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VENDOR-BUYER INTEGRATED PRODUCTION-INVENTORY SYSTEM FOR IMPERFECT QUALITY ITEM UNDER TRADE CREDIT FINANCE AND VARIABLE SETUP COST

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Abstract. This paper model a vendor-buyer integrated production-inventory system by considering issues of imperfect quality of the item, trade credit finance, setup cost reduction and shortages including partial backlogging and lost sale. The vendor produces a lot in one production setup and sends to the buyer in multiple shipments to fulfill customers' demand. Due to imperfect production and/or unsafe transportation, the received lot of the buyer contains the imperfect quality of the item, which is detected through the screening process, and is sold at a discounted price in a single batch at the end of the process. To accelerate bulk purchasing, the vendor offers a trade credit period to the buyer to settle the amount. In this regard, we develop a methodology to account the opportunity cost and opportunity gain. Depending upon the screening period μ, trade credit period M, shortage beginning time t and the buyer's scheduling period T, we consider four cases: (1) $M < \mu < t < T$, (2) $\mu < M < t < T$, (3) $\mu < t < M < T$ and (4) $\mu < t < T < M$. The proposed integrated model is testified with numerical experiment and sensitivity analysis by changing the value of key parameters.

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1. Introduction

In a supply chain (SC), adequate supply of material at the adequate time and inventory management are the most important issues, because its help in satisfaction of customers' expectation, increase the service level and avoid shortage at minimum cost. SC partners are indented to increase their business sharing, consequently, they adopt tactics which help in it. Trade credit policy is one of such a tactic. According to an estimate, more than 80% of business-to-business (B2B) transactions in the United Kingdom (UK), and about 80% of United States (US) firms offer their product on trade credit (Seifert *et al.* [23]). Such a worldwide practice encourages researchers to model the trade credit or permissible delay in payment while developing the mathematical models. This study is intended to develop a two-echelon production-inventory system under trade credit finance, and to account, the opportunity cost and opportunity gain for item undergo through the screening process to detect imperfect quality.

Keywords and phrases: Inventory, supply chain, screening, shortage, variable setup cost, trade credit.

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Trade credit period in form of permissible delay in payment is a time span between purchasing and settling the amount. Generally, it is offered to a lower-stair partner by an upper-stair partner in an SC hierarchy. The lower-stair partner can earn interest on the sale revenue gained during the trade credit period and may pay interest for delayed settlement after the end of the credit period. Goyal [8] first derived an economic order quantity (EOQ) formula by considering a permissible delay in payment. Dave [4] and Teng [25] amended Goyal [8] by distinguishing opportunity cost (interest paid) and opportunity gain (interest earn) which are calculated on purchasing cost and selling price, respectively. After that, trade credit and its variants such as partial trade credit and two-level trade credits have been modeled in different environment such as deterioration, price or stock dependent demand, trade credit period dependent demand, etc. We now focus on recent researches on trade credit, which tends to our proposed model. Teng *et al.* [26] and Sheen and Tsao [24] modeled trade credit policy in inventory control problems wherein demand rate were considered as price dependent. Ouyang *et al.* [16] developed an integrated production-inventory model by considering lot size dependent trade credit policy. Teng *et al.* [27] addressed an SC model by considering the linear trend in demand, the wherein upstream party offered permissible delay to downstream party. Sarkar *et al.* [21] modeled a vendor-buyer integrated inventory system by considering stochastic lead-time and permissible delay in payment. Chen *et al.* [3] trade credit modeling includes the order quantity dependent permissible delay in payment. Jaggi *et al.* [11] derived an EOQ model by considering trade credit finance and shortage, and discussed the different condition on interest terms. Zhou *et al.* [32] developed an EOQ model for imperfect quality items by considering inspection error, trade credit finance, and shortage. Giri and Sharma [7] considered time-dependent linear demand into an inventory problem, wherein two level of trade credit policy and shortage are also included.

In real life business situation, due to many undesirable reasons such as defective production, equipment failure, natural disaster, and damage or breakage in transit, the lot size produced/received may contain some defective items. Thus, before meeting up the customers' demand, screening of the item to detect the imperfect quality, is essential. Salameh and Jaber [20] derived an EOQ formula for items with imperfect quality, wherein screening process has been proposed. After that screening process in single-echelon inventory control problems has been widely considered. Goyel and Cárdenas-Barrón [9] amended Salameh and Jaber [20] and presented simpler model by using renewal-reward theorem. Papachristos and Konstantaras [17] amended Salameh and Jaber [20] by providing a suitable inequality to restrict the shortage during the screening period. Wee *et al.* [29] and Eroglu and Ozdemir [6] extended Salameh and Jaber [20] by considering shortages which were fully backlogged. Chang and Ho [2] amended Wee *et al.* [29], and used renewal-reward theorem to find the expected cost function. Maddah and Jaber [13] rigorously analyzed Salameh and Jaber [20] model to ensure 100% screening process before meeting up the customer demand. Maddah *et al.* [14] suggested order overlapping in order to ensure 100% screening. Khan *et al.* [12] presented an elaborative review on Salameh and Jaber [20] model, and rigorously discussed different situations. Vörös [28] skipped a common assumption of the above researchers, a random fraction of imperfect quality $< 1-$ demand rate/screening rate, and rigorously analyzed the mathematical model by considering two cases of a random cycle: (1) independent cycle and (2) connecting cycle. Recently, Moussawi *et al.* [15] developed a production-inventory model for imperfect quality of an item by incorporating twice screening process in a production cycle, and imperfect quality items are sent back for rework.

In two-echelon SC inventory management (SCIM), some researchers have been carried out by considering screening process under different business scenarios. Huang [10] addressed an integrated production-inventory system by considering a random fraction of an imperfect quality item, wherein buyer sends back the imperfect quality to the vendor for rework. Wu and Zhao [30] considered stock and time-dependent demand rate into an imperfect quality item of SC which undergoes both cooperative and non-cooperative environments. Dey and Giri [5] considered a stochastic vendor-buyer integrated production-inventory system for an imperfect quality item, wherein all items are screened before meeting up customers demand. The model also considered that the vendor invests money to improve the production quality.

According to Porteus [18], through an initial investment, setup cost for the setting of machines can be reduced which finally reduces the total cost. Porteus [18] proposed logarithm investment function to reduce the setup cost. In recent trend of two-echelon SC modeling problem, setup cost reduction is used by many researchers such as [1, 19, 22, 31].

