RAIRO-Oper. Res. 52 (2018) 1201–1217
https://doi.org/10.1051/ro/2018018

**RAIRO Operations Research**
www.rairo-ro.org

# BI-OBJECTIVE MEAN–VARIANCE METHOD BASED ON CHEBYSHEV INEQUALITY BOUNDS FOR MULTI-OBJECTIVE STOCHASTIC PROBLEMS

Hannan Amoozad Mahdiraji*, Seyed Hossein Razavi Hajiagha, Shide Sadat Hashemi and Edmundas Kazimieras Zavadskas

**Abstract.** Multi-objective programming became more and more popular in real world decision making problems in recent decades. There is an underlying and fundamental uncertainty in almost all of these problems. Among different frameworks of dealing with uncertainty, probability and statistic-based schemes are well-known. In this paper, a method is developed to find some efficient solutions of a multi-objective stochastic programming problem. The method composed a process of transforming the stochastic multi-objective problem to a bi-objective equivalent using the concept of Chebyshev inequality bounds and then solving the obtained problem with a fuzzy set based approach. Application of the proposed method is examined on two numerical examples and the results are compared with different methods. These comparisons illustrated that the results are satisfying.

**Mathematics Subject Classification.** 60H99

Received November 8, 2016. Accepted February 23, 2018.

## 1. Introduction

Selecting the best option among alternatives is often a difficult process, especially when the evaluation criteria are vague or qualitative, besides the objectives vary in importance and scope. The decision-maker (DM) usually combines the vague criteria with known quantitatively criteria to obtain the best possible alternative. Without systematic approaches to the process one cannot be sure that the proper decision has been made [23]. Optimization has always been of great importance and interest particularly in solving complex real-world problems. Basically, the optimization process is defined as finding a set of values for a vector of design variables leading to an optimum value of an objective or cost function [26].

Meanwhile, many decision and planning problems involve multiple conflicting objectives which should be considered simultaneously. Such problems are generally known as multiple criteria decision making (hereafter MCDM) problems. We can classify MCDM problems in many ways depending on the characteristics of the problem in question [33]. This class is further divided into multi-objective decision making (henceforth MODM) and multi-attribute decision making (henceforward MADM) [12]. Real-world decision-making problems often consist

*Keywords and phrases:* Multi-objective decision-making, stochastic programming, mean–variance criterion, fuzzy set based approach.

University of Tehran, Tehran, Iran.

* Corresponding author: h.amoozad@ut.ac.ir

in considering multiple and antithetic objectives; therefore, MODM is a practical framework in implicational areas [39]. Cohon [13] believed that considering more than one objective have the following advantages:

- promotes more appropriate roles for the participants in decision-making process;
- a wider range of alternatives is usually identified;
- models or the analyst's perception of a problem will be more realistic.

Different frameworks are introduced in response to modelling and analyzing the uncertainty of systems. Liu and Lin [29] classified the uncertainty frameworks into three distinct fields, probability and statistics, fuzzy sets and grey systems theory. A key note in modeling uncertain systems is that a complicated model is not always necessary to deal with incomplete information and inaccurate data [38].

Conventional parameter optimization methods seek to find a single optimized solution based upon a weighted sum of all objectives. If all objectives get better or worse entirely, this conventional approach can effectively find the optimal solution; however, if the objectives conflict, then there is not a single optimal solution available. In this case, a multi-objective optimization (hereafter MOO) study should be performed that provides multiple solutions representing the tradeoffs among the objectives [9]. In MOO problems, we are optimizing several conflicting objective functions simultaneously. Due to the conflicting nature of the objectives, we can identify compromise solutions, so-called Pareto optimal solutions, where we cannot improve the value of any objective function without impairing at least one of the others [16, 18].

A considerable amount of research has been reported in this area over the last 20 years. The concept of MOO is attributed to the economist, Pareto [37]. After several decades, this concept was recognized in operations research and has recently become popular [7]. In MOO the set of feasible solutions is not explicitly known in advance but it is restricted by constraint functions, alongside with its characteristic that no unique solution exists but a set of mathematical solutions which are equally good can be identified. These solutions are known as non-dominated, efficient, non-inferior or Pareto optimal solutions. In *a priori* method, the DM first articulates preference information and one's aspirations; afterwards, the resolvent process tries to find a Pareto optimal solution satisfying them as well as possible. This is a straightforward approach but the difficulty maintains since the DM does not necessarily know the possibilities and limitations of the problem beforehand, additionally, may have too optimistic or pessimistic expectations. Alternatively, it is possible to use *a posteriori* method, where a representation of the set of Pareto optimal solutions is generated primarily and then the DM is supposed to select the most preferred one. This approach gives the DM an overview of different solutions available; nonetheless, if there are more than two objectives in the problem, it may be difficult for the DM to analyze the large amount of information. On the other hand, generating the set of Pareto optimal solutions may be computationally expensive [33]. Several decades ago, the works by the economists Francis Edgeworth and Vilfredo Pareto had laid the formal foundations for the analysis of MOO problems, but it was in the 50s of the 20th century when researchers started to design computer-based algorithms to provide DM with concrete solutions to such problems [22]. A generic multi-objective design optimization problem may be formulated as below [15].

$$\begin{aligned} &\text{Min } J(x,p)\\ &S.T.\\ &g(x,p)\leq 0\\ &h(x,p)=0\\ &x_{i,LB}\leq x_i\leq X_{i,UB}\quad (i=1,...,n)\\ &x\in S, \end{aligned} \tag{1.1}$$

where $J=[J_1(x),\ldots,J_l(x)]^T$, $x=[x_1,\ldots,x_n]^T$, $g=[g_1(x),\ldots,g_{m_1}(x)]^T$, and $h=[h_1(x),\ldots,h_{m_2}(x)]^T$.

Here, $J$ is a column vector of $l$ objectives, whereby $J_i\in R$. The individual objectives are dependent on a vector $x$ of $n$ design variables as well as vector of fixed parameters, $p$. The individual design variables are assumed continuous and can be changed independently by a designer within upper and lower bounds, $X_{UB}$ and $X_{LB}$, respectively. In order to a particular design $x$ be in the feasible domain $S$, both a vector of $m_1$

inequality constraints, $g$, and $m_2$ equality constraints, $h$, have to be satisfied. The problem is to minimize – simultaneously – all elements of the objective vector. A number of names have been given to this type of problem: vector minimization, multi-criteria optimization, multi-attribute maximization and so forth [15].

Stochastic modeling is an interesting and challenging area of probability and statistics.

Although both stochastic optimization and MOO are well studied subjects in Operational Research and Machine Learning, much less is developed for stochastic multiple objective optimization. This is amazing, since in economic and managerial applications, the features of multiple decision criteria and uncertainty are very frequently co-occurring. Stochastic optimization and MOO saw a rapid impressive and extremely fruitful development in recent decades; nevertheless, these two areas evolved to a good part separately, and even today, their intersection is addressed by only a comparably small fraction of publications; even though, real-life decision making frequently encompasses both uncertainty and multiple objectives [22].

