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REDUCING THE BULLWHIP EFFECT IN A SUPPLY CHAIN NETWORK BY APPLICATION OF OPTIMAL CONTROL THEORY

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Abstract. Controlling the bullwhip effect and reducing the propagated inventory levels throughout the supply chain layers has an important role in reducing the total inventory costs of a supply chain. In this study, an optimal controller that considers demand as control variable is designed to dampen propagated inventory fluctuations for each node throughout the supply chain network. The model proves to be very useful in revealing the dynamic characteristics of the chain and provides a proper interface to study decisions taken into account at each node of the supply chain in different periods by decision makers (DMs). In the proposed approach, two feedback loops and online updated values of net stock quantities are used for calculation of the orders. To investigate the efficiency of the proposed approach, a real case of bicycle industry is conducted. The acquired results justify the efficiency of the proposed approach in controlling and dampening the bullwhip effect and reducing inventory levels, net stock quantities and inventory attributed costs throughout the supply chain network layers.

Mathematics Subject Classification. 49N90, 37N40, 47N10, 78M50

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1. Introduction

There are four major causes for emerging bullwhip effect [35] and the most important one is propagated demand forecasts caused by each different layer and echelon of the supply chain (SC) network. These forecasted demands impose devastatingly high inventory holding costs to the SC. Consequently, this increases the price of products and leads to inefficiencies in SC management. The dynamic nature of SC makes it a complex system with two major flows. The first is material flow from upstream (suppliers) to downstream (customers), and the other is information flow from customers toward suppliers. Neglecting the dynamic behavior of the SC and considering it as a steady state system, can cost a lot for its members. This behavior is thoroughly investigated by Towill [65]. Modeling supply chain by potentially evolving equations over the time, can immunize the system against demand fluctuation costs. It is worthy to note that inventory management includes approximately over than 30 percent of logistics costs as the American Logistics Aid Network (ALAN) claims. Therefore, an efficient

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decision making tool which helps managers to efficiently control inventory levels throughout the SC would reduce remarkable percent of logistics costs.

A SC is a complex network and its goal is to produce and distribute goods or services to the customers in order to satisfy their demands. Fluctuation of the demand and uncertainties in the market forces SC managers to study and investigate the dynamics of the SC. Accordingly, modeling the dynamics of a SC has become an interesting challenge among academics and practitioners in recent years [2, 33]. One of the most important phenomena of a SC is the amplification in demand rate which called bullwhip effect. This phenomenon declares that small fluctuation in demand in the downstream will be amplified drastically through the chain when it moves toward upstream. So the upstream members will face the propagated demand. This phenomenon was firstly remarked by Forrester [21]. The bullwhip effect increases the costs of inventory handling of a SC, unnecessarily. Approaches that are used by researchers to tackle with this evil are wide from developments in inventory policies [41], to modelling the supply chain as a centralized system [11, 35]. An early but informative review of the models and methods used in the literature can be found in Riddalls *et al.* [54]. Researchers have presented several approaches to eliminate the bullwhip effect such as increasing the buffers in the supply chain and developed mathematical models to optimize the SC as a centralized system. Holweg and Bicheno [28] mentioned the two different major approaches in dealing with SC dynamics: first, control theoretic based approaches, and second, the behavioral science based approach, so called beer game. We will propose analyzes, comparing these two major avenues and their results. Operation research based models have undeniable advantages in modeling and solving man-made problems, which is warranted by virtue of their widespread use among practitioners. But, these approaches fail to give insights into the dynamics of supply chain systems. For instance, Ivanov *et al.* [2] have investigated the body of the literature in order to find the gap between practical efforts and academic studies in supply chain dynamics and control. They mentioned that there is an essential need for an efficient decision making tool which works properly in such complex dynamic systems. Control theory is one the most important decision making tools widely used for modeling the dynamic behavior of a system from the very early studies up far [1, 3, 6, 10, 15, 47, 72]. This theory has all the required mathematical tools for studying and monitoring dynamic systems. Applications of control theory to production planning and inventory control were first studied by Simon [59] and extended by other researchers. One of the most well-known procedures of the control theory is optimal control. This application is mostly applied for optimizing and controlling complex and over time evolving systems. The applications of optimal control are in production scheduling, finance and economics, marketing and other areas [57]. Here, we apply these applications to a full-stage supply chain and calculate the order quantities for each node through the planning horizon. The contributions of this paper that differentiates it from those existing in the literature are:

- applying the optimal control theory to a full-stage supply chain from suppliers to customers;
- considering the five major elements in modeling the bullwhip effect, namely; demand, forecasting, time delay, ordering policy and information sharing;
- conducting a real case study in an Iranian bicycle company;
- comparing the proposed approach with prevalent methods available in the literature for controlling the bullwhip effect;
- redefining control theory concepts, which are mostly defined in electrical engineering, into common concepts in supply chain management literature;
- Providing a proper interface to study the decisions taken into account in the nodes of the chain (behavior of them), regarding supply chain dynamics and revealing the interactions among the key factors.

To the best of our knowledge, there is no research paper in the literature addressing the above mentioned extensions within a simple yet effective mathematical modeling approach. The body of the paper is organized as follows. Next section is about the background of the bullwhip effect and control theory applications in supply chain management and gap analyses. In Section 3, the problem is described and required control theory concepts are redefined in to prevalent concepts in supply chain management philosophy. Also, modeling the dynamic nature of supply chain through optimal control theory to control the bullwhip effect is addressed in this section. In Section 4, computational results are presented. The results of both, proposed model and the beer game simulation approaches are compared and thoroughly analyzed, and further, used for controlling the demand variation. Furthermore, a real case of an Iranian bicycle company is studied and the acquired results of application are described. Finally, Section 5 concludes this paper and offers some useful directions for future researches.

2. Literature review

Through the past decades, many studies have been conducted on the bullwhip effect reduction. But there is a lack of appropriate models for studying and controlling the supply chain dynamics, especially when addressing the reduction of the bullwhip effect is needed. This section briefly discusses about the background of bullwhip effect, related studies and mainly addresses the gaps between steady state models and dynamic models, representing supply chain networks and those whom are appropriate for tackling with dynamic properties of supply chain systems. This section is subcategorized into three subsections, first the early works on bullwhip effect are investigated from both academia and practitioner's perspective. Then, control theory applications and optimal control theoretic based models are thoroughly investigated and analyzed. At the end, the gap in the literature is pointed out and contributions of this study are clarified.

2.1. Early works on bullwhip effect identification and quantification

As mentioned earlier, the first research on the bullwhip effect was done by Forrester [21]. Furthermore, many researchers have studied this phenomenon to quantify, find and classify the reasons. Early discussions on the areas of bullwhip effect were carried out by authors such as Chen *et al.* [13], Chen *et al.* [12], Lee *et al.* [35], Metters [41] and Fransoo and Wouters [22]. These early studies just cover the identifying and discovering the reasons of happening this effect or dampening the bad consequences of this evil. Sucky [62] divided the researches on bullwhip effect into six main classifications:

- papers attempting to measure and quantify the bullwhip effect;
- papers aiming to find the causes of bullwhip effect;
- works studying the bullwhip effect in industries and factories;
- *papers focusing on finding the ways to reduce or eliminate the bullwhip effect*;
- *works studying simulation of system behavior*;
- papers aiming to validate bullwhip effect by experimental results.