In this paper we consider a two-echelon integrated production-inventory model for a single-vendor and a single-buyer, who deal with a single item contains a random fraction of the imperfect quality. Shortages are permissible in the buyer's inventory system and are the mixture of partially backlogging and lost sale. In one setup, the vendor produces a lot and delivers to the buyer in multiple shipments to fulfill the customer demand. The buyer 100% screens the received lot before meeting up the customer demand. Furthermore, the vendor offers a trade credit period to the buyer to settle the purchasing amount. In this regard, we develop a methodology to calculate the opportunity cost and opportunity gain. Moreover, we consider that the vendor setup cost is dynamic in nature which can be reduced through initial investment, and use logarithmic investment function as of Porteus [18]. Thus, the proposed model captures the more suitable real-life business situations. As an evidence of literature survey and best of our knowledge, no such an SC model has been developed till now.

The rest of the paper is arranged as follows: In Section 2, all notations and assumptions have been mentioned which are used throughout the paper. Mathematical formulation of the proposed integrated SC is presented in Section 3. In Section 4, we show that Chang and Ho [2] and Maddah and Jaber [13] models are special cases of our model. Section 5 provides solution procedure and proposes an algorithm towards the global optimal policy. Section 6 is the illustrative example section, wherein we present a numerical example to validate the mathematical formulation and solution procedure. The section also provides managerial insights through the sensitivity analysis for changing the value of the key parameters. Finally, the discussion is ended in Section 7 by delineating the concluding remarks, finding and future direction of the research.

2. Notations and assumptions

The following notations are used throughout the paper.

2.1. Notations

2.1.1. Parameters

r	Demand rate for the buyer
F	Fixed transportation cost per trip for the buyer
A	Ordering cost per order for the buyer
p	Production rate for vendor, $p > r$
d	Loss on per unit imperfect item
s	Screening cost per item
x	Percentage of defective items, a random variable
y	Screening rate
h_v	Vendor's inventory carrying cost per item per unit time
h_b	Buyer's inventory holding cost per item per unit time
η	Fractional charges per unit setup cost
i_e	Interest rate earned by the buyer
i_c	Interest rate charged on the buyer by the vendor
c_b	Unit purchasing cost per item for the buyer
s_b	Unit selling price per item for the buyer
α	Percentage of the shortage to be partially backlogged
B	Back-ordering cost per item per unit time
L	Loss of sale cost per item
M	Trade credit period offered to the buyer

2.1.2. Decision variables

n	Shipment frequency to the buyer in one production cycle, a positive integer
t	Time when the buyer's inventory reaches at zero level and shortage starts $(0 < t < T)$

q Order quantity of the buyer
K Variable setup cost of the vendor

2.1.3. Dependant variables

μ Screening period to ensure the 100% screening of received lot of the buyer
T Scheduling period of the buyer
$f_v(K)$ Investment function for setup cost reduction
$EJTRC$ Expected joint total relevant cost.

The following assumptions are made while developing the proposed integrated SC.

2.2. Assumptions

(1) A single vendor produces a single product and deals with a single buyer. The vendor and the buyer share all information with each other, *i.e.*, they work in collaborative environment.
(2) Instantaneous shipment to the buyer. *i.e.*, lead time is zero.
(3) The vendor produces the nq quantity of the item in one production cycle, and ships to the buyer for n shipments, each of size q.
(4) The buyer screens the received lot before meeting up the customers' demand and sells the imperfect quality of items at some discounted price (called salvage value) in a single batch at the end of the process. d is per unit loss due to the imperfect item, *i.e.*, d equals to purchasing cost minus salvage value.
(5) Shortage is allowed at the buyer's inventory system and is a mixture of partial backlogging and lost sale. Even though the 100% screening process has not been completed while receiving the products, the back ordered quantity is delivered without any defects (see, Chang and Ho [2]).
(6) Fraction of the defective items contained in each batch is a random variable x $(0 \leq x < 1)$.
(7) The screening rate y is greater than the demand rate r, and in order to avoid shortage during screening period, a constraint has been imposed as $(1-x)y > r, \ \forall \ x \Rightarrow E[1-x]y > r$ (see, Papachristos and Konstantaras [17]; Khan *et al.* [12]).
(8) The vendor's setup cost is reduced through an initial investment. We consider logarithmic investment function Porteus [18] as $f_v = \frac{\eta}{\delta} \ln\left(\frac{K_o}{K}\right)$, $0 \leq K \leq K_0$, where K_0 is the original setup cost, K is the reduced setup cost, δ is percentage decrease in setup cost per unit price increase in f_v and η is the annual fractional cost of capital investment.
(9) A trade credit period M is offered to the buyer to settle the purchasing amount. During this period, the buyer's cumulative sale revenue is deposited in an interest-bearing account. At the end of the period, the revenue and earned interest are used in payment of purchasing cost. If trade credit period is shorter than of scheduling period T, then the buyer has to pay interest on his/her investment in inventory for the period $[M, T]$.

3. Integrated production-inventory model

In this section, we mathematically formulate the proposed vendor-buyer integrated production-inventory model, and also calculate the opportunity terms. The vendor is the manufacturer who occupies the top position in SC hierarchy as shown in Figure 1. The vendor produces a lot of sizes nq in each production run and sends to the buyer in n shipments each of size q. During production uptime, the inventory level of the vendor is saw type while during downtime it looks like a ladder. Figure 1 also delineates that the vendor starts production at such a time that no inventory left after the first shipment, which reduces his/her the holding quantity. Production and demand satisfaction is a synchronized process. Inventory position of the buyer is shown in lower part of Figure 1. As shown in the figure, the shortage is permissible in the buyer's system and is a mixture of partial backlogging and lost sale. In each time span T, the buyer receives a lot of size q, which may contain a random fraction x of imperfect quality.

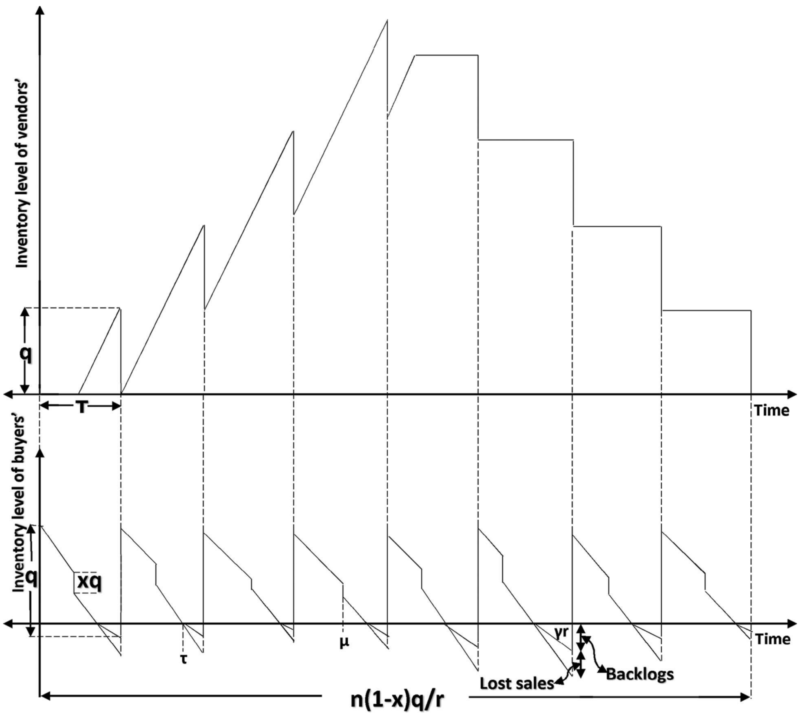


FIGURE 1. Inventory position of the vendor and the buyer.