In this paper, an interactive approach is designed to analyze and solve the stochastic MOO problem (hereafter SMOO). This approach transforms the stochastic objectives and constraints into equivalent deterministic forms. Subsequently, the transformed configuration of the problem is solved using available optimization methods by an interaction with DM to determine his/her satisfaction level(s).

The rest of paper is organized as follows: a detailed description of SMOO problems is given in Section 2 alongside with a comprehensive review on related studies in this area. Next, the proposed approach to solve SMOO is developed in Section 3 with proving some of its supporting lemmas and theorem. Some numerical examples besides comparing the results of proposed method with previous techniques are discussed in Section 4. Ultimately, the research is concluded in Section 5.

## 2. Stochastic multi-objective linear programming

SMOO problems are a special type of MOO problems where their objective and/or constraint functions are stated stochastically. Mathematically, an SMOO problem can be stated as

$$\begin{array}{l} \text{Max}\, f(w,x) = (f_1(w,x),\ldots,f_m(w,x)) \\ S.T. \\ x \in X(w) = \{g_j(w,x) \le b_j(w),\ j=1,\ldots,k\} \\ x \in D, w \in \Omega, \end{array} \tag{2.1}$$

where $D$ is a deterministic convex set, the feasible set $X(w)$ is random, *e.g.* the constraints $g_j$ and the parameters $b_j$ are random, the $m$ objectives $(f_1(w,x),\ldots,f_m(w,x))$ are random, $(\Omega, E, p)$ is a probability space, and $x = (x_1,\ldots,x_n)$ is an $n$-dimensional vector.

If the objectives and constraints are linear, problem (1.1) can be written as follows:

$$\begin{array}{l} \text{Max}\ C(w)\cdot x \\ S.T. \\ A(w)\cdot x \le b(w) \\ x \in D, w \in \Omega, \end{array} \tag{2.2}$$

where $C$ is a random $m \times n$ matrix, $A$ is a random $K \times n$ matrix and $b$ is a random $K$-column vector defined on $(\Omega, E, p)$ [5].

SMOO problems are not mathematically well defined. Current approaches of solving SMOO are usually upon application of some transformations on objective functions and constraints. Goicoechea *et al.* [21] introduced PROTRADE method that inspired from deterministic STEM method. Leclercq [28] procedure transformed the SMOO problem into an equivalent deterministic multi-objective problem on chance constrained method and two-stage programming. Teghem and Kunsch [41] and Teghem *et al.* [42] proposed the STRANGE scheme to solve SMOO problems where both objective function coefficients and right-hand side parameters are random.

Urli and Nadeau [44, 45] proposed a sketch for solving SMOO problems when all parameters are determined randomly. They also built a Decision Support System which enables the DM to identify many current stochastic contexts: risky situation and situations of partial uncertainty [36]. Other methods dealing with transformation of stochastic objective functions and/or constraints can be found in [34, 35, 40]. A group of authors inspired from goal programming approach to solve SMOO problems [3, 14]. Hulsurkar *et al.* [24] proposed a fuzzy programming approach for SMOO problems. Also, Ben Abdelaziz *et al.* [6] proposed a Chance Constrained Compromise Programming (CCCP) technique. Moreover, Caballero *et al.* [8] presented stochastic approach *versus* multi-objective approach for obtaining efficient solutions in stochastic multi-objective programming problems.

Beyond its theoretical aspects, SMOO is widely used in different implicational area. These applications are in different fields from financial programming, resource management, supply chain management, etc. In 1987, Kunsch and Teghem used the stochastic linear programming approach for a best fuel cycle policy including four criteria: production costs, the supply of raw material, the commercial balance and employment. Gobbi *et al.* [20] applied SMOO for designing automotive suspension systems. Ben Abdelaziz *et al.* [6] applied it in portfolio selection; furthermore, he presented solution approaches for the multi-objective stochastic programming in 2012. Azaron *et al.* [2] developed a SMOO model considering uncertainty of demands, supplies, processing, transportation, shortage, and capacity expansion costs. Chen *et al.* [10] developed three SMOO models for designing transportation network under demand uncertainty. Zhang *et al.* [47] has used the fuzzy-robust stochastic multi-objective programming approach for managing the petroleum waste. They integrated fuzzy-robust and stochastic linear programming into a general multi-objective programming framework. Two satisfied conflict objectives were minimization of system costs and minimization of waste flows directly to landfill. Masri [32] studied the agent portfolio problems under uncertainty and deal with this problem with a stochastic goal programming approach. Alizadeh Afrouzi *et al.* [1] used the fuzzy SMOO model in order to design a multi-echelon, multi-objective supply chain model that considers new product development and its effects on SC arrangement. Recently, Masmoudi and Abdelaziz [31] presented a survey and reviewed deterministic and stochastic multiple objective programming models for the portfolio selection problems in their research.

## 3. An exact approach for solving SMOOP

Liu and Lin [29] pointed that non-existence of an exact solution is an axiom for uncertain problems; thus, identifying an approximated solution in a certain level of satisfaction is the best possibility. Considering the above axiom, a method is developed to find a satisfactory solution for the considered SMOOP. This procedure includes the following steps:

- transforming stochastic objectives to non-stochastic equivalents;
- transforming stochastic constraints to non-stochastic equivalents;
- solving the obtained non-stochastic model at a certain level of satisfaction.

These steps are detailed at the forthcoming subsections.

### 3.1. Transforming stochastic objectives

In equation (2.2), the considered SMOO problem consists of a set of $l$ stochastic objectives, *i.e.* $\tilde{Z} = \{\tilde{z}_1, \tilde{z}_2, \ldots, \tilde{z}_l\}$, where each objective $\tilde{z}_k, k = 1, 2, \ldots, l$ can be considered as a stochastic linear function of the form.

$$\tilde{z}_k = \sum_{j=1}^{n} \tilde{c}_{kj} x_j, \tag{3.1}$$

where the objective function parameters of $\tilde{c}_{kj}, j = 1, 2, \ldots, n$, statistically distributed with a given distribution function $F_{kj}, j = 1, 2, \ldots, n$, with expected values of $\mu_{kj}, j = 1, 2, \ldots, n$ and bounded variances of $\sigma^2_{kj}, j = 1, 2, \ldots, n$.