Considering the above categorization, developing a proper controlling scheme for orders and demands is an attempt to reduce or even eliminate the bullwhip effect while studying the system behavior (italic items). As already mentioned, system behavior is the decisions made by DMs in each level of the chain in response to the new conditions of each period. These decisions will determine the status of each node in the chain, characteristics like inventory on hand and shortage. By the time, researchers noticed a lack of control theoretic based models in supply chain management scope, specifically models that are controlling or trying to dampen the bullwhip effect. Control theoretic based models have the advantages of revealing the interactions caused by the dynamics of the system. Earlier studies focused on production planning control and applying control theory to this area, later the main core of these works was on inventory control policies known as IOBPCS family of decision support systems. Disney and Towill [16] discussed on these control systems and classified them by considering three control drivers, feed forwarded customer demand, target inventory and target pipeline. Lin *et al.* [40] reviewed citations to Towill's works on the dynamics of IOBPCS and APIOBPCS. This direction of study is still open and many valuable studies are publishing on that, for instance, see; Parsanejad and Matsukawa [46] and White and Censlive [68]. Applications of control theory to SC management have been widely increased in recent years because of the high mathematical analyses that these applications can potentially provide. Operation research based approaches are commonly used in both human science and business studies and the evidence of this claim is the numerous case studies and practical usage of them. But these models cannot deal with the dynamic nature of the decision making procedures.

2.2. Optimal control theoretic based efforts on bullwhip effect

One of the well-known control theories, is the optimal control, but there is not many studies regarding SC and production scheduling by means of optimal control. Studies have been began in early 1990, Lobo Pereira and Borges De Sousa [48], Bai and Elhafsi [4] and Egilmez and Sharifnia [18], are such earlier works that apply the optimal control approach to tackle with the dynamic nature of scheduling and production scheduling problems and not the behavior of the supply chain networks and such complicated systems. Note that, we assume the DMs act rationally. This means they will behave in such manner to increase their profit while decreasing their costs and satisfying the customer's demand. Riddalls and Bennett [53] presented a method to optimally control aggregate production-inventory systems, by applying a novel optimal control algorithm to a differential equation model of a production-inventory system in which batch production costs have a strong influence on the overall production-inventory cost structure. Sethi and Thompson [57] discussed the applications of optimal control theory in management and economy with details. These applications are extended to achieve proper modeling approaches for investigating the supply chain network in this study. Cheng and Duran [14] developed a decision support system (DSS) to improve the interactive inventory and transportation system in the world-wild crude oil industry. Discrete event simulation and optimal control are used to formulate the DSS, but as they have also mentioned their proposed model lacks the sufficient computational efficiency, so there is a need for a mathematical and practical approach to investigate the SC networks. Giglio *et al.* [25] proposed a hybrid model which includes continuous variables with discrete-event dynamics. The main aim of their study was to optimize the dynamic behavior of production center and they did not consider other nodes of the system to be optimized. Anderson Jr *et al.* [2] investigated the dynamic behavior of customized service supply chains by developing a two-stage dynamic staffing model. They developed optimal control policies to balance backlog costs against hiring and firing personnel cost under both centralized and decentralized control.

2.3. Control theory applications on supply chain dynamics

During the past decade, some other authors have also proposed models studying the dynamics of supply chains by means of control theory and specially, optimal control principles (for instance see, Bemporad and Giorgetti [5], Buwalda *et al.* [7], Fagunde and Facó [19], Zhang and Lv [71], Dong and Li [17], Sun *et al.* [63], Miranbeigi *et al.* [42], Udenio *et al.* [66], and Pinho *et al.* [49]). The current work presents a practical modeling approach to deal with the dynamics of the supply chains by the considerations of the cost and inventory level fluctuations. Ivanov *et al.* [31] showed that by years of academic research on operation research and planning supply chains, remarkable advantages have been achieved. They also reported that there has been a growing volition for work on SCs and integration of production and transportation planning along with considering and answering the needs to controlling the SC dynamics. They developed an integrated framework based on modern optimal control theory and operation research method to model a transportation and production planning problem with some simplifying assumptions. Ivanov *et al.* [29] reviewed the most important issues that describe dynamics in supply chains, and categorized the different streams in application of control theory to production, logistics, and SC management. Their investigations were based on the basic principles of control theory and experimental modeling approach. They reported that with the aid of control theory, robustness, adaptability and quick recovery of SCs can be achieved. They also mentioned that there is a lack of direct application of control theory for SCs control and execution. This gap in the literature can be fulfilled by studies like present study. Ivanov and Sokolov [30] categorized the components of research on supply chain dynamics and their relations in a triangular scheme, including methods of operation research and control theory, supply chain management and information systems. As here, these components are considered to achieve better results. Along with these attempts, developing and applying heuristic and metaheuristic algorithms to deal with the complexity of the problem was investigated by many researchers, for instance see O'donnell *et al.* [45] and Tosun *et al.* [64]. Later Xu *et al.* [69] investigated and analyzed the bullwhip effect from the viewpoint of an order up-to inventory policy in SC systems. They transformed the bullwhip effect problem to a discrete-time optimal control problem and showed the usefulness of the proposed approach in dampening the bullwhip effect. If the system is naturally continues, this approach may cause in inappropriate results. Wang and Disney [67] investigated and

Table 1. Recent trends in bullwhip research.

Research trends and directions	Pols of the factors		
Structure of the SC	Linear and simple	Nonlinear and complex	
Product type	Tangible	Intangible	
Flow type	Logistics	Information	Financial
Market power	Monopoly	Competing	
Sustainability	Economic	Environmental	
Concept	Operations	Finance, regulatory	

comprehensively reviewed the bullwhip effect literature. They have proposed comparisons between the results of both simulation and empirical based approaches. They have also pointed out the common assumptions in the modeling of the inventory-production systems. They categorized the adopted methodologies in bullwhip research into four main streets: empirical, experimental, analytical and simulation-based, and introduced five major elements in modeling the bullwhip effect including demand, forecasting, time delay, ordering policy and information sharing. In this paper, all these five components are considered. Table 1 shows recent trends in the area of bullwhip effect dampening. Our proposed model studies a linear SC with material and information flows. The proposed model is indifferent towards the product type (tangible or intangible). One of the advantages of proposed model is that it can be easily altered to a nonlinear SC.