3.1. Buyer's relevant cost

The buyer's scheduling period (time between two replenishment) is

$$T = t + \frac{(1-x)q - rt}{\alpha r} = \frac{(1-x)q - (1-\alpha)rt}{\alpha r}. \tag{3.1}$$

The total relevant cost of the buyer includes costs of ordering, holding, transportation, screening and shortage cost including backlogging and lost sales. The buyer orders nq quantity once, and receives it in n shipments each of size q. Hence, ordering cost per replenishment cycle for him/her is A/n. As we said all received items are screened, hence screening cost sq is incurred. Transportation cost F is incurred for each shipment. The buyer's cumulative inventory during the period $[0,t]$ is $rt^2/2 + xq^2/y$ (see, Wee *et al.* [29]). Hence, holding cost per replenishment cycle of the buyer is $h_b(rt^2/2 + xq^2/y)$. In each lot, a random quantity xq is imperfect quality. Hence, lost cost due to imperfect quality in per cycle is dxq. Shortage starts at time t and continue up to T. The time-weighted backlogged quantity during stock-out period $[t,T]$ is $(T-t)(T-t)\alpha r/2 = [(1-x)q - rt]^2/2\alpha r$ and lost sales quantity is $(T-t)(1-\alpha)r = [(1-x)q - rt](1-\alpha)/\alpha$. The total relevant cost of the buyer is a function of random variable x, and is

$$ATC_b = \frac{A}{n} + F + sq + dxq + h_b\left(\frac{rt^2}{2} + \frac{x}{y}q^2\right) + B\frac{1}{2\alpha r}(q - rt - xq)^2 + L\frac{1-\alpha}{\alpha}(q - rt - xq). \tag{3.2}$$

We now use the renewal-reward theorem to find the expected per unit relevant cost of the buyer.

$$EATC_b = \frac{E[ATC_b]}{E[T]}. \tag{3.3}$$

3.2. Vendor's relevant cost

In each production run, the vendor incurs setup cost K. The time-weighted inventory carried in the vendor's system is shown in Figure 1, and is $nq^2\big((2-n)\alpha r/p+(n-1)(1-x)\big)/2\alpha r - n(n-1)(1-\alpha)qt/2\alpha$. The relevant cost of the vendor consists of costs of setup and inventory carrying. Hence, the vendor's relevant cost per production cycle is as follows:

$$ATC_v = K + h_v\frac{nq^2}{2\alpha r}\left((2-n)\frac{\alpha r}{p}+(n-1)(1-x)\right) - h_v\frac{n(n-1)(1-\alpha)}{2\alpha}qt. \tag{3.4}$$

Expected per unit relevant cost of the vendor is

$$EATC_v = \frac{E[ATC_v]}{nE[T]}. \tag{3.5}$$

As we discussed earlier, original setup cost can be reduced through an investment which finally reduces the overall cost of SC. Therefore, it is quite appropriate for the vendor to make an investment to reduce the original set cost. Assuming logarithmic investment function in the form $\eta\ln(K_0/K)/\delta$, the expected total cost of the vendor is obtained as

$$EATC_v = \frac{E[ATC_v]}{nE[T]} + \frac{\eta}{\delta}\ln\left(\frac{K_o}{K}\right). \tag{3.6}$$

3.3. Opportunity cost and opportunity gain

As is discussed in the introduction section, the vendor offers a trade credit period M to the buyer to settle the purchasing amount, buyer earns/pays interest depending upon M and T. In this section, we mathematically model the same and calculate the opportunity cost and opportunity gain. The buyer deposits the sale revenue of the period $(0, M]$ is an interest-bearing account and earns interest with the rate i_e. Moreover, he/she has to pay interest on the investment in the remaining inventory of the period $(M, T]$. Depending upon values of M, T, screening completion time μ and shortage starting time t, four cases arise as shown in Figure 2. It is clear from the figure, he/she earns as well as pays interest in first two cases while in last two cases he/she earns, only.

Case 1. $M < \mu < t < T$: In this case, trade credit period is less than the buyer's scheduling period. Hence, as shown in Figure 2A, the buyer earns interest during the period $[0, M]$ while pays for the period $(M, T]$. The interest earned by the buyer is

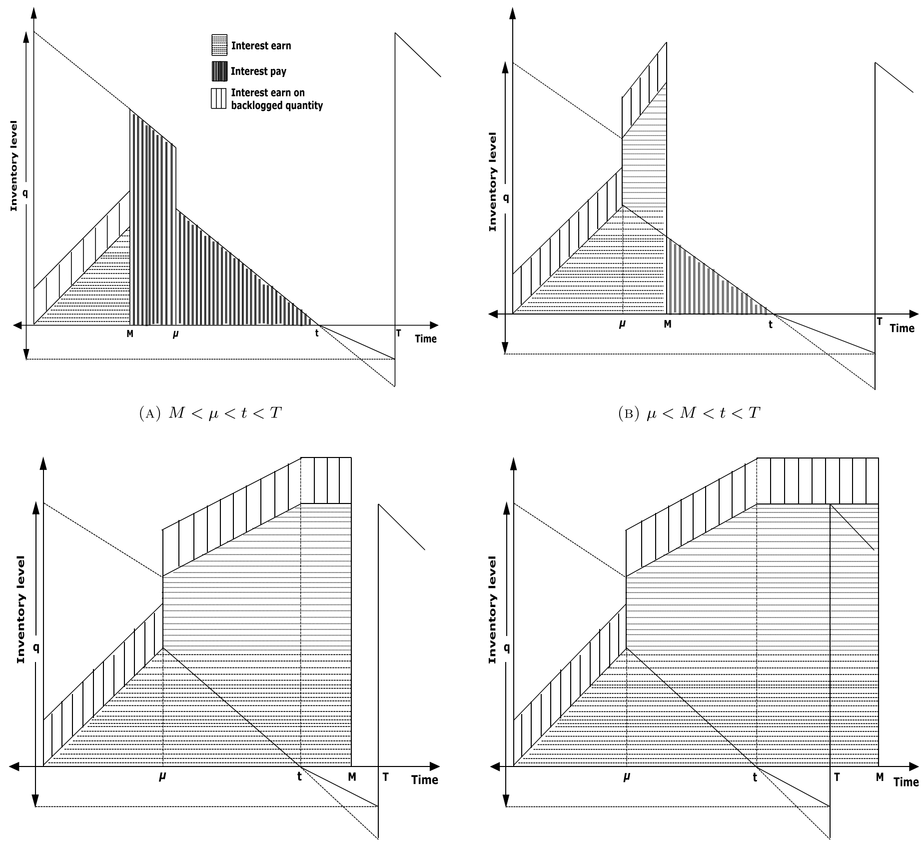
$$IE_1 = s_b i_e\left[\frac{1}{2}rM^2 + ((1-x)q - rt)M\right]. \tag{3.7}$$