According to linear combination of random variables, the $k$th objective $\tilde{z}_k$ has the following expected value:

$$\bar{z}_k = \sum_{j=1}^{n} \mu_{kj} x_j. \tag{3.2}$$

Moreover, since decision variables are independent in linear programming framework [11, 19], the variance of $\tilde{z}_k$ can be computed as:

$$\sigma_k^2 = \sum_{j=1}^{n} \sigma_{kj}^2 x_j^2. \tag{3.3}$$

Following the Chebyshev inequality, a random variable $\tilde{z}_k$ with a mean $\overline{z}_k$ and variance $\sigma_k^2$ will take its mean with a probability greater than $\left(1 - 1/k^2\right)$ in $\pm k\sigma$ above and below the average of the random variable, *i.e.*

$$P\left(\bar{z}_k - k\sigma_k \leq \mu_k \leq \bar{z}_k + k\sigma_k\right) \geq 1 - \frac{1}{k^2}. \tag{3.4}$$

Therefore, in a satisfaction level of $\left(1 - 1/k^2\right)$, the $k$th objective is optimized when the above interval is optimized. According to ordering relations of Ishibuchi and Tanaka [25], an interval like $(\underline{z}_k, \overline{z}_k)$ is minimized when its lower bound $\underline{z}_k$ and its center $(\underline{z}_k + \overline{z}_k/2)$ are maximized. Similarly, an interval $(\underline{z}_k, \overline{z}_k)$ is minimized when its upper bound $\overline{z}_k$ and its center $(\underline{z}_k + \overline{z}_k/2)$ are minimized. Therefore, maximization of the stochastic function $\tilde{z}_k, k = 1, 2, \ldots, l$ is transformed to optimization of the interval $\bar{z}_k - k\sigma_k \leq \mu_k \leq \bar{z}_k + k\sigma_k$. Applying Ishibuchi and Tanaka [25] rules, this interval will be maximized when $\overline{z}_k$ and $\overline{z}_k - k\sigma_k$, simultaneously. Considering the lower bound $\overline{z}_k - k\sigma_k$, since $\overline{z}_k$ is maximized, the lower bound will be maximized if $\sigma_k$ and consequently, $\sigma_k^2$ is minimized. Hence, the maximization of stochastic function $\tilde{z}_k$ at any satisfaction level is transformed into maximization of $\overline{z}_k$ and minimization of $\sigma_k^2$, which is the well-known mean–variance measure in portfolio optimization [30].

If the objective is to minimize the function, then $\overline{z}_k$ and $\overline{z}_k + k\sigma_k$ should be minimize simultaneously. This will be occurring if $\overline{z}_k$ and $\sigma_k^2$ are minimized, contradicting with mean–variance approach. Considering a set of $l$ stochastic functions to be maximized, the equivalent problem will be as follows:

$$\begin{array}{l} \text{Max } \{\bar{z}_1, \bar{z}_2, \ldots, \bar{z}_l\} \\ \text{Min } \left\{\sigma_1^2, \sigma_2^2, \ldots, \sigma_l^2\right\}. \end{array} \tag{3.5}$$

According to equation (3.5), it is clear that SMOLP problem is transformed into a multi-objective non-linear problem with $2l$ objective functions. In the next subsections, a method is proposed to treat the above objective functions.

### 3.2. Transforming stochastic constraints

Consider the $i$th stochastic constraint of the problem as below

$$\sum_{j=1}^{n} \tilde{a}_{ij} x_j \,(\leq, =, \geq)\, \tilde{b}_i,\; i = 1, 2, \ldots, m, \tag{3.6}$$

where the random variables $\tilde{a}_{ij}$, $i = 1, 2, \ldots, m; j = 1, 2, \ldots, n$ follow different probability distributions with given means of $\gamma_{ij}$, $i = 1, 2, \ldots, m; j = 1, 2, \ldots, n$ and given variances of $\delta_{ij}$, $i = 1, 2, \ldots, m; j = 1, 2, \ldots, n$, and random variables $\tilde{b}_i$, $i = 1, 2, \ldots, m$ statistically distributed with means $\overline{b}_i$, $i = 1, 2, \ldots, m$ and variances of $\sigma_{b_i}^2$, $i = 1, 2, \ldots, m$.

The proposed approach for dealing with stochastic constraints is developed based on considering the amount of DM(s) risk attitude. Suppose that the $i$th constraint in equation (3.6) is of the $\leq$ type. Now, in a satisfaction level of $\alpha$, following Chebyshev inequality, the $j$th variable's coefficient in this constraint will occur in the interval of $[\gamma_{ij} - k\delta_{ij},\ \gamma_{ij} + k\delta_{ij}]$, where $k = \sqrt{1/\alpha}$. Comparably, the right hand side of the constraint situates in the interval of $\left[\bar{b}_i - k\sigma_{b_i},\ \bar{b}_i + k\sigma_{b_i}\right]$.

These confidence intervals are replaced in equation (3.6) and the following interval constraint is obtained:

$$\sum_{j=1}^{n} [\gamma_{ij} - k\delta_{ij}, \gamma_{ij} + k\delta_{ij}]\, x_j\, (\leq, =, \geq) \left[\bar{b}_i - k\sigma_{b_i}, \bar{b}_i - k\sigma_{b_i}\right],\ i = 1, 2, \ldots, m. \tag{3.7}$$

The constraint presented in equation (3.7) provides DM(s) a wide range of options to find their decisions, while considering the stochastic nature of constraints. This constraint is a transformation of stochastic constraint to an equivalent interval constraint. These interval constraints can be analyzed applying the order ranking relation of Ishibuchi and Tanaka [25].

Suppose a lower than or equal type constraint. This constraint is replaced by a pair of constraints as below:

$$\begin{cases} \sum_{j=1}^{n} (\gamma_{ij} + k\delta_{ij})\, x_j \leq \bar{b}_i + k\sigma_{b_i} \\ \sum_{j=1}^{n} \gamma_{ij} x_j \leq \bar{b}_i. \end{cases} \tag{3.8}$$

For greater than or equal type constraints, this equivalency is hold as follows:

$$\begin{cases} \sum_{j=1}^{n} (\gamma_{ij} - k\delta_{ij})\, x_j \geq \bar{b}_i - k\sigma_{b_i} \\ \sum_{j=1}^{n} \gamma_{ij} x_j \geq \bar{b}_i. \end{cases} \tag{3.9}$$

### 3.3. Solving the transformed problem

Integrating the above transformations on objective functions and constraints, the SMOLP problem, equation (2.2), is transformed into the following multi-objective non-linear programming (SMONLP) problem:

$$\begin{aligned} &\text{Max } \{\bar{z}_1, \bar{z}_2, \ldots, \bar{z}_l\} \\ &\text{Min } \left\{\sigma_1^2, \sigma_2^2, \ldots, \sigma_l^2\right\} \\ &\text{S.T.} \\ &\sum_{j=1}^{n} (\gamma_{ij} + k\delta_{ij})\, x_j \leq \bar{b}_i + k\sigma_{b_i},\ i = 1, 2, \ldots, m_1 \\ &\sum_{j=1}^{n} \gamma_{ij} x_j \leq \bar{b}_i,\ i = 1, 2, \ldots, m_1 \\ &\sum_{j=1}^{n} (\gamma_{ij} - k\delta_{ij})\, x_j \geq \bar{b}_i - k\sigma_{b_i},\ i = m_1 + 1, 2, \ldots, m \\ &\sum_{j=1}^{n} \gamma_{ij} x_j \geq \bar{b}_i,\ i = m_1 + 1, 2, \ldots, m \\ &x_j \geq 0,\ j = 1, 2, \ldots, n, \end{aligned} \tag{3.10}$$

where the first $m_1$ constraints are less than or equal type and the next $m-(m_1-1)$ constraints are greater than or equal type.