Wang and Disney [67] clarify that, although most of the above mentioned categories are not new but they have remained underdeveloped due to the complexity and technical challenges. We will discuss more about them in the case study. Regarding the directions and categories discussed by these authors, newly published studies can be classified under these guidelines and directions. Interested readers can refer to Garcia *et al.* [23], Garcia Salcedo *et al.* [55], Li and Liu [38], Javadian and Tavakkoli-Moghaddam [32], Cao *et al.* [8], Yan *et al.* [70], and Cao *et al.* [9]. Ponte *et al.* [50] investigated nonlinear supply chains considering capacity limitations with the order-up-to replenishment policy. For more details on nonlinear supply chains interested readers can refer to Giacomo and Patrizi [24]. Their results illustrate that nonlinearity has a crucial impact on production smoothing. They introduced the concept of a settling capacity, and noticed when the available capacity is less than the settling capacity, the nonlinear effects will have a huge impact and should be taken into account. But, as already mentioned most of the works just consider one or two of the main assumptions and here we will investigate the supply chain system by studying these components and their possible dependence. Li [37] considered a limitation on the level of information sharing among the supply chain members with respect to uncertainty. Their study aimed to reduce the inventory level fluctuations by developing a SC state transition model along with a new inventory control method, but it did not consider the network nature of the SC. Monostori *et al.* [44] overviewed the pros and cons of cooperative control approaches to production and logistics systems. They investigated six main challenging characteristics of complex systems and proposed two case studies to investigate the cooperative control approaches. Miranbeigi *et al.* [43] applied an offline application of distributed control to a supply chain network system. In industries where long-term historical data on demands are available, the utilization of online controller system may seem noneconomic and unnecessary, therefore each node of the network can be controlled separately by a local offline controller and then share its information with other nodes in the cascade network. The authors claimed that this separate controlling system leads to an effective economic solution for large SCs. Here, this approach to the SC planning problem is extended to achieve the benefits of the offline controlling and reducing the costs associated with a full online controlling systems to dampen the bullwhip effect. Our proposed model focuses on the material and information flows in a linear supply chain which is easily extendable to a non-linear supply chain network. Also, this paper studies the behavior and decisions of each node of the SC during planning horizon from the cost perspective in order to dampen the demand fluctuations and reduces related inventory holding costs.

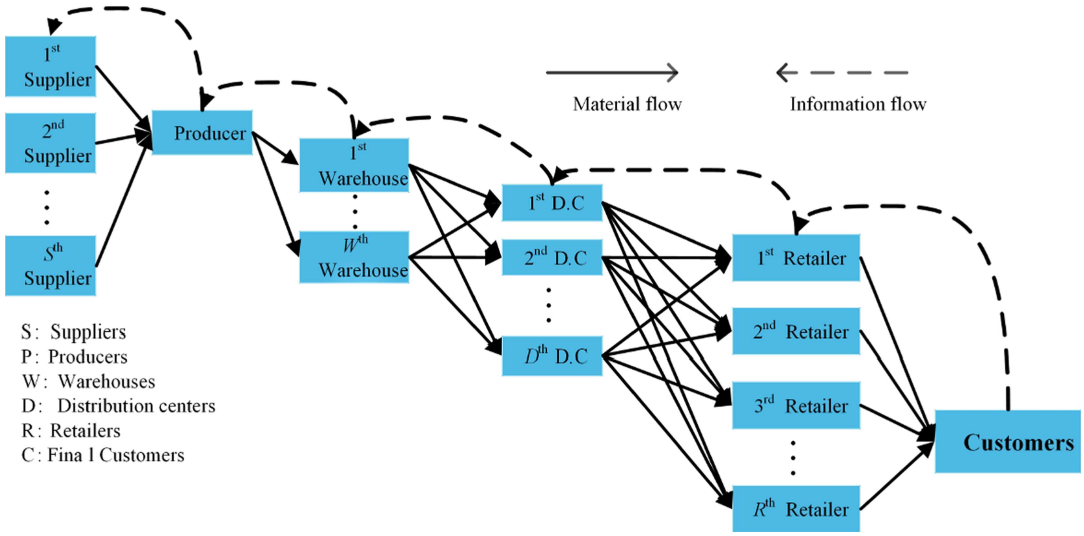

FIGURE 1. Scheme of a supply chain.

3. Problem description

In this section, an optimal control approach is applied to a five-stage SC. Ordinarily, an optimal control problem consists of the following three main sub-problems. First the system should be defined well. Second, we define the logical constraints that define our system and make it reasonable and feasible, and third is the establishment of a proper objective function to be optimized through the control process. Generally, optimal control deals with the problem of finding a control law for a given system such that a certain optimality criterion is achieved. A control problem includes an objective function (almost cost function) including state and control variables. An optimal controller is a set of differential equations describing the paths of the control variables that minimize the cost function. The optimal control can be derived using *Pontryagin's maximum principle* (a necessary condition), or by solving the *Hamilton–Jacobi–Bellman equation* (a sufficient condition). There are also other approaches for solving and analyzing the system. For a simple production-inventory systems, see Sethi and Thompson [57]. In this study, basically we employ the *Pontryagin's maximum principle* [51] to apply the optimal control capability in controlling and dampening the bullwhip effect.

3.1. The proposed model

Consider a supply chain with five major stages from suppliers to the retailers illustrated in Figure 1. In the existing literature, the maximum number of stages of a supply chain network is considered to be five. Really, many real world supply chains have such structure (see [20, 26, 27, 56, 58, 60, 61]). The main aim is minimizing the high variability and fluctuations of the demand and inventory levels of each node throughout the network and eventually dampening bullwhip effect in the SC within a pre-defined time horizon. The proposed model is a linear SC model, but it could be used in nonlinear SC planning by adopting some modifications. In this study, linear SCs mean chains in which each echelon nodes do not have inner-echelon relationships, they just receive the material flow from next upstream echelon and transmit the information flow to it. This assumption will able us to use the benefits of the exponential smoothing forecasting method in our proposed model.

Generally, the sequence of events in a linear SC is as follows; at the beginning of each time period t, the retailer receives his order and satisfies the customer's demand. Then the retailer reviews the new level of his inventory and places an order to a distributor. The distributor gets the order in period $t+1$, with one unit of time delay because of order processing and paperwork. The distributor himself, places orders to its upstream node and this cycle goes on till the end of the chain. The assumptions used in developing the proposed model are as follows:

- the studied SC is a Decentralized SC;
- demand is uncertain and must be forecasted;
- lead time in delivery can take various amounts each period;
- ordering policy is periodic review, order-up-to level;
- production parameters, ordering amounts, demand quantities and variations are limited and bounded;
- forecasting method is the classic exponential smoothing (see Razmi and Sabbaghnia [52]);
- the system is considered to be linear and the equations creating the system are linear difference equations;
- unsatisfied demands are back-ordered with a penalty cost parameter;
- equations clarify the system dynamics are developed for the producer node, but the procedure is same for all the other nodes (we will examine the model for a D.C in case study);
- system is considered to be discrete time.

At the following, the indices, parameters and variables used in the proposed model are defined. As depicted in Figure 1, the abbreviations S, P, W, D, R stand for Suppliers, Plants, Warehouses, Distribution centers and Retailers, respectively.

Indices
j, k: index of supply chain network nodes, $j, k \in \{S, P, W, D, R\}$
t: index of time period $t \in \{1, 2, \ldots, T\}$

Parameters
$TNS_j(t)$: target net stock in node j at time period t
$TWIP_j(t)$: target work in process in node j at time period t
T_p: production lead time
G: control gain
$D_{kj}(t)$: demand of downstream node k placed to upstream node j in time period t
$\hat{D}_{kj}(t)$: estimated values of $D_{kj}(t)$

Decision variables
$NS_j(t)$: net stock in node j at time period t
$WIP_j(t)$: work in process in node j at time period t
$ENS_j(t)$: error net stock in node j at time period t
$EWIP_j(t)$: error work in process in node j at time period t
$OR_j(t)$: order rate in node j at time period t
$R_j(t)$: receive rate in node j at time period t
$BO_j(t)$: back order amount in node j at time period t
$I_j(t)$: inventory amount in node j at time period t
$Y_{jk}(t)$: amount of goods delivered by upstream node j to downstream node k in time period t
$J(t)$: cost function of optimal control

According to the above-mentioned descriptions, the difference equations for node j in time period t can be presented as follows. These equations model the sequence of events in the system as discussed earlier.

