimal model of supply chain is as follows:

$$\begin{aligned} \min \quad & SC_2(Q, X_S, \theta) = \frac{(K_R+K_M)D}{Q} + \frac{\left(h_R+\frac{h_M D}{P}\right)Q}{2} + (c + p_M)D - p_s X_s + \frac{r\theta^2}{2} \\ \text{s.t.} \quad & \frac{(f_R+f_M)D}{Q} + \frac{\left(g_R+\frac{g_M D}{P}\right)Q}{2} + (e_R + a - a\theta)D + X_s = C_R + C_M \\ & 0 < \theta < 1 \\ & Q > 0 \\ & X_s > 0. \end{aligned}$$

Then, we set up the Lagrange function with Lagrange multipliers $\hat{\mu}_1$, $\hat{\mu}_2$, $\hat{\mu}_3$, $\hat{\mu}_4$, $\hat{\mu}_5$:

$$\begin{aligned} L_2\left(Q, X_S, \theta, \hat{\mu}_1, \hat{\mu}_2, \hat{\mu}_3, \hat{\mu}_4, \hat{\mu}_5\right) = & \frac{\left(K_R+K_M\right) D}{Q}+\frac{\left(h_R+\frac{h_M D}{P}\right) Q}{2}+\left(c+p_M\right) D-p_s X_s+\frac{r \theta^2}{2} \\ & +\hat{\mu}_1\left[\frac{\left(f_R+f_M\right) D}{Q}+\frac{\left(g_R+\frac{g_M D}{P}\right) Q}{2}+\left(e_R+a-a \theta\right) D\right. \\ & \left.+X_s-C_R-C_M\right]+\hat{\mu}_2 \theta+\hat{\mu}_3(1-\theta)+\hat{\mu}_4 Q+\hat{\mu}_5 X_s. \end{aligned} \tag{A.13}$$

The Kuhn-Tucker conditions for equation (A.13) are:

$$\left\{\begin{array}{l} \frac{\partial L_2}{\partial Q}=-\frac{\left(K_R+K_M\right) D}{Q^2}+\frac{h_R+\frac{h_M D}{P}}{2}+\hat{\mu}_1\left[-\frac{\left(f_R+f_M\right) D}{Q^2}+\frac{g_R+\frac{g_M D}{P}}{2}\right]+\hat{\mu}_4=0 \\ \frac{\partial L_2}{\partial \theta}=r \theta-\hat{\mu}_1 a D+\hat{\mu}_2-\hat{\mu}_3=0 \\ \frac{\partial L_2}{\partial X_s}=-p_s+\hat{\mu}_1-\hat{\mu}_5=0 \\ \hat{\mu}_1\left[\frac{\left(f_R+f_M\right) D}{Q}+\frac{\left(g_R+\frac{g_M D}{P}\right) Q}{2}+\left(e_R+a-a \theta\right) D+X_s-C_R-C_M\right]=0 \\ \hat{\mu}_2 \theta=0 \\ \hat{\mu}_3(1-\theta)=0 \\ \hat{\mu}_4 Q=0 \\ \hat{\mu}_5 X_s=0 \\ \hat{\mu}_i \geq 0, i=1,2,3,4,5. \end{array}\right. \tag{A.14}$$

Since $\theta \neq 0$, $\theta \neq 1$, $Q \neq 0$, $X_s > 0$, then $\hat{\mu}_2 = \hat{\mu}_3 = \hat{\mu}_4 = \hat{\mu}_5 = 0$. From equation (A.14) we derive that $\hat{\mu}_1 = p_s$, and $\theta = \frac{a p_s D}{r}$, $Q = \sqrt{\frac{2\left[K_R+K_M+p_s\left(f_R+f_M\right)\right] D}{h_R+\frac{h_M D}{P}+p_s\left(g_R+\frac{g_M D}{P}\right)}}$,

$X_s = C_R + C_M - \left(e_R + a - \frac{a^2 p_s D}{r}\right) D - \frac{\left(f_R+f_M\right) D}{\sqrt{\frac{2\left[K_R+K_M+p_s\left(f_R+f_M\right)\right] D}{h_R+\frac{h_M D}{p}+p_s\left(g_R+\frac{g_M D}{P}\right)}}} - \frac{\left(g_R+\frac{g_M D}{P}\right) \sqrt{\frac{2\left[K_R+K_M+p_s\left(f_R+f_M\right)\right] D}{h_R+\frac{h_M D}{p}+p_s\left(g_R+\frac{g_M D}{P}\right)}}}{2}$ at the condition of $0 < \theta < 1$ and $X_s > 0$.

Therefore, the optimal solutions exist only when $D < \frac{r}{a p_s}$ and

$C_R + C_M > \left(e_R + a - \frac{a^2 p_s D}{r}\right) D + \frac{\left(f_R+f_M\right) D}{\sqrt{\frac{2\left[K_R+K_M+p_s\left(f_R+f_M\right)\right] D}{h_R+\frac{h_M D}{p}+p_s\left(g_R+\frac{g_M D}{P}\right)}}} + \frac{\left(g_R+\frac{g_M D}{P}\right) \sqrt{\frac{2\left[K_R+K_M+p_s\left(f_R+f_M\right)\right] D}{h_R+\frac{h_M D}{p}+p_s\left(g_R+\frac{g_M D}{P}\right)}}}{2}$ hold.

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