In this subsection, a fuzzy set based approach is developed to solve the above problem. Consider the $k$th objective's mean $\bar{z}_k$ and variance $\sigma_k^2$. First of all, the ideal value of means and anti-ideal value of variances are computed by solving the below linear programming problem:

$$\begin{aligned}
&\text{Max}\,\bar{z}_k,\ k=1,2,\ldots,l\\
&\text{S.T.}\\
&\sum_{j=1}^{n}\left(\gamma_{ij}+k\delta_{ij}\right)x_j \le \bar{b}_i + k\sigma_{b_i},\ i=1,2,\ldots,m_1\\
&\sum_{j=1}^{n}\gamma_{ij}x_j \le \bar{b}_i,\ i=1,2,\ldots,m_1\\
&\sum_{j=1}^{n}\left(\gamma_{ij}-k\delta_{ij}\right)x_j \ge \bar{b}_i - k\sigma_{b_i},\ i=m_1+1,2,\ldots,m\\
&\sum_{j=1}^{n}\gamma_{ij}x_j \ge \bar{b}_i,\ i=m_1+1,2,\ldots,m\\
&x_j\ge 0,\ j=1,2,\ldots,n.
\end{aligned}\tag{3.11}$$

With the optimal solution of $\bar{z}_k^*$, $k=1,2,\ldots,l$ and the non-linear programming of:

$$\begin{aligned}
&\text{Max}\,\sigma_k^2,\ k=1,2,\ldots,l\\
&\text{S.T.}\\
&\sum_{j=1}^{n}\left(\gamma_{ij}+k\delta_{ij}\right)x_j \le \bar{b}_i + k\sigma_{b_i},\ i=1,2,\ldots,m_1\\
&\sum_{j=1}^{n}\gamma_{ij}x_j \le \bar{b}_i,\ i=1,2,\ldots,m_1\\
&\sum_{j=1}^{n}\left(\gamma_{ij}-k\delta_{ij}\right)x_j \ge \bar{b}_i - k\sigma_{b_i},\ i=m_1+1,2,\ldots,m\\
&\sum_{j=1}^{n}\gamma_{ij}x_j \ge \bar{b}_i,\ i=m_1+1,2,\ldots,m\\
&x_j\ge 0,\ j=1,2,\ldots,n.
\end{aligned}\tag{3.12}$$

With the optimal value of $\sigma_{k*}^2, k=1,2,\ldots,l$. The above problems can be solved easily by the current solvers, and as shown in the following sections, both problems have global optima. The anti-ideal value of means are supposed to be $\bar{z}_{k*}=0$, $k=1,2,\ldots,l$, while the ideal value of variances are assumed to be $\sigma_k^{2*}=0$. Accordingly, the below membership function is constructed for means:

$$\mu\left(\bar{z}_k\right)=\begin{cases}0, & \bar{z}_k\le 0\\ \frac{\bar{z}_k}{\bar{z}_k^*}, & 0\le \bar{z}_k\le \bar{z}_k^*\\ 1, & \bar{z}_k\ge \bar{z}_k^*.\end{cases}\tag{3.13}$$

The membership functions of variances are constructed as:

$$\mu\left(\sigma_k^2\right)=\begin{cases}1, & \sigma_k^2 \leq 0\\ \frac{\sigma_{k*}^2-\sigma_k^2}{\sigma_{k*}^2}, & 0\leq \sigma_k^2 \leq \sigma_{k*}^2\\ 0, & \sigma_k^2 \geq \sigma_{k*}^2.\end{cases} \tag{3.14}$$

Aggregating the equations (3.14) and (3.15), an objective function is obtained as:

$$\sum_{k=1}^{l}\mu\left(\bar{z}_k\right)+\mu\left(\sigma_k^2\right). \tag{3.15}$$

If a weight vector $(w_1, w_2, \ldots, w_l)$, $w_k \geq 0, k=1,2,\ldots,l$ and $\sum_{k=1}^{l} w_k = 1$ is assigned to objective functions, the weighted objective function is constructed as:

$$\sum_{k=1}^{l} w_k\left(\mu\left(\bar{z}_k\right)+\mu\left(\sigma_k^2\right)\right). \tag{3.16}$$

Eventually, the following problem is solved to find a solution for the primal problem of equation (2.2):

$$\begin{aligned}
&\text{Max}\sum_{k=1}^{l} w_k\left(\mu\left(\bar{z}_k\right)+\mu\left(\sigma_k^2\right)\right)\\
&\text{S.T.}\\
&\sum_{j=1}^{n}\left(\gamma_{ij}+k\delta_{ij}\right)x_j \leq \bar{b}_i + k\sigma_{b_i},\ i=1,2,\ldots,m_1\\
&\sum_{j=1}^{n}\gamma_{ij}x_j \leq \bar{b}_i,\ i=1,2,\ldots,m_1\\
&\sum_{j=1}^{n}\left(\gamma_{ij}-k\delta_{ij}\right)x_j \geq \bar{b}_i - k\sigma_{b_i},\ i=m_1+1,2,\ldots,m\\
&\sum_{j=1}^{n}\gamma_{ij}x_j \geq \bar{b}_i,\ i=m_1+1,2,\ldots,m\\
&x_j \geq 0,\ j=1,2,\ldots,n.
\end{aligned} \tag{3.17}$$

In addition, the risk attitude of DM can be implied by considering different weights for mean and variance of objective function. Let $\lambda_m$ and $\lambda_v$ be the weights of mean and variance of different objectives, respectively, where $\lambda_m, \lambda_v \geq 0$ and $\lambda_m+\lambda_v= 1$. These weights can be classified into three conditions:

- for a risk averse DM, $\lambda_m \leq \lambda_v$;
- for a risk neutral DM, $\lambda_m=\lambda_v$;
- for a risk seeking DM, $\lambda_m \geq \lambda_v$.

Then, the objective function of equation (3.17) becomes as follows:

$$\text{Max}\sum_{k=1}^{l} w_k\left(\lambda_m\mu\left(\bar{z}_k\right)+\lambda_v\mu\left(\sigma_k^2\right)\right). \tag{3.18}$$

### 3.4. Algorithmic scheme

In this section, the above presented stages are summarized in a procedure including six steps:

**Step 1.** Formulate the primal problem according to SMOLP structure of equation (2.2);
**Step 2.** Construct the mean and variance functions of different objectives according to equations (3.2) and (3.3), respectively.
**Step 3.** Solve the problem in equation (3.11) detecting the ideal values of means, $\bar{z}_k^*$, $k = 1, 2, \ldots, l$.
**Step 4.** Solve the problem in equation (3.12) finding the anti-ideal values of variances, $\sigma_{k*}^2, k = 1, 2, \ldots, l$.
**Step 5.** Construct the membership functions of means and variances, applying equations (3.13) and (3.14), respectively.
**Step 6.** Construct and solve the problem in equation (3.17) or equation (3.18) to acquire the optimal solution of the primal problem.

The above method requires solving $2l+1$ problems, with similar constraints and different objectives. Common solvers, *e.g.* Lingo, AIMS, etc., can be used easily to solve the foregoing problems. The following lemmas represent some important characteristics of the obtained solutions.