$$ENS_j(t) = TNS_j(t) - NS_j(t) \tag{3.1}$$

$$EWIP_j(t) = TWIP_j(t) - WIP_j(t) \tag{3.2}$$

$$WIP_j(t) = WIP_j(t-1) + OR_j(t-1) - R_j(t) \tag{3.3}$$

$$OR_j(t) = Optimiser + \hat{D}_{kj}(t) \tag{3.4}$$

$$NS_j(t) = NS_j(t-1) + R_j(t) - Y_{jk}(t) \tag{3.5}$$

$$TNS_j(t) = G * \hat{D}_{kj}(t) \tag{3.6}$$

$$TWIP_j(t) = T_p * \hat{D}_{kj}(t) \tag{3.7}$$

$$R_j(t) = OR(t - T_p - 1) \tag{3.8}$$

$$NS_j(t) = I_j(t) - BO_j(t) \tag{3.9}$$

$$BO_j(t) = D_{kj}(t-1) - Y_{jk}(t) \tag{3.10}$$

The proposed model deals with dynamic nature of the supply chain. Recall that operation research-based models fail to investigate the dynamics of SCs, properly. In this model, the input signal of the system is the demand signals and by imposing delays and control actions calculates the output signals. Reminding that the most important cause of uncertainty in the system is driven by demand forecasting. Equations (3.1) and (3.2) show the error amount calculated for *Net stock* and *Work in process*. Work in process generally means partially processed items, especially in manufacturing literature; here, we use both work in process and work in progress terms interchangeably based on the characteristics of the problem. Equation (3.3) shows the calculation of Work in process in period t by adding the difference between order rate of $t-1$ and received rate (in period t) to the previous amount of work in process. Equation (3.4) shows the order rate of node j which is equal to estimated demand amount plus the output of Error Optimizer block (see Fig. 2). The mechanism of this block will be thoroughly described in the following. Equation (3.5) calculates the Net stock in node j. Equation (3.6) is calculating the Target Net Stock of each node in each period, which is determined by the amounts of the delivered goods, pervious period's realized net stock and material receive rate. In this study, we have considered the Target Net Stock value as a function of mean satisfied demand on that period. This setting is one of the contributions in this work, so the objective amount of Target Net Stock is dynamic and evolves each period, which will able the system to mimic the behavior of the nodes more realistically. In this system, we have Work in Process feedback loop and *TWIP* which is calculated by equation (3.7). Orders placed by node j at time period t faces T_p delay and one unit delay because of paper works and order processing, so the production rate or orders plan in time period t equals with orders placed in time period $k - T_p - 1$, which is shown in equation (3.8). Equations (3.9) and (3.10) are developed based on the inventory policy and do not participate directly in our system's difference equations. The sequence of events and developed difference equations could be shown by Figure 2. According to this figure, the system includes two feedback loops. The first one is net stock loop and the second one is work in process loop. The assembly of our system can accept any development especially the ones on the dynamic loops by defining the proper difference equations and adding them to the respected function. This simplicity allows managers to understand the behavior of the system while the achieved results are proven to be effective and efficient. As discussed earlier, the behavior of the system is the decisions made by

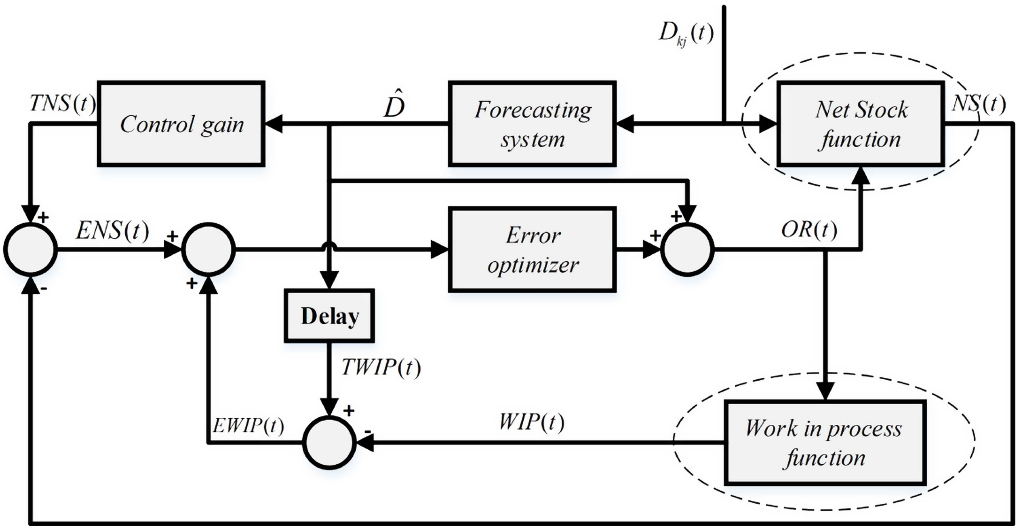

FIGURE 2. Block diagram representing node j.

each node as a decision maker unit along the chain. For demand forecasting, exponential smoothing method is utilized which is adequately consistent with control systems according to Dejonckheere *et al.* [1]. This method is an efficient and well-known method for forecasting linear systems.

Figure 2 represents the dynamic procedure of supply chain and order chain in node j, net stock and work in process controlling loops define the dynamic evolution in each time period. Objective function and constraints of the optimizer block is as follow:

$$\text{Min} \sum \sum w(Error^t_{j_{NS}} + Error^t_{j_{WIP}}) \tag{3.11}$$

$$OR^{t,j}_{\min} \leq OR(t) \leq OR^{t,j}_{\max} \forall t, j \tag{3.12}$$

$$NS^{t,j}_{\min} \leq NS(t) \leq NS^{t,j}_{\max} \forall t, j \tag{3.13}$$

$$0 \leq WIP(t) \leq WIP^{t,j}_{\max} \forall t, j \tag{3.14}$$

The *Error* values are equal to the quadratic matrix of errors of net stock and work in process. Constraints are the control constraints of the control variables (order signals). These constraints would be satisfied with solving procedure otherwise they should be considered as a penalty term with proper penalty weight factor added to the objective function. In equations (3.12)–(3.14), the boundary conditions are defined by both physical and legal constraints, like available budget, warehousing capacity, processing capacity, government quota for production size and etc. To this point, SC characteristics have been modeled using differential equations. The mechanism of the optimizer block has been revealed, too. Here, by defining the cost function of the optimal controller our model is completed. The objective function of control system is presented as follow; this function is about to

TABLE 2. Demand amount for 10 periods.

	Time period									
	1	2	3	4	5	6	7	8	9	10
Demand	72	126	102	90	127	115	104	106	161	125

minimize simultaneously the target net stock, holding costs and shortage costs.

$$J(t) = \int_{0}^{T} (NS(t) + WIP(t) + I(t).H + BO(t).H' + w.|OR(t-1) - OR(t)|)\mathrm{d}t \tag{3.15}$$

The last term of above function considers the differences between two consecutive periods as a penalty and tries to minimize the order level turbulences between two preceding periods by weight factor w. Function (3.15) is constrained by constraint equations (3.12)–(3.14), along with the difference equations of the nodes denoted by (3.1)–(3.10). The computational advantages of control theoretic approaches allow us to modify and develop the complex function under the integral and yet reach the optimality in a reasonable time. This will be discussed in the following section.