The expected interest earned per unit time is

$$EIE_1 = \frac{s_b i_e \alpha r}{qE[1-x]-(1-\alpha)rt}\left[\frac{1}{2}rM^2 + (E[1-x]q - rt)M\right]. \tag{3.8}$$

The interest paid by the buyer is calculated as follows:

$$IC_1 = c_b i_c\left[\frac{r}{2}(t-M)^2 + \frac{xq}{y}(q - yM)\right]. \tag{3.9}$$



(A) $M < \mu < t < T$ (B) $\mu < M < t < T$

(C) $\mu < t < M < T$ (D) $\mu < t < T < M$

FIGURE 2. Time weighted amount for opportunity cost and opportunity gain.

The expected interest paid per unit time is

$$EIC_1 = \frac{c_b i_c \alpha r}{qE[1-x] - (1-\alpha)rt}\left[\frac{r}{2}(t-M)^2 + \frac{E[x]q}{y}(q - yM)\right]. \tag{3.10}$$

Case 2. $\mu < M < t < T$: Similar to Case 1, the random interests earned and paid for this case are calculated as follows:

$$IE_2 = s_b i_e \left[\frac{1}{2}rM^2 + ((1-x)q - rt)M + \frac{xq}{y}(yM - q)\right] \tag{3.11}$$

and

$$IC_2 = c_b i_c \frac{r}{2}(t-M)^2. \tag{3.12}$$

The expected values of interest earned and interest paid per unit time are

$$EIE_2 = \frac{s_b i_e \alpha r}{qE[1-x] - (1-\alpha)rt}\left[\frac{1}{2}rM^2 + (E[1-x]q - rt)M + \frac{E[x]q}{y}(yM - q)\right] \tag{3.13}$$

and

$$EIC_2 = \frac{c_b i_c \alpha r^2 (t-M)^2}{2(qE[1-x] - (1-\alpha)rt)}. \tag{3.14}$$

Case 3. $\mu < t < M < T$: In this case the trade credit period is longer than t. Hence, no interest is charged on the buyer, he/she only earns interest during the period $[0, M]$. The interest terms for this case are as follows:

$$IE_3 = s_b i_e \left[\frac{xq}{y}(yM - q) + \frac{rt}{2}(2M - t) + ((1-x)q - rt)M\right] \tag{3.15}$$

and

$$IC_3 = 0. \tag{3.16}$$

The expected values of interest earn per unit time is

$$EIE_3 = \frac{s_b i_e \alpha r}{qE[1-x] - (1-\alpha)rt}\left[\frac{xq}{y}(yM - q) + \frac{rt}{2}(2M - t) + ((1-x)q - rt)M\right]. \tag{3.17}$$

Case 4. $\mu < t < T < M$: The chargeable and earned interests for this case are shown in Figure 2D, and are as same as obtained in Case 3.

3.4. The expected joint total relevant cost

We above discussed four cases of opportunity cost and opportunity gain for the integrated vendor-buyer system and found that interest terms of Case 4 were same as of Case 3. Hence, a random total cost for Case 4 is same as of Case 3. We now integrate all cases in order to find the expected joint total relevant cost (EJTRC) as follows:

$$EJTRC(K,q,t,n) = \begin{cases} EJTRC_1 = EATC_v + EATC_b + EIC_1 - EIE_1, \\ M \le \mu \le t \le T; \\ EJTRC_2 = EATC_v + EATC_b + EIC_2 - EIE_2, \\ \mu \le M \le t \le T; \\ EJTRC_3 = EATC_v + EATC_b + EIC_3 - EIE_3, \\ \mu \le t \le M \le T, \end{cases} \tag{3.18}$$

where

$$E_1 = 1 - E[x], E_2 = E[(1-x)^2] \tag{3.19}$$

$$
\begin{aligned}
EJTRC_1 = \frac{\alpha r}{qE_1-(1-\alpha)rt} & \left\{ \frac{A}{n} + F + sq + dE[x]q + h_b\left(\frac{rt^2}{2} + \frac{E[x]}{y}q^2\right) \right. \\
& + L\frac{1-\alpha}{\alpha}(E_1q - rt) + \frac{B}{2\alpha r}\left(E_2q^2 - 2E_1rqt + r^2t^2\right) \\
& + h_v\frac{q^2}{2\alpha r}\left((2-n)\frac{\alpha r}{p} + (n-1)E_1\right) \\
& - h_v\frac{(n-1)(1-\alpha)}{2\alpha}qt + \frac{K}{n} - s_bi_e\left(\frac{r}{2}M^2 + (E_1q-rt)M\right) \\
& \left. + c_bi_c\left(\frac{r}{2}(t-M)^2 + \frac{E[x]q}{y}(q-yM)\right)\right\} + \frac{\eta}{\delta}\ln\left(\frac{K_o}{K}\right),
\end{aligned} \tag{3.20}
$$

$$
\begin{aligned}
EJTRC_2 = \frac{\alpha r}{qE_1-(1-\alpha)rt} & \left\{ \frac{A}{n} + F + sq + dE[x]q + h_b\left(\frac{rt^2}{2} + \frac{E[x]}{y}q^2\right) \right. \\
& + \frac{B}{2\alpha r}\left(E_2q^2 - 2E_1rqt + r^2t^2\right) + h_v\frac{q^2}{2\alpha r}\left((2-n)\frac{\alpha r}{p} + (n-1)E_1\right) \\
& + \frac{K}{n} - s_bi_e\left(\frac{rM^2}{2} + (E_1q-rt)M + \frac{E[x]q}{y}(yM-q)\right) + c_bi_c\frac{r}{2}(t-M)^2 \\
& \left. + L\frac{1-\alpha}{\alpha}(E_1q-rt) - h_v\frac{(n-1)(1-\alpha)}{2\alpha}qt\right\} + \frac{\eta}{\delta}\ln\left(\frac{K_o}{K}\right),
\end{aligned} \tag{3.21}
$$