**Lemma 3.1.** *The feasible space obtained from intersection of constraints of the form in equation* (3.12) *is convex.*

*Proof.* Considering linearity of constraints, the proof is straightforward. □

**Lemma 3.2.** *The objective functions in equations* (3.11)*,* (3.12) *and* (3.17) *are convex functions.*

*Proof.*

The proof is given in three points:

(1) The objective function of equation (3.11) is linear and therefore its convexity is clear.
(2) The objective function of equation (3.12) is a quadratic function with a diagonal Hessian of $H_k = diag\left(2\sigma_{k1}^2, 2\sigma_{k2}^2, \ldots, 2\sigma_{kn}^2\right)$ which is clearly positive, therefore, is convex.
(3) Since equation (3.17) is a positive linear combination of convex functions; thus, it is a convex function [43]. □

**Theorem 3.3.** *The optimal solution obtained from solving models in equations* (3.11)*,* (3.12) *and* (3.17) *are global solutions.*

*Proof.* Since the considered problems associated with maximizing a convex function over a convex set of constraints, the proof is straightforward [4]. □

## 4. Numerical examples

To shed more light on the advantages of our proposed method, some numerical examples are solved and the obtained results are compared with previously proposed techniques.

### 4.1. Case 1

Ekhtiari and Ghoseiri [17] solved a manpower allocation problem, dealing with allocation of 30 people to 5 work stations. Three objectives are defined as (1) stochastic objective of maximizing final outputs, (2) deterministic objective of minimizing daily wage cost objective of workers and (3) stochastic objective of minimizing total unallowable idle times of workers. The SMOO problem is represented in equation (4.1):

$$
\begin{aligned}
&\text{Max}\, \tilde{f}_1(x) = \sum_{j=1}^{5} \tilde{o}_j x_j \\
&\text{Min}\, f_2(x) = \sum_{j=1}^{5} w_j x_j \\
&\text{Min}\, \tilde{f}_3(x) = \sum_{j=1}^{5} \tilde{t}_j x_j \\
&\text{S.T.} \\
&x_1 + x_2 + x_3 + x_4 + x_5 = 30 \\
&3 \le x_1 \le 9 \\
&3 \le x_2 \le 9 \\
&4 \le x_3 \le 9 \\
&2 \le x_4 \le 9 \\
&3 \le x_5 \le 9,
\end{aligned}
\tag{4.1}
$$

where $x_j$ is the decision variable of the number of manpower in $j$th unit, $\tilde{o}_j$ the random variable of output quantity per worker in $j$th unit, $w_j$ the daily wage/worker employed in $j$th unit and $\tilde{t}_j$ is the random variable of unallowable idle time/worker employed in $j$th unit.

The means and variances of random variables are expressed in Table 1.

Ekhtiari and Ghoseiri [17] solved the above problem by chance-constrained global criterion (CCGC) and chance-constrained compromise programming (CCCP). This problem is solved by our proposed method. The step by step solution approach is detailed below.

**Step 1.** The problem is formulated in equation (4.1).

**Step 2.** The mean and variance functions are constructed. For the first objective, the mean and variance functions are, respectively, as follows:

$$
\begin{aligned}
\overline{O} &= 8x_1 + 5x_2 + 12x_3 + 6x_4 + 10x_5 \\
\sigma_O^2 &= 0.000125x_1 + 0.000324x_2 + 0.000469x_3 + 0.000521x_4 + 0.000324x_5.
\end{aligned}
$$

Since the second objective is a deterministic one, it has only a mean function as

$$\overline{W} = 20x_1 + 15x_2 + 17x_3 + 12x_4 + 18x_5.$$

Finally, for the third object it follows that:

$$\overline{T} = 40x_1 + 60x_2 + 35x_3 + 50x_4 + 45x_5$$

$$\sigma_O^2 = 0.000162x_1 + 0.00021x_2 + 0.000135x_3 + 0.000222x_4 + 0.000198x_5.$$

**Steps 3 and 4.** In this step, the problem consists of optimizing the mean and variance functions to determine their ideal and anti-ideal values. The first problem is to maximize $\overline{O}$ with the certain constraints of equation (3.18) Table 2.

TABLE 1. Random variables mean and variance.

| $j$ | 1 | | 2 | | 3 | | 4 | | 5 | |
|---|---|---|---|---|---|---|---|---|---|---|
| | Mean | Variance | Mean | Variance | Mean | Variance | Mean | Variance | Mean | Variance |
| $\tilde{o}_j$ | 8 | 0.000125 | 5 | 0.000324 | 12 | 0.000469 | 6 | 0.000521 | 10 | 0.000324 |
| $w_j$ | 20 | 0 | 15 | 0 | 17 | 0 | 12 | 0 | 18 | 0 |
| $\tilde{t}_j$ | 40 | 0.000162 | 60 | 0.00021 | 35 | 0.000135 | 50 | 0.000222 | 45 | 0.000198 |

TABLE 2. Ideal and anti-ideal values for different objectives.

| | $\overline{O}$ | $\sigma_O^2$ | $\overline{W}$ | $\overline{T}$ | $\sigma_T^2$ |
|---|---|---|---|---|---|
| Ideal | 279 | 0 | 0 | 0 | 0 |
| Anti-ideal | 0 | 0.09589 | 525 | 1485 | 0.04479 |

**Step 5.** Determining the ideal and anti-ideal values, the membership functions of mean and variance are constructed. The results of the first objective are formed as:

$$\mu\left(\overline{O}\right) = \frac{8x_1+5x_2+12x_3+6x_4+10x_5}{297},$$

and

$$\mu\left(\sigma_O^2\right) = \frac{0.09589 - 0.000125x_1-0.000324x_2-0.000469x_3-0.000521x_4-0.000324x_5}{0.09589},$$

respectively, since the second objective is deterministic, the membership function is just constructed for its mean. This membership function is as follows:

$$\mu\left(\overline{W}\right) = \frac{20x_1+15x_2+17x_3+12x_4+18x_5}{525}.$$

Finally, the membership functions associated with the third objective's mean and variance are respectively as below:

$$\mu\left(\overline{T}\right) = \frac{8x_1+5x_2+12x_3+6x_4+10x_5}{297},$$

and

$$\mu\left(\sigma_T^2\right) = \frac{0.04479 - 0.000162x_1-0.00021x_2-0.000135x_3-0.000222x_4-0.000198x_5}{0.09589}.$$

**Step 6.** Considering equal weights for different objectives, and a risk neutral DM, the problem in equation (3.17) becomes as follow in this example.