4. Computational results

For evaluating the applicability of the proposed model, test problem is generated based on equation (4.1) and Table 2, in which the demand amounts are presented. Test problem and proposed case study are solved using MATLAB software on a computer configured by Intel® Core™ i7-5500U CPU @ 2.40GHz. The test problem consists 10 periods of planning and the amount of demand for each period is calculated by equation (4.1), where $Demand_t$ denotes demand in period t, *Base* term is demand average, *Slope* term is the demand variation trend, $SeasonCycle()$ is the number of periods, *noise* term represents the system noises and $Snormal()$ is a random normal number generating function and we have:

$$Demand_t = Base + Slope * t + season * \sin\left(\frac{2\pi}{Season\ Cycle} * t\right) + noise * snormal() \tag{4.1}$$

Sensitivity analysis about the parameters of the above mentioned equation is investigated in Razmi and Sabbaghnia [52]. Table 2 represents the amount of demand of test problem for 10 periods for a supply chain consisting four stages from retailers to producers.

These data are used in two solving procedures, first we have solved the problem using beer game simulation (see Li [39]), then the same data is applied to the proposed model and results of both procedures are calculated. Simulations in beer game are calculated for 50 iterations in a four stage supply chain and 10 planning periods. Demand trends, order variations and fluctuations of the test problem for each layer of the supply chain (representative of a node in the network) is shown in Figure 3. As it is clear in this figure, a trifle fluctuation in customer's demand is drastically propagated along the chain. These amplified demands will cause the upstream of the chain to conduct with high levels of inventories and non-optimal production plans.

Table 3 shows some important statistical results driven from the beer game solution. These results help managers to have a brief summary of solution on test problem. This table carries important statistical insights of our system, which will be compared with the simulation results of the proposed approach. Bullwhip effect amount for each node can be easily driven out from this table by dividing each node's variations to customer's demand variation. Along with that, this table has interesting managerial insights itself, for instance, demand variation grows severely from customer end of the chain to the producer end. Figure 4 provides more depictive illustration of demand and placed orders behavior of the nodes (number one is the customer).

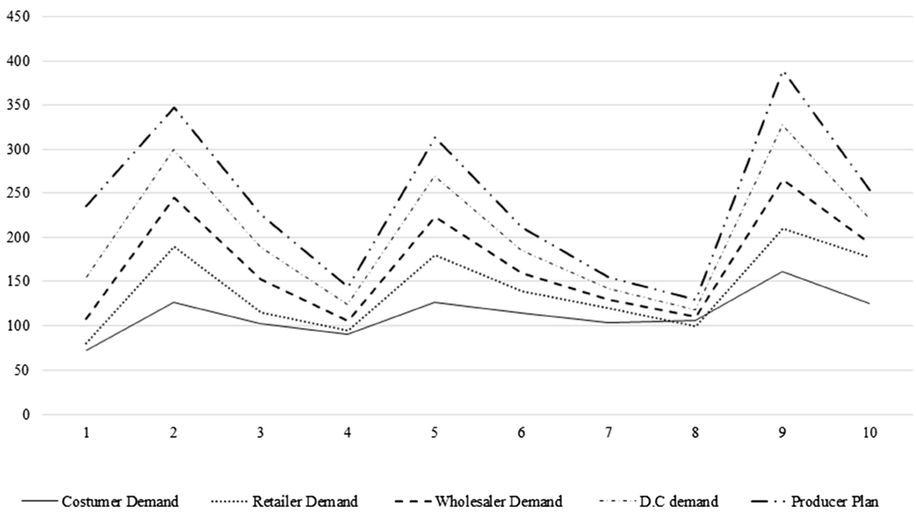

FIGURE 3. Trend of demand in each node of supply chain (beer game).

TABLE 3. Beer game solution statistic results.

	Customer	Retailer	Wholesaler	Distributer	Producer
Demand mean	113	141	169	203	241
Maximum demand	161	210	265	327	390
Minimum demand	72	80	105	118	130
Demand variation	586	2073	3523	5517	7719
Standard deviation	24	46	59	74	88

Figure 4 depicts the behavior of demand signals in the system. In this figure, we can trace the differences of demand between maximum and minimum demand quantities in each stage of supply chain and see that the demand fluctuation increases by moving upward the chain. Difference between demand quantities has reached from 89 units to 260 units in producer stage which is about 300 percent increase while the customer's average demand is about 112.8 units.

Now, we will solve the same test problem with our proposed approach. Remember, we have two major dynamic controlling loop, equations (3.3) and (3.5) represent these dynamic loops. Optimal controller signal or the ordering signal must be calculated in such manner to minimize the differences from the target signals. This setting will lead us to a lower amount of bullwhip effect. Time limitation is considered as the stopping criterion in solving process. Test problem has reached optimality in 47 iterations before reaching one hour limit on running time. The initial condition is considered to be zero for all the parameters and variables, which means there is no stock in the system at the beginning of the planning and also the lead time for the first placed orders as well. Trend of demand fluctuation in each period is shown in Figure 5. As shown in this figure, the proposed method rapidly responds to the variations compared to the basic solution method (see Figure 3). In this figure, by mentioning the periods 6 to 8, it can be seen when the demand signals go smoother, response signals converge more and reach the target stock in following planning period.

Figure 6 shows the control performance cost function in each iteration and converge speed of this objective function. Table 4 shows the results of the proposed optimal control approach for different nodes of the supply chain network. The achieved results indicate that the proposed method is able to reduce demand variation significantly respect to beer game solution approach. Figure 7 shows the differences between maximum demand and minimum demand amounts in each stage of the chain. This amount has increased from 89 units to 131 units

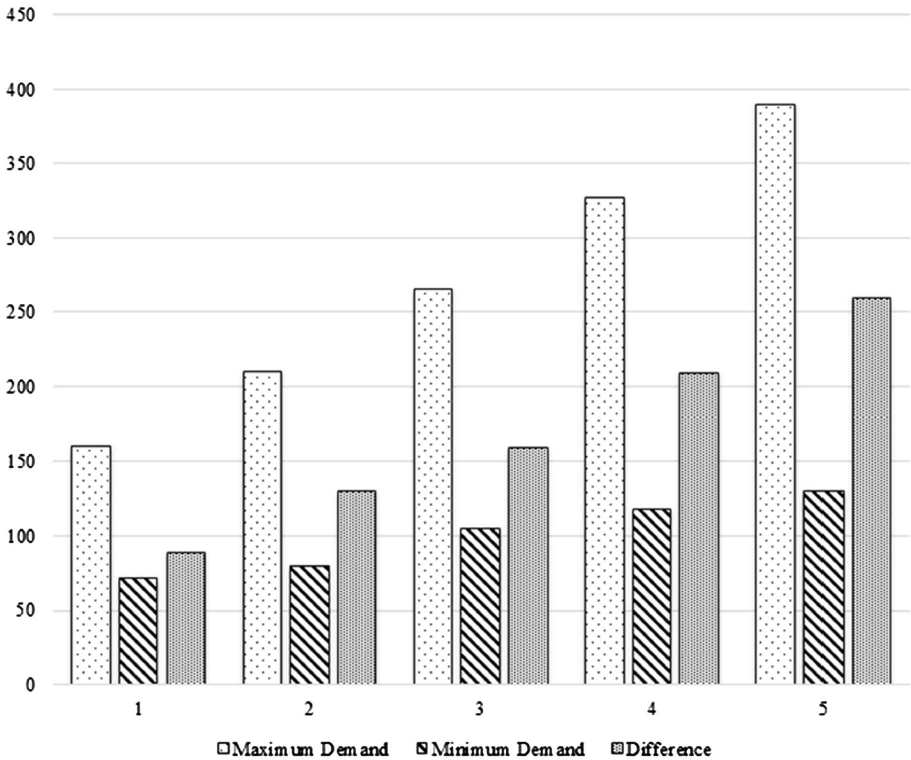

FIGURE 4. Demand difference in beer game solution.