$$
\begin{aligned}
E(JTRC_3) = \frac{\alpha r}{qE_1-(1-\alpha)rt} & \left\{ \frac{A}{n} + F + sq + dE[x]q + h_b\left(\frac{rt^2}{2} + \frac{E[x]}{y}q^2\right) \right. \\
& + \frac{B}{2\alpha r}\left(E_2q^2 - 2E_1rqt + r^2t^2\right) + h_v\frac{q^2}{2\alpha r}\left((2-n)\frac{\alpha r}{p} + (n-1)E_1\right) \\
& + \frac{K}{n} - s_bi_e\left(\frac{E[x]q}{y}(yM-q) - \frac{rt^2}{2} + E_1qM\right) + L\frac{1-\alpha}{\alpha}(E_1q-rt) \\
& \left. - h_v\frac{(n-1)(1-\alpha)}{2\alpha}qt\right\} + \frac{\eta}{\delta}\ln\left(\frac{K_o}{K}\right).
\end{aligned} \tag{3.22}
$$

4. Analysis

If we consider an inventory system of a buyer, only, without trade credit finance and without set up cost reduction investment, where shortages are allowed and are fully backlogged. For this situation we substitute $K = K_0 \to 0$, $h_v = 0$, $i_v = 0$, $i_c = 0$, $n = 1$, $\alpha = 1$ and $F = 0$ in the above integrated model, then equation (3.18) becomes as

$$
EJRTC(q,t) = \frac{r}{qE_1}\left[A + sq + dE[x]q + h_b\left(\frac{rt^2}{2} + \frac{E[x]q^2}{y}\right) + \frac{B}{2r}\left(E_2q^2 - 2E_1rtq + r^2t^2\right)\right] \tag{4.1}
$$

Furthermore, if we assume that w be the shortage quantity as of Chang and Ho [2], then t can be expressed as $t = (E_1q - w)/r$. Thus equation (4.1) can be written as

$$EJRTC(q,t) = \frac{1}{qE_1}\left[Ar + \left(\frac{h_b + B}{2}\right)(E_1q - w)^2\right] + \frac{1}{E_1}\Big[rs + rdE[x] - BE_1(E_1q - w)\Big]$$
$$+\frac{1}{E_1}\left[\frac{rb_bE[x]}{y} + \frac{BE_2}{2r}\right]q. \tag{4.2}$$

$\partial EJRTC/\partial w = 0 = \partial EJRTC/\partial q$ give the backordered quantity and order quantity as

$$w^* = \frac{h_bE_1q^*}{h_b + B} \quad \text{and} \quad q^* = \sqrt{\frac{2rA}{h_b\left(E_2 + \frac{2rE[x]}{y} - \frac{h_bE_1^2}{h_b+B}\right)}}. \tag{4.3}$$

The order and backordered quantities obtained in equation (4.3) coincide with Chang and Ho [2]. When shortage is not allowed, *i.e.*, backordering cost increases infinitive ($B \to \infty$), then from equation (4.3),

$$w^* = 0 \quad \text{and} \quad q^* = \sqrt{\frac{2rA}{h_b\left(E_2 + \frac{2rE[x]}{y}\right)}} \tag{4.4}$$

coincide with Maddah and Jaber [13].

5. Solution approach

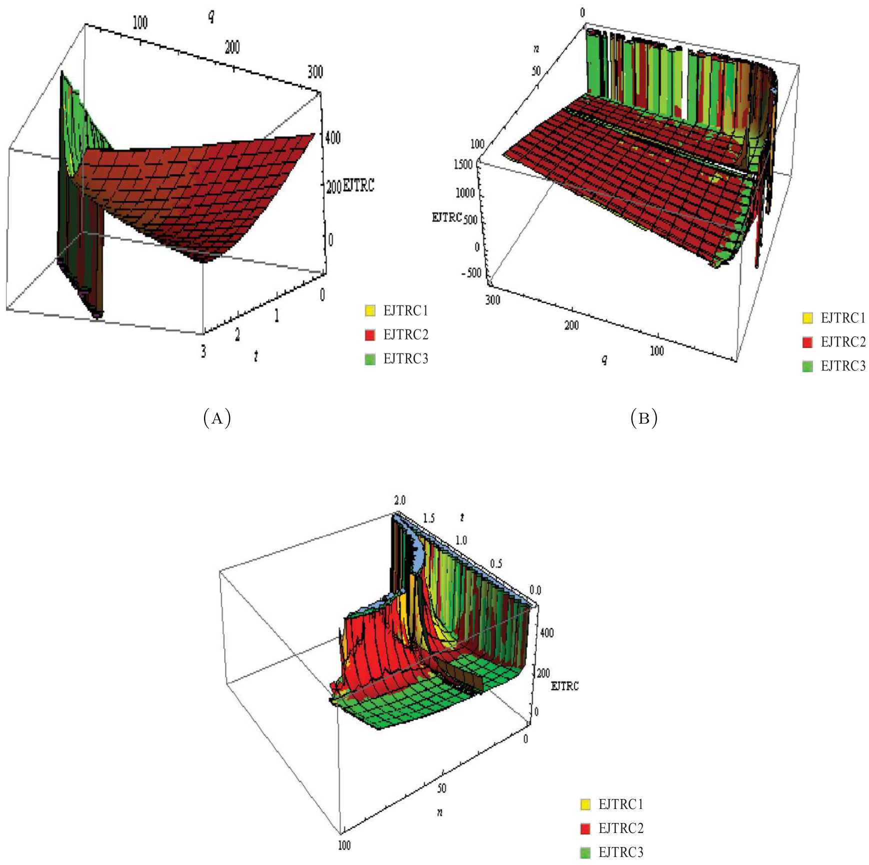
$EJTRC$ is a function of three continuous variables q, t, K and a positive integer variable n. Our aim is to find a feasible solution (K^*, q^*, t^*, n^*) that minimizes the $EJTRC$.

Table 1. Cost factors and other parameters of the manufacturer and the retailer.