TABLE 3. The results of different methods.

| | Proposed method | CCGC | CCCP |
|---|---|---|---|
| $x_1$ | 9 | 9 | 9 |
| $x_2$ | 4 | 3 | 3 |
| $x_3$ | 7 | 9 | 9 |
| $x_4$ | 4 | 2 | 2 |
| $x_5$ | 6 | 7 | 7 |
| Final outputs mean | 258 | 277 | 277 |
| Final outputs variance | 0.05829 | 0.06899 | 0.06899 |
| Wage cost | 512 | 528 | 528 |
| Unallowable idle times mean | 1324 | 1270 | 1270 |
| Unallowable idle times variance | 0.03378 | 0.03654 | 0.03654 |

$$\begin{aligned}
&\text{Max}\, \mu\left(\bar{O}\right) + \mu\left(\sigma_O^2\right) + \mu\left(\bar{W}\right) + \mu\left(\bar{T}\right) + \mu\left(\sigma_T^2\right)\\
&\text{S.T.}\\
&x_1 + x_2 + x_3 + x_4 + x_5 = 30\\
&3 \le x_1 \le 9\\
&3 \le x_2 \le 9\\
&4 \le x_3 \le 9\\
&2 \le x_4 \le 9\\
&3 \le x_5 \le 9.
\end{aligned} \tag{4.2}$$

The problem in equation (4.2) is solved with LINGO. The obtained results with the achieved level of means and variances are summarized in Table 3.

It is clear from Table 3 that the result of the proposed method is a Pareto optimal solution of the stochastic multi-objective linear programming problem of equation (4.1), comparing with the results of CCGC and CCCP methods. On the other hand, it is evident that the variances of optimal solutions of CCGC and CCCP methods in two stochastic objectives of final outputs maximization and unallowable idle times minimization are smaller at the proposed method solution, regard to two other methods. It is worth noting here that this variance minimization can be considered as an advantage of the proposed method in solving stochastic problems.

### 4.2. Case 2

Muñoz and Ruiz [34] solved the following problem using their ISTMO approach:

$$\begin{aligned}
&\text{Max}\, Z_1\left(x_1, x_2\right) = -3x_1 - 16x_2 + \tilde{t}_1\left(4x_1 + x_2\right)\\
&\text{Max}\, Z_2\left(x_1, x_2\right) = 9x_1 + 3x_2 + \tilde{t}_2\left(x_1 + 30x_2\right)\\
&\text{S.T.}\\
&x_1 + x_2 \ge 6, \quad -2x_1 + 3x_2 \le 6,\\
&-x_1 + 3x_2 \le 17, \quad x_1 + 4x_2 \le 39,\\
&2x_1 + 3x_2 \le 43, \quad x_1 - x_2 \le 9,\\
&x_1 - 2x_2 \le 6, \quad x_1 + 4x_2 \ge 12,\\
&x_1, x_2 \ge 0,
\end{aligned} \tag{4.3}$$

where $\tilde{t}_1$ is a random variable that follows uniform distribution $\tilde{t}_1 \sim U\left(1,\ 5\right)$ and $\tilde{t}_2$ is a random variable whose distribution is exponential with parameter $\lambda = 3.4$. They solved this problem in 3 iterations and the optimal solution is obtained as $\left(x_1, x_2\right) = (13.634,\ 4.634)$. If this problem is solved *via* the proposed method, with

TABLE 4. The results of different methods.

| | Proposed method | ISTMO |
|---|---|---|
| $x_1$ | 8.4244 | 13.634 |
| $x_2$ | 1.2122 | 4.634 |
| First objective's mean | 60.061 | 62.464 |
| First objective's variance | 1515.995 | 3994.199 |
| Second objective's mean | 139.1765 | 340.1467 |
| Second objective's variance | 204.539 | 1475.742 |

TABLE 5. The results of the proposed method for DMs with different risk attitude.

| | Risk seeking $\lambda_m$= 0.6, $\lambda_v$ = 0.4 | Risk averse $\lambda_m$= 0.4, $\lambda_v$ = 0.6 |
|---|---|---|
| $x_1$ | 11.403 | 8 |
| $x_2$ | 2.701 | 1 |
| First objective's mean | 67.508 | 59 |
| First objective's variance | 2783.803 | 1366.67 |
| Second objective's mean | 234.0041 | 125.667 |
| Second objective's variance | 620.4433 | 167.1111 |

equal weights of objectives and for a risk neutral DM, the optimal solution will be obtained as $(x_1, x_2) =$ (8.4244, 1.2122). The objective functions mean and variance values for this problem are summarized in Table 4.

It can be seen in Table 4 that the proposed method's solution has smaller variances in both objectives and therefore its obtained solution is less risky. The results of Table 4 are found for a risk neutral DM. For a moderately risk aversion DM with $\lambda_m$= 0.4 and $\lambda_v = 0.6$, and a moderately risk seeking DM with $\lambda_m$= 0.6 and $\lambda_v = 0.4$, the results are represented in Table 5.

It is obvious that, by increasing DM's attitude toward risk, the objective function means are increased while their variances are decreased. Remark that the ISTMO approach result is obtained for a moderately high risk seeking DM, where $\lambda_m$ between 0.7 and 0.8.

To elaborate the proposed model and for more verification, a stochastic binary bi objective transportation model is employed in Iranian steel industry network. The steel industry main buyers or retailers in Iran with nearly 80% market share are Z.A ($R_1$) and F.M ($R_2$) since 1980. The only domestic mining system being capable to meet the buyers demand is G.G ($S_1$); nonetheless, in the vast majority of cases the remained required iron ore is imported from mines in Russia ($S_2$), China ($S_3$) and Kazakhstan ($S_4$). Figure 1 illustrates the supply network. Reducing transportation costs and transportation time are retailers and steel supply network controversial issues alongside with supplying the retailers demand.

Accordingly, the steel industry supply network encompasses 4 suppliers and 2 retailers or buyers. Deterministic parameters and the stochastic demand related to aforementioned network are presented in Table 6. Two main objective functions are minimizing transportation costs and times.

Moreover, uncertain quantities compromising transportation costs and transferring time are delineated in Tables 7 and 8 upon stochastic information emanated from managerial brain storming sessions and calculations, on the basis of different scenarios.

On the basis of above information and in conjunction with proposed model, the stochastic bi objective model for the considered case study is presented as below.

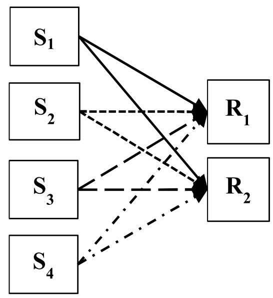


FIGURE 1. Iranian steel industry network.