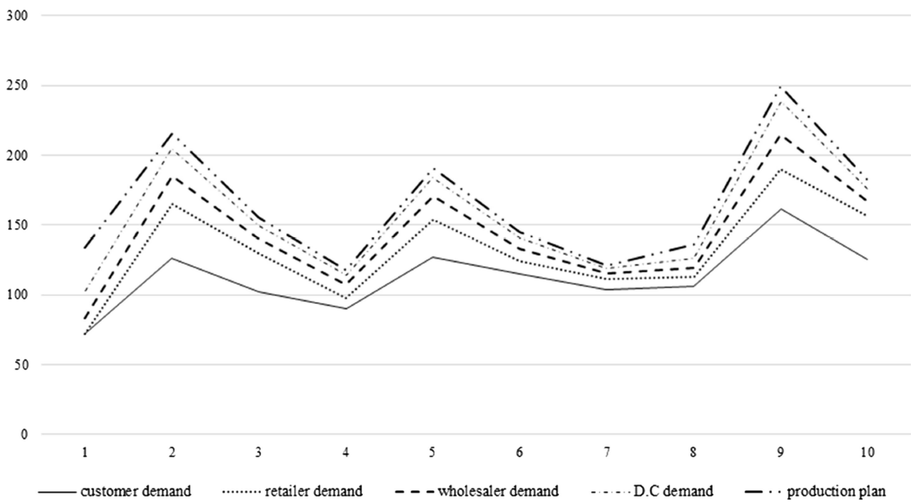

FIGURE 5. Trend of demand in each node of supply chain (control approach).

which is only about 47 percent increment, while 300 percent increment is happening when using the beer game solution approach.

Figure 8 compares maximum, minimum and the difference amounts of demand in optimal control model (OC) and beer game (BG) solution. Bars representing difference amounts in both solutions display efficiency of

TABLE 4. The results of the proposed optimal control approach.

	Customer	Retailer	Wholesaler	Distributer	Producer
Demand mean	113	131	144	155	165
Maximum demand	161	190	215	238	249
Minimum demand	72	72	83	103	118
Demand variation	586	1237	1620	1935	1895
Standard deviation	24	35	40	44	44

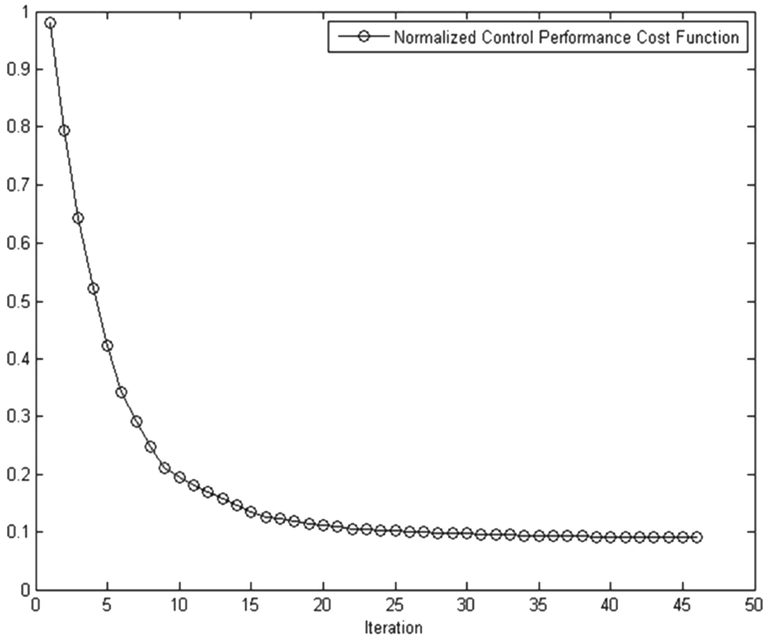

FIGURE 6. Converge speed of control performance function.

TABLE 5. Bullwhip effect in two solving method.

Approaches		Customer (first stage)	Producer (final stage)
Beer game	Demand variation	586	7719
	Bullwhip effect	*13.17*	
Proposed model	Demand variation	586	1895
	Bullwhip effect	*3.32*	

optimal control method. As proceeding along the chain to the upstream nodes, the optimal control difference grows less than the beer game difference quantities. This stabilized performance not only leads SC managers to efficient inventory-production planning, but also decreases the inventory handling costs of the chain.

Finally, bullwhip effect is calculated and represented in Table 5. The procedure of calculating is dividing the demand variation of end node (producer) of the chain to demand variation of the first node (customer) of the chain. As discussed earlier, even in this small-size problem the performance of the proposed approach is significant. Two feedback loops for controlling the order rates along with the online updated order signals increases the efficiency of the proposed approach.

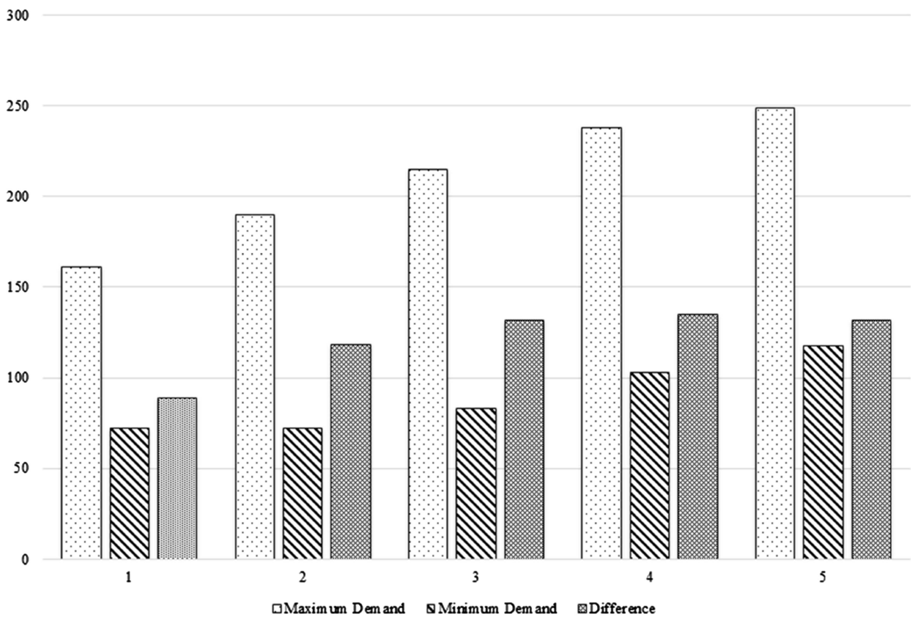

FIGURE 7. Demand difference in optimal control solution for all stages of the SC.