Notation	Description	Value
p	Manufacturing rate of the pot	320 pots
m_v	Per unit manufacturing cost	\$6
c_b	Retailer purchasing cost per pot	\$10
s_b	Retailer selling price	\$15
c_d	Discounted price of an imperfect pot	\$9
$d = c_b - c_d$	Lost due to per imperfect pot	\$1
K_0	Set up cost of the manufacturer	\$100
F	Transportation cost of the retailer	\$30
A	Ordering cost for the retailer	\$50
r	Per month demand rate of the retailer	100 pots
h_v	Per month per pot holding cost of the manufacturer	\$0.1
h_b	Per month per pot holding cost of the retailer	\$0.2
y	Screening rate per month	350 pots
α	Percentage of backlogging	70%
B	Backlogging cost of the retailer	\$2
L	Lost of sale of the retailer for unsatisfied demand	\$1
M	Trade credit period offered to the retailer	0.75 month
i_e	Interest rate earned by the retailer	0.05
i_c	Interest rate charged on the retailer	0.07

Theorem 5.1. *If we consider n as a real valued variable, then for fixed values of q, t and K, $EJTRC$ is a convex function of n.*

See Appendix A for proof. However Theorem 5.1 is proved for the continuous variable, but, it ensures that there exists a unique integer n where $EJTRC$ attains minimum value for given values of q, t and K. We now write a solution procedure to find the unique global optimal policy.



(A) (B) (C)

Figure 3. Optimal EJTRC.

Table 2. Sensitivity analysis of trade credit period.

M	i^*	$EJTRC^*$	n^*	K^*	q^*	μ^*	t^*	T^*	f_v^*
0	1	143.225	5	58.3771	97.4204	0.414555	0.741898	1.16754	5.38246
0.25	1	128.088	5	57.1805	97.8519	0.416391	0.774287	1.14361	5.58958
0.50	2	112.308	5	55.2636	97.5324	0.415032	0.806364	1.10527	5.93056
0.75	2	95.7374	6	59.5829	91.2145	0.388147	0.794756	0.993048	5.17802
1	4	78.3159	6	57.1937	90.224	0.383932	0.815163	0.953228	5.58727
1.25	4	60.1958	6	53.7762	88.5143	0.376656	0.83861	0.896269	6.2034

5.1. Solution procedure

Step 1: Set $i = 1$.
Step 2: $i = i + 1$.
Step 3: $n = 0$ and $EJTRC_i = \infty$.
Step 4: $n = n + 1$.
Step 5: Minimize $EJTRC_i$ by using the command NMinimize in Mathematica software.
Step 6: If $EJTRC_i(q_i, t_i, K_i, n) < EJTRC_i(q_i, t_i, K_i, n-1)$ then go to step 4. Otherwise $EJTRC_i^* = EJTRC_i(q_i, t_i, K_i, n-1)$ and if $i < 3$ then go to step 2.
Step 7: $i^* = \arg\left(\min_{i\in\{1,2,3\}} EJTRC_i^*\right)$.
Step 8: Optimal policy is $EJTRC^* = EJTRC_{i^*}$, $q = q_{i^*}$, $t^* = t_{i^*}$, $K^* = K_{i^*}$ and $n^* = n$.
Step 9: Stop.

6. Illustrative example

A small decorative stuff manufacturer company manufactures flower pots and sends to a decorative retail store to fulfill the demand of the customer. The associated costs and other input parameters are given in Table 1. The retailer's record says that a fraction of the received lot of pots is imperfect quality, and is randomly varied lot to lot. The fraction of imperfect quality is estimated to follow the probability distribution function $f(x) = 25,\ 0 \le x \le 0.04$. Furthermore, the manufacturer makes an investment to reduce the setup cost. The investment function is $\eta \ln(K_0/K)/\delta$ with $\eta = 0.2,\ \delta = 0.02$. The problem is to determine the optimal production lot size of the manufacturer, shipment frequency, and the optimal backorder quantity in order to minimize the total relevant cost of the integrated production-inventory system.

The proposed algorithm with MATHEMATICA 9.0 software gives the solution as follows: $i^* = 2$, $n^* = 6$, $K^* = 59.5829$, $EJTRC^* = 95.7374$, $q^* = 91.2145$, $\mu^* = 0.388147$, $t^* = 0.794756$, $T^* = 0.993048$, $f^* = 5.17802$. For the given data set, the Hessian matrix of $EJTRC$ is positive definite as shown in Appendix B. Hence, the solution is optimal and unique. The uniqueness of optimality can be also fixed from Figure 3.

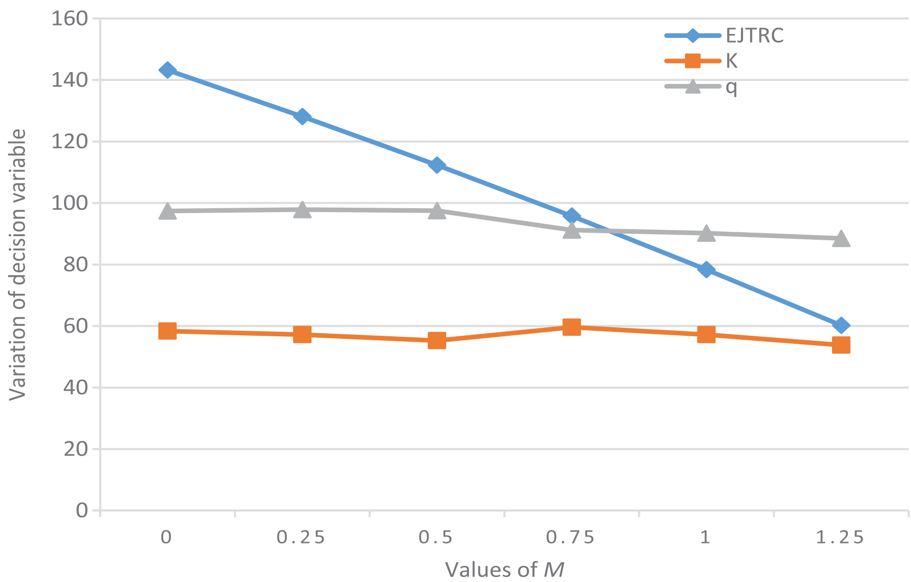


Figure 4. Effect of M on the decision variables.

TABLE 3. Sensitivity analysis under variation of various parameter.