TABLE 6. Parameters of steel supply network (per year).

| Definition | Symbol | Average quantity | Random variable distribution |
|---|---|---|---|
| Demand for $j$th retailer | $D_1$ | 50 million ton | Poison distribution |
| | $D_2$ | 75 million ton | Poison distribution |
| The production capacity of $i$th supplier | $Ca_1$ | 30 million ton | Parameter |
| | $Ca_2$ | 90 million ton | Parameter |
| | $Ca_3$ | 80 million ton | Parameter |
| | $Ca_4$ | 40 million ton | Parameter |

TABLE 7. Stochastic transportation costs from supplier to retailer ($C_{ij}$ USD per ton).

| Random variable name | $C_{11}$ | $C_{12}$ | $C_{21}$ | $C_{22}$ | $C_{31}$ | $C_{32}$ | $C_{41}$ | $C_{42}$ |
|---|---|---|---|---|---|---|---|---|
| Random variable distribution type | Triangular distribution | | | | | | | |
| Random variable information | (300,390,420) | (310,395,500) | (1250,1400,1600) | (1350,1450,1500) | (2040,2200,2400) | (2080,2180,2250) | (950,990,1020) | (990,1020,1200) |

$$
\begin{aligned}
&\text{Min } \tilde{f}_1(x) = \sum_{i=1}^{4}\sum_{j=1}^{2} \tilde{C}_{ij}.x_{ij}\\
&\text{Min } \tilde{f}_2(x) = \sum_{i=1}^{4}\sum_{j=1}^{2} \tilde{T}_{ij}.x_{ij}\\
&\text{S.T.}\\
&x_{11} + x_{12} \le 30;\\
&x_{21} + x_{22} \le 90;\\
&x_{31} + x_{32} \le 80;\\
&x_{41} + x_{42} \le 40;\\
&x_{11} + x_{21} + x_{31} + x_{41} = \overset{\approx}{50};\\
&x_{12} + x_{22} + x_{32} + x_{42} = \overset{\approx}{75};\\
&x_{ij} \ge 0.
\end{aligned}
$$

TABLE 8. Stochastic transfer time of transported goods from supplier to retailer ($T_{ij}$ per week).

| Random variable name | $T_{11}$ | $T_{12}$ | $T_{21}$ | $T_{22}$ | $T_{31}$ | $T_{32}$ | $T_{41}$ | $T_{42}$ |
|---|---|---|---|---|---|---|---|---|
| Random variable distribution type | Uniform distribution | | | | | | | |
| Random variable information | 2–5 | 2–4/5 | 2/5–3/5 | 2/5–4 | 2/5–4 | 2/5–4 | 2–3 | 2–3/5 |

TABLE 9. The results of the real world case study by proposed model compared with other approaches.

| Variable | Proposed method | Chance-constrained global criterion (CCGC) Ekhtiari and Ghoseiri [17] | Chance-constrained compromise programming (CCCP) Abdelaziz *et al.* [6] |
|---|---|---|---|
| $x_{11}$ | 2.974258 | 1.243141 | 0 |
| $x_{12}$ | 0 | 1.234568 | 0 |
| $x_{21}$ | 26.1519 | 26.26029 | 50 |
| $x_{22}$ | 19.18571 | 16.24657 | 0 |
| $x_{31}$ | 1.01E−09 | 1.12E+01 | 0 |
| $x_{32}$ | 11.31167 | 16.24657 | 0 |
| $x_{41}$ | 20.87385 | 11.24829 | 0 |
| $x_{42}$ | 18.35072 | 16.24657 | 25 |
| Cost optimal mean | 130425.6 | 150078.1 | Infeasible |
| Cost optimal variance | 4847467 | 5404401 | |
| Time optimal mean | 290.6312 | 302.1026 | |
| Time optimal variance | 256.0869 | 242.1592 | |

Comparing the results in Table 9, it is clear that the proposed method results are preferred to CCGC and CCCP because in cost objective, both mean and variance of the proposed method are better than CCGC while in time objective, mean time is better than CCGC while the obtained variance is greater. On the other hand, CCCP method in this case results in an infeasible solution. Also, the normality assumption is necessary for both CCGC and CCCP methods while the proposed method is not based on normality assumption that is another feature of it.

## 5. CONCLUSION

Multi-objective problems are a type of common problems in engineering, management and science. This commonality arises from the fact that human decisions, in both personal and professional life, are made by considering several objectives. As Vincke [46] believed, multi-criteria problems are hard since they are "ill-defined mathematical problems, without any objective solution, *i.e.* a solution which is better than all others for all the criteria". This difficulty of multi-criteria problems are intensified by implying the uncertainty of information. In this case, stochastic programming is an accepted framework for dealing with the challenge of uncertainty in programming type problems. Therefore, in this paper a model is proposed for solving stochastic multi-objective programming problems. The stochastic uncertainty of objective functions and constraints are handled using the concept of confidence intervals. Since, different parameters of the model, including the objective functions coefficients and technological coefficients of the constraint, can follow different statistical distributions, determining an exact statistical distribution for objective functions and constraints become more and more difficult. Therefore, the Chebyshev inequality bound is applied to transform the stochastic objectives and constraints to equivalent intervals. Afterwards, it is shown that maximization of a given stochastic objective in a certain confidence level is equivalent to maximizing its mean and simultaneously minimizing its variance. This mean maximization and variance minimization scheme is the classic mean–variance concept in portfolio optimization. Next, a transformation is introduced for stochastic constraints applying the inequality bounds and interval numbers ordering relations. The performance of the proposed method is investigated in two different numerical examples and it is compared with two previously presented models. It is shown that considering the variance minimization in the proposed method, the obtained results of SMOO problems tend to have smaller variances regard to other methods. This advantage leads to results that are more robust than previous methods. Furthermore, implying the risk attitude of DM in solving the problem is another advantage of the proposed method which provides the chance of finding different results for an uncertain problem according to riskiness of DM. Finally, the ability to solve problems where both objective functions and constraints are stochastic is really a marked difference of our proposed method, making it an appropriate approach to solve SMOO problems in an uncertain environment with a semi-interactive method.

## References


[1] Z. Alizadeh Afrouzy, S.H. Nasseri, I. Mahdavi and M.M. Paydar, A fuzzy stochastic multi-objective optimization model to configure a supply chain considering a new product development. *Appl. Math. Model.* **40** (2016) 7545–7570.

[2] A. Azaron, K.N. Brown, S.A. Tarim and M. Modarres, A multi-objective stochastic programming approach for supply chain design considering risk. *Int. J. Prod. Econ.* **116** (2008) 129–138.

[3] E. Ballestero, Stochastic goal programming: a mean–variance approach. *Eur. J. Oper. Res.* **131** (2001) 476–481.

[4] M.S. Bazaraa, H.D. Sherali and C.M. Shetty, Nonlinear Programming: Theory and Algorithms. John Wiley & Sons, New Jersey (2006).

[5] F. Ben Abdelaziz, Solution approaches for the multi objective stochastic programming. *Eur. J. Oper. Res.* **216** (2012) 1–16.

[6] F. Ben Abdelaziz, B. Aouni and R. El Fayedh, Multi-objective stochastic programming for portfolio selection. *Eur. J. Oper. Res.* **177** (2007) 1811–1823.

[7] V. Bhaskar, G.K. Santosh and R.K. Ajay, Applications of Multi objective Optimization in Chemical Engineering. Department of Chemical Engineering, Indian Institute of Technology, Kanpur, India (2013).

[8] R. Caballero, E. Cerda, M. Del Mar, L. Rey, Stochastic approach versus multiobjective approach for obtaining efficient solutions in stochastic multiobjective programming problems. *Eur. J. Oper. Res.* **158** (2002) 633–648.

[9] N. Chase, M. Rademacher and E. Goodman, A Benchmark Study of Multi-Objective Optimization Methods. Michigan State University, Red Cedar Technology, East Lansing, Michigan (2009).