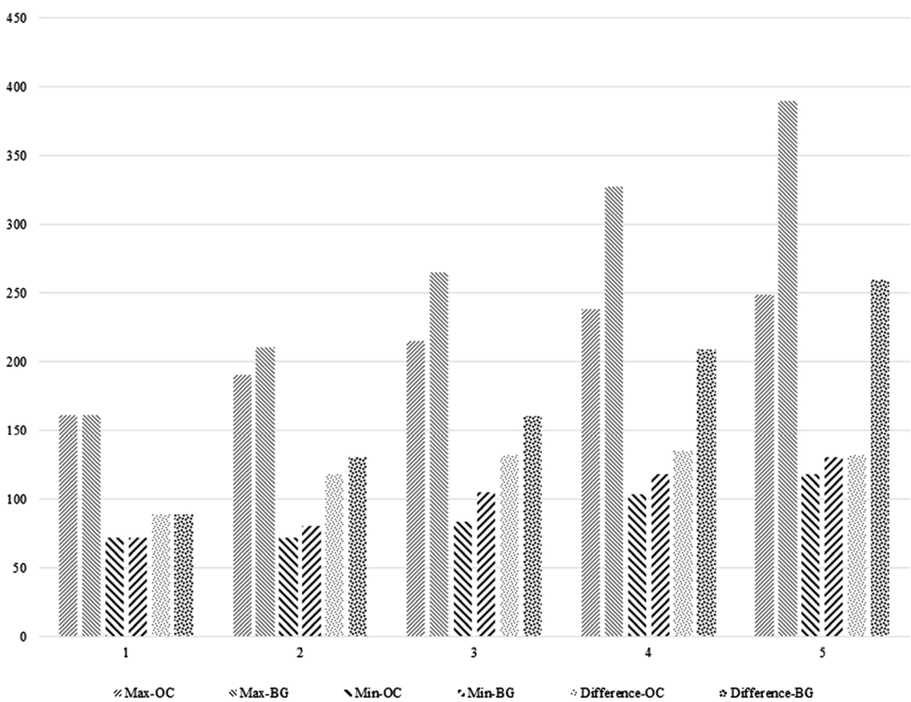

FIGURE 8. Demand differences in optimal control solution.

TABLE 6. Bullwhip effect in two solving method.

Period	Demand	Period	Demand	Period	Demand	Period	Demand	Period	Demand
1	105	12	70	23	185	34	65	45	165
2	70	13	90	24	200	35	130	46	100
3	180	14	145	25	116	36	164	47	104
4	185	15	210	26	65	37	85	48	200
5	100	16	95	27	65	38	85	49	120
6	140	17	100	28	88	39	70	50	120
7	200	18	195	29	90	40	155	51	180
8	165	19	135	30	186	41	80	52	75
9	150	20	200	31	65	42	160	53	165
10	70	21	72	32	140	43	115		
11	120	22	60	33	165	44	105		

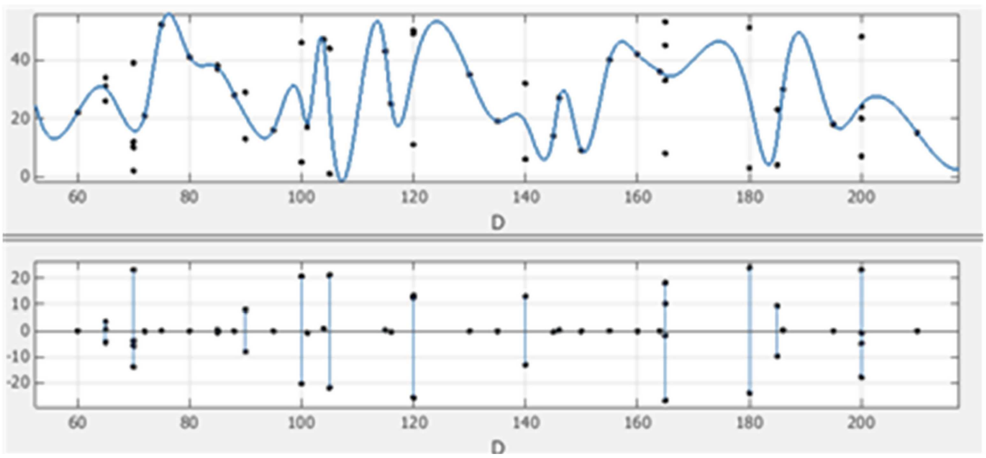

FIGURE 9. Case study's demand fitness.

4.1. Case study: an Iranian Bicycle company

Validation of our proposed model is done on a case study. *Nikanpoor Bicycle Business Cooperation* is in Tabriz, Iran and is a well-known company in importing the bicycle, relevant tools, and accessories. Demand data are related to orders of size 26' ring of bicycles from April 2013 till September 2014. This company is considered as the *distributor* and *wholesaler* node in our supply chain network. Table 6 represents the data of demand provided by this company.

For doing more processing on these data we have fitted them on MATLAB software with least sum square errors equals 7400 and piecewise linear estimation. Table 7 illustrates the statistical results of solution of case study and the bullwhip effect is calculated to be 2.35. This table also has some important insights discussed at the following:

- Total orders of the chain are not increased, so the total change in demand is 638 units over the 53 time periods. In other words, there is only 8 percent difference between customers demand and supplier demand, while many primary studies report 1:2 ratios for this fluctuation (see Lee and Whang [34]; Lee *et al.* [36]).
- The above analysis is also attributable to the case of maximum demand. The range of the changes seems to be significantly controlled, but in case of minimum demand, the excessive inventories from prior periods cause the production/order rate be zero in some periods.

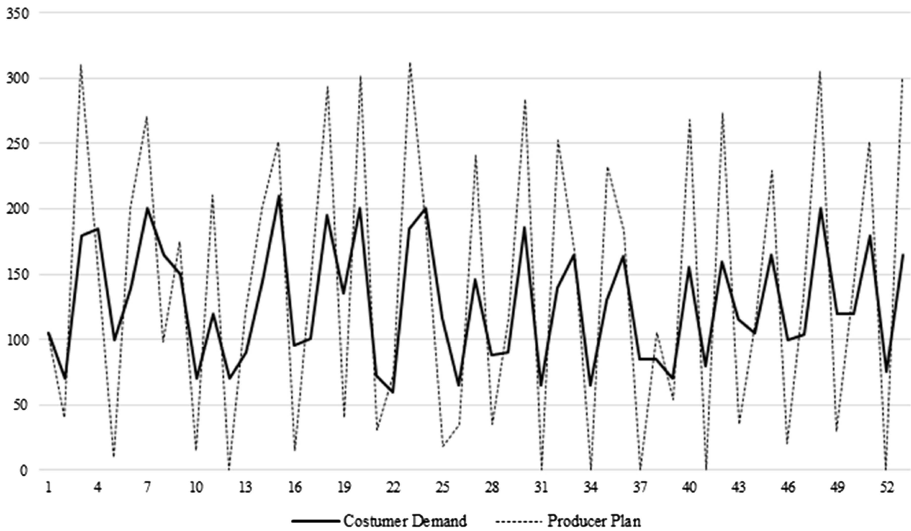

FIGURE 10. Demand fluctuations in last stage of chain (case study).