Para meter	% change	i^*	$EJTRC^*$	n^*	q^*	μ^*	t^*	T^*	K^*	f_v^*
A	+50	2	99.7867	6	95.3335	0.405675	0.820376	1.04816	62.8896	4.63789
	+25	2	97.7889	6	93.3055	0.397045	0.807762	1.02103	61.2615	4.90018
	−25	2	93.429	5	93.7334	0.398866	0.814444	1.02273	51.1366	6.7067
	−50	2	90.9627	5	90.9919	0.3872	0.797214	0.986228	49.3114	7.07015
F	+50	2	110.647	4	119.422	0.508177	0.974886	1.36578	54.631	6.04568
	+25	2	103.622	5	104.994	0.446783	0.878541	1.17934	58.9671	5.28191
	−25	3	86.2713	7	77.8795	0.331402	0.702802	0.823636	57.6545	5.50702
	−50	4	73.915	9	60.6901	0.258256	0.579163	0.594537	53.5084	6.25332
h_b	+50	3	98.7413	6	88.7496	0.377658	0.745572	0.993921	59.6352	5.16924
	+25	2	97.2842	6	89.8903	0.382512	0.772449	0.989402	59.3641	5.21481
	−25	2	94.1056	6	92.6229	0.39414	0.818421	0.996987	59.8192	5.13844
	−50	2	92.3605	5	99.6009	0.423834	0.883196	1.06898	53.4491	6.2644
h_v	+50	2	102.795	4	96.3983	0.410205	0.826346	1.06306	42.5224	8.55139
	+25	2	99.5357	5	92.5817	0.393965	0.801929	1.01267	50.6336	6.80555
	−25	2	91.0453	7	91.9387	0.391229	0.802537	0.999461	69.9623	3.57214
	−50	2	84.9819	9	92.2725	0.392649	0.807892	1.00065	90.0585	1.0471
B	+50	2	98.2383	6	88.5266	0.376709	0.796948	0.938174	56.2904	5.74646
	+25	2	97.0515	6	89.7486	0.381909	0.797168	0.961904	57.7142	5.49666
	−25	2	94.0952	5	98.6893	0.419955	0.823552	1.11076	55.5379	5.88104
	−50	2	91.8608	5	102.214	0.434954	0.80492	1.19848	59.9238	5.12096
α	+50	2	89.349	5	101.905	0.433637	0.774229	1.07348	53.6739	6.22244
	+25	2	92.5847	5	99.2756	0.422449	0.804166	1.07414	53.7071	6.21625
	−25	2	98.8889	6	88.0965	0.374879	0.819172	0.936968	56.2181	5.75932
	−50	2	102.055	7	80.2237	0.341378	0.814917	0.900018	49.0013	7.13324
K_o	+50	2	99.792	6	91.2145	0.388147	0.794756	0.993048	59.5829	9.23267
	+25	2	97.9688	6	91.2145	0.388147	0.794756	0.993048	59.5829	7.40946
	−25	2	92.8606	6	91.2145	0.388147	0.794756	0.993048	59.5829	2.3012
	−50	2	88.8961	5	96.3674	0.410074	0.830998	1.0578	50	0
δ	+50	2	93.0097	5	92.795	0.394873	0.808546	1.01024	33.6745	7.25619
	+25	2	94.3122	5	94.2065	0.400879	0.817417	1.02903	41.1612	7.10139
	−25	2	96.8575	6	94.6558	0.402791	0.816161	1.03909	83.1275	2.46394
	−50	2	97.1658	6	101.924	0.43372	0.861369	1.13634	100	0
η	+50	2	96.9793	7	91.8304	0.390768	0.794619	1.00526	100	0
	+25	2	96.679	6	93.7832	0.399078	0.810734	1.02742	77.0563	3.25792
	−25	2	93.8548	5	93.6745	0.398615	0.814074	1.02195	38.323	7.1934
	−50	2	90.9625	5	91.0637	0.387505	0.797665	0.987183	24.6796	6.99597

6.1. Sensitivity of trade credit period

As is mentioned earlier, accounting of opportunity cost and opportunity gain for the integrated production-inventory system is one of the main aims of this research. Opportunities cost and gain, and hence $EJRTC$ depend upon the trade credit period. So, we now find the effect of M by taking a set different value as $M \in \{0, 0.25, 0.50, 0.75, 1, 1.25\}$. The optimal policies for this changes are provided in Table 2. When M is increased, $EJTRC^*$ rapidly decreases while q^* and K^* slightly decrease. It is because, for larger trade credit period, the buyer earns more interest, and hence the integrated SC. Furthermore, when $M = 0$, *i.e.*, when no trade credit