[10] A. Chen, J. Kim, S. Lee and Y. Kim, Stochastic multi-objective models for network design problem. *Expert Syst. Appl.* **37** (2010) 1608–1619.

[11] R.M. Chiulli, Quantitative Analysis: An Introduction. Gordon and Breach Science Publishers, Amsterdam (1999).

[12] J. Climaco, Multi Criteria Analysis. Springer, New York (1997).

[13] J.L. Cohon, Multi Objective Programming and Planning. Dover Publication, New York (2004).

[14] B. Contini, A stochastic approach to goal programming. *Oper. Res.* **16** (1968) 576–586.

[15] O.L. De Weck, Multi Objective Optimization: History and Promise. Massachusetts Institute of Technology, Dept. of Aeronautics & Astronautics, Engineering Systems Division, Cambridge, Massachusetts (2004).

[16] K. Deb, Multi-Objective Optimization Using Evolutionary Algorithms: An Introduction. Department of Mechanical Engineering, Indian Institute of Technology Kanpur, Kanpur, India (2011).

[17] M. Ekhtiari and K. Ghoseiri, Multi-objective stochastic programming to solve manpower allocation problem. *Int. J. Adv. Manuf. Technol.* **65** (2013) 183–196.

[18] P. Eskelinen, K. Miettinen, Trade-off analysis approach for interactive nonlinear multiobjective optimization. *OR Spectr.* **34** (2011) 803–816.

[19] F.R. Giordano, W.P. Fox and S.B. Horton, A First Course in Mathematical Modeling. Cengage Learning, Boston (2013).

[20] M. Gobbi, F. Levi and G. Mastinu, Multi-objective stochastic optimization of the suspension system of road vehicles. *J. Sound Vib.* **298** (2006) 1055–1072.

[21] A. Goicoechea, L. Duckstein and M.M. Fagel, Multiple objectives under uncertainty: an illustrative application of PROTRADE. *Water Resour. Res.* **15** (1979) 203–210.

[22] W.J. Gutjahr and A. Pichler, Stochastic multi-objective optimization: a survey on non-scalarizing methods. *Ann. Oper. Res.* **236** (2016) 475–499.

[23] T.L. Hardy, Multi-Objective Decision-Making Under Uncertainty: Fuzzy Logic Methods. NASA Technical Memorandum, Cleveland, Ohio (1995).

[24] S. Hulsurkar, M.P. Biswal and S.B. Sinha, Fuzzy programming approach to multi-objective stochastic linear programming problems. *Fuzzy Sets Syst.* **88** (1997) 173–181.

[25] H. Ishibuchi and H. Tanaka, Multi objective programming in optimization of the interval objective function. *Eur. J. Oper. Res.* **48** (1990) 219–225.

[26] A. Khalkhali, M. Sadafi, J. Rezapour and H. Safikhani, Pareto based multi-objective optimization of solar thermal energy storage using genetic algorithms. *Trans. Can. Soc. Mech. Eng.* **34** (2010) 463–475.

[27] P.L. Kunsch and J. Teghem, Nuclear fuel cycle optimization using multi-objective stochastic linear programming. *Eur. J. Oper. Res.* **31** (1987) 240–249.

[28] J.-P. Leclercq, Stochastic programming: an interactive multicriteria approach. *Eur. J. Oper. Res.* **10** (1982) 33–41.

[29] S. Liu and Y. Lin, Grey Information: Theory and Practical Applications. Springer, London (2006).
[30] H.M. Markowitz, Portfolio selection. *J. Finance* **7** (1952) 77–91.
[31] M. Masmoudi and F. Abdelaziz, Portfolio selection problem: a review of deterministic and stochastic multiple objective programming models. *Ann. Oper. Res.* **267** (2018) 335–352.
[32] H. Masri, A multiple stochastic goal programming approach for the agent portfolio selection problem. *Ann. Oper. Res.* **251** (2017) 179–192.
[33] K. Miettinen, Introduction to Multiobjective Optimization: Noninteractive Approaches. Department of Mathematical Information Technology, University of Jyväskylä, Springer, Finland (2012).
[34] M.M. Muñoz and F. Ruiz, ISTMO: an interval reference point-based method for stochastic multiobjective programming problems. *Eur. J. Oper. Res.* **197** (2009) 25–35.
[35] M.M. Muñoz, M. Luque and F. Ruiz, Interest: a reference-point-based interactive procedure for stochastic multiobjective programming problems. *OR Spectr.* **32** (2010) 195–210.
[36] R. Nadeau, B. Uril and L.N. Kiss, PROMISE: a DSS for multiple objective stochastic linear programming problems. *Ann. Oper. Res.* **51** (1994) 45–59.
[37] V. Pareto, Cours d'Économie Politique. Professé à l'Université de Lausanne. Vol. I, edited by F. Roug. Lausanne (1896).
[38] S.H. Razavi Hajiagha, H. Amoozad Mahdiraji and S.S. Hashemi, Multi-objective linear programming with interval coefficients. *Kybernetes* **42** (2013) 482–496.
[39] S.H. Razavi Hajiagha, H. Amoozad Mahdiraji, E.K. Zavadskas and S.S. Hashemi, Fuzzy multi-objective linear programming based on compromise vikor method. *Int. J. Inf. Tech. Decis. Mak.* **13** (2014) 679–698.
[40] M. Sakawa, K. Kato and I. Nishizaki, An interactive fuzzy method for multiobjective stochastic linear programming problems through an expectation model. *Eur. J. Oper. Res.* **145** (2003) 665–675.
[41] J. Teghem and P. Kunsch, Multi objective decision making under uncertainty: an example jór power systems, in Decision Making with Multiple Objective, edited by Y.Y. Haimes and V. Chankong. Springer-Verlag, Berlin (1985) 443–456.
[42] J. Teghem, D. Dufrane and M. Thauvoye, Strange: an interactive method for multi-objective linear programming under uncertainty. *Eur. J. Oper. Res.* **26** (1986) 65–82.
[43] T. Udriste, Convex Functions and Optimization Methods on Riemannian Manifolds. Kluwer Academic Publishers, Dordrecht (1994).
[44] B. Urli and R. Nadeau, Multi-objective stochastic linear programming with incomplete information: a general methodology, in Stochastic *versus* Fuzzy Approaches to Multi objective Mathematical Programming Under Uncertainty, edited by R. Slowinski and J. Teghem. Kluwer Academic Publishers, Springer Netherlands (1990) 131–161.
[45] B. Urli and R. Nadeau, PROMISE/scenarios: an interactive method for multi objective stochastic linear programming under partial uncertainty. *Eur. J. Oper. Res.* **155** (2004) 361–372.
[46] P. Vincke, Multicriteria Decision-Aid. John Wiley, New York (1992).
[47] X. Zhang, G.H. Huang, C.W. Chen, Z. Liu and Q. Lin, A fuzzy-robust stochastic multi objective programming approach for petroleum waste management planning. *Appl. Math. Model.* **34** (2010) 2778–2788.