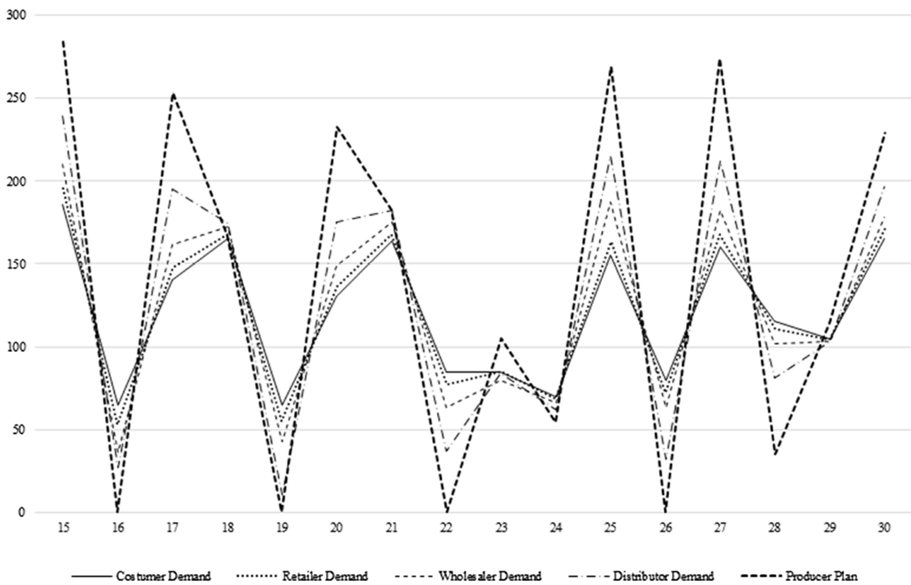

FIGURE 11. Trend of demand in each node of supply chain for 15 periods (case study).

– Calculated amount of bullwhip effect shows significant and logical reduction in comparison with previous studies. First, the rate of reduction is tempting and persuading for managers and decision makers to take advantage of this model and apply it in their organizations. In other words, it could be concluded that the presented model is a practical one and its application did not need more extra costs. Actually, this is a proper and well defined way of improving inventory and material planning system.

The response speed and propagation of demand fluctuations is shown in Figure 10, the third term we have added to the performance function causes output signals be smoother and act as an anti-propagator.

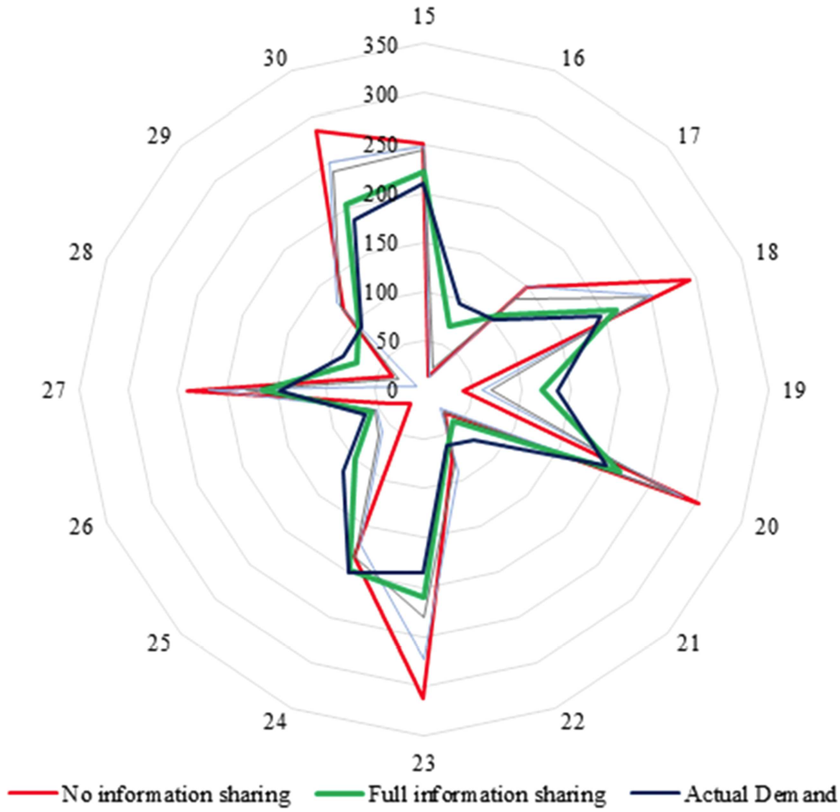

FIGURE 12. Impact of information sharing on the manufacturer's decisions.

TABLE 7. Statistic results of case study.

	Customer	Retailer	Wholesaler	Distributor	Producer
Sum of demands	6747	6750	6786	6881	7385
Maximum demand	210	210	225	225	312
Minimum demand	60	53	36	10	0
Demand variation	2098	2606	3654	6154	11 643
Bullwhip effect			*2.35*		

In Figure 11, we have shown the trend of demand fluctuations for all the stages of supply chain in 15 random time periods. According to this figure:

- By moving upward the chain, accuracy and response speed decrease and the minimum fluctuations in retailer and wholesaler stages are occurred.
- By reduction in severity of changes in customer's actual demand, convergence of orders into the target point is increased.
- Since the proposed model controls the loops, it necessarily do not follow the customer demand's jumps and create an equilibrium relationship between target net-stock, work in process and received demand.

At the next step, the impact of information sharing is illustrated in the case study for a window of periods (see Fig. 12). When there is no information sharing, each node in the chain must predict the downstream demand. Eventually, this will not lead to a proper production plan for the upper nodes in the chain. As the information sharing rate increases, the deviations from actual demand decreases. Here, information sharing is defined by the level of access to the next down-stream's information throughout the chain. It is obvious from the figure that as the level of shared information is boosting the behavior (production plan decisions) of the manufacturer is getting more alike the end-user (actual demand).

5. Discussion and conclusions

Control engineering concepts in supply chain management have been widely studied by researchers, especially in inventory-production problems. Incremental need of investigating the dynamics of supply chains and tackling with issues arising from them have triggered a sense among researchers to develop appropriate tools to deal with this problem. Over the years, control engineering has been developed for controlling mechanical and electrical devices and machines. But, their ability and strong mathematical tools made them adaptable to other non-electrical but over time evolving systems. The modeled system in this study is based on optimal control concepts. In this paper, first the supply chain system is modeled using difference equations and then is evaluated by a test problem. Comparison of the acquired results of the proposed model with respect to the beer game simulation, justifies the efficiency of the proposed model. Finally, the proposed model is validated through studying a real case of bicycle industry in Iran. The output signal of the controller (order rate of each node) shows a significant reduction in the amount of bullwhip effect. This reduction will directly affect inventory handling costs and other costs driven by bullwhip effect. The main contributions of this work can be summarized as follows:

- Target net stock is variable in each time period and is a function of mean satisfied demand. This updateable variable will allow the system to follow the fluctuations and reduce the demand variation through the chain.
- Taking inventories of work in process or on the way orders (WIP) as a feedback loop in the system.
- Proposing a novel modelling approach to the dynamics of the supply chain and creating a mechanism for understanding the control theory applications in problems of inventory management and production planning.

Further extensions of the current model can be considered in both modeling approaches and solving algorithms as follows:

- Evaluating the performance of the model with some long-term case studies.
- Considering multiple connections between nodes in each supply chain stage and running sensitivity analysis on the number of connections.
- Utilizing control theory approaches on centralized supply chains and in the case where there is a full information sharing between stages.

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