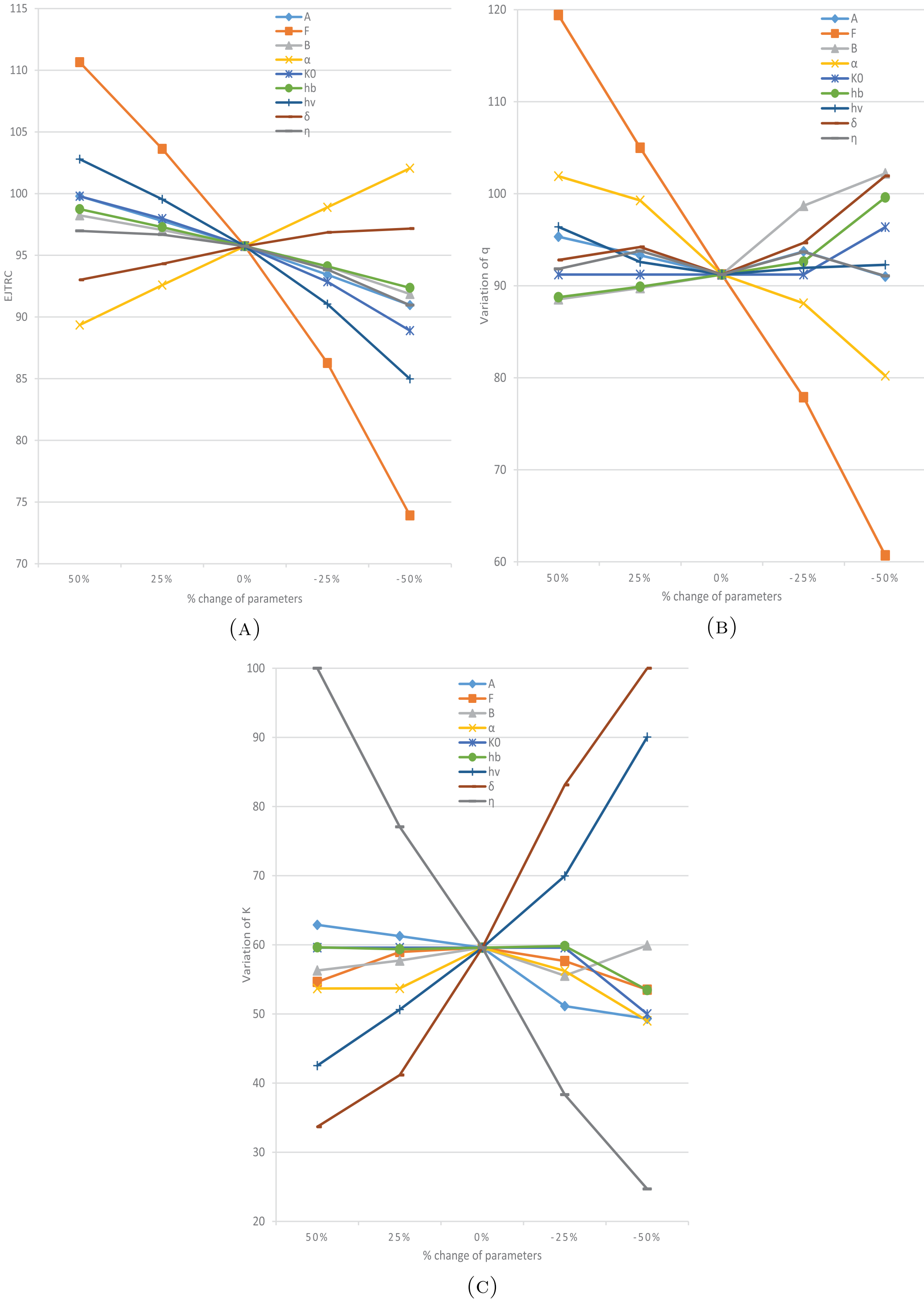


Figure 5. Effects of key parameters on (a) EJTRC, (b) order quantity and (c) setup cost.

period is offered then $EJTRC^*$ is maximum. Thus, we can conclude that longer trade credit period earns more interest and pays less interest, and hence is beneficial for the integrated production-inventory system. How M effects decision variables? is also delineated in Figure 4.

6.2. Sensitivity of other parameters

Sensitivity of the key parameters A, F, B, α, K_o, h_b, h_v, δ, η are also examined here by changing its values as -50%, -25%, $+25\%$ and $+50\%$. The optimal policies for these variations are given in Table 3, and are depicted in Figure 5.

When ordering cost A is decreased, $EJTRC^*$, q^*, n^* and K^* decrease. The decreasing transportation cost F sharply decrease q^* and $EJTRC^*$, and as expected, increases n^*. When the buyer's holding cost h_b is decreased then as expected, $EJTRC^*$ decreases, q^* increases (because decreasing holding cost encourages to keep more stock), consequently, shipment frequency decreases. When the vendor's holding cost is decreased, $EJTRC^*$ and q^* decrease, consequently, n^* increases. Both holding costs also influence the setup cost K^*, but the vendor's holding cost is more. When backlogging cost is decreased, then as expected, $EJTRC^*$ and n^* decrease, while q^* and T^* increase and t^* are almost constant, which means shortage quantity increases. When the fraction of backlogging α is decreased, $EJTRC^*$ increases, because of increment in the lost sale. Figure 5 indicates that F highly influences $EJTRC^*$ and q^*, while δ and η make high influence on K^*.

As Table 3 and Figure 5 indicates, parameters of setup cost reduction function significantly influence the optimal policy. When the value of initial setup cost K_0 is decreased, $EJTRC^*, q^*, n^*$ and K^* are remains unchanged for some limit after that slightly changed. When the value of δ is decreased, $EJTRC^*, q^*, n^*$ and K^* increase. When the value of η is decreased, $EJTRC^*, q^*, n^*$ and K^* decrease. For both variations of δ and η, K^* is tremendously changed. Overall we can say that setup cost reduction parameters highly influences the optimal policy.

7. Conclusions

In this paper, we presented an integrated SC inventory management model for a single vendor and a single buyer, who deal with trade credit policy for the imperfect quality of items, wherein shortages are permissible in the buyer's inventory and are partially backlogged. The paper also includes setup cost reduction and screening facility to detect imperfect quality item and examines its effect on optimal decision policy. In this regards, we have developed a methodology to calculate the opportunity cost and opportunity gain. To the best of our knowledge and as an evidence of literature survey no such integrated production-inventory model has been developed till now. In order to validate the mathematical formulation, we showed Chang and Ho [2] and Maddah and Jaber [13] models were special cases of our model. The mathematical model is analyzed and optimized in order to find a unique global optimal policy. A numerical experiment is also presented to illustrate the foregoing discussion. Through sensitivity analysis, managerial insights and authenticity of the mathematical formulation are established. We found that for large trade credit period the total cost of the integrated production-inventory is comparatively less. Furthermore, we found that instead of a fixed setup cost, consideration of reducible setup cost minimizes the total cost.

In this model, we have considered shortages are partially backlogged, and a fraction of backlogging is constant, but it is not true at all because it is difficult to predict the number of impatience customers. Thus, this model can further be extended by considering backlogging rate is a random variable instead of a fixed constant. Consideration of partial trade credit is another potential extension of this research.

Appendix A

$$\frac{\partial^2 EJTRC(K,q,t,n)}{\partial n^2} = \frac{\partial^2 EJTRC_i(K,q,t,n)}{\partial n^2} = \frac{2r(A+K)}{qE_1 n^3} > 0, \qquad i = 1,2,3.$$

Appendix B

It is not possible to show analytically that the Hessian matrix of $EJTRC$ with respect to q, K and t is positive definite. Hence, for the given data set, we here show that the Hessian matrix is positive definite. For this data we get a solution through proposed algorithm using MATHEMATICA software as $i^* = 2$, $n^* = 6$, $K^* = 59.5829$, $q^* = 91.2145$, $t^* = 0.794756$. For this solution, all principal minors of the Hessian matrix are $\frac{\partial^2 EJTRC}{\partial q^2} = 0.0494376$, $\frac{\partial^2 EJTRC}{\partial t^2} = 500.276$, $\frac{\partial^2 EJTRC}{\partial K^2} = 0.0028168$, $\begin{vmatrix} \frac{\partial^2(EJTRC)}{\partial q^2} & \frac{\partial^2(EJTRC)}{\partial t \partial q} \\ \frac{\partial^2(EJTRC)}{\partial q \partial t} & \frac{\partial^2(EJTRC)}{\partial t^2} \end{vmatrix} = 6.27596$,

$$\begin{vmatrix} \frac{\partial^2(EJTRC)}{\partial q^2} & \frac{\partial^2(EJTRC)}{\partial t \partial q} & \frac{\partial^2(EJTRC)}{\partial K \partial q} \\ \frac{\partial^2(EJTRC)}{\partial q \partial t} & \frac{\partial^2(EJTRC)}{\partial t^2} & \frac{\partial^2(EJTRC)}{\partial K \partial t} \\ \frac{\partial^2(EJTRC)}{\partial q \partial K} & \frac{\partial^2(EJTRC)}{\partial t \partial K} & \frac{\partial^2(EJTRC)}{\partial K^2} \end{vmatrix} = 0.0155868.$$

Therefore, Hessian matrix is positive. Hence, $EJTRC$ is convex function with respect to q, t and K. Furthermore, in section 5, we proved the robustness of the model, and deducted Chang and Ho [2] and Maddah and Jaber[13] models from ours model.

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