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THE SUPER EDGE CONNECTIVITY OF KRONECKER PRODUCT GRAPHS

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Abstract. Let G_1 and G_2 be two graphs. The Kronecker product $G_1 \times G_2$ has vertex set $V(G_1 \times G_2) = V(G_1) \times V(G_2)$ and edge set $E(G_1 \times G_2) = \{(u_1, v_1)(u_2, v_2) : u_1u_2 \in E(G_1) \text{ and } v_1v_2 \in E(G_2)\}$. In this paper we determine the super edge–connectivity of $G \times K_n$ for $n \geq 3$. More precisely, for $n \geq 3$, if $\lambda'(G)$ denotes the super edge–connectivity of G, then at least $\min\{n(n-1)\lambda'(G), \min_{xy\in E(G)}\{\deg_G(x) + \deg_G(y)\}(n-1)-2\}$ edges need to be removed from $G \times K_n$ to get a disconnected graph that contains no isolated vertices.

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1. Introduction

Let G be a finite and simple graph, where $V(G)$ and $E(G)$ denote the set of vertices and the set of edges of G, respectively. If there is an edge $e = uv \in E(G)$, then u and v are *adjacent vertices*, while u and e are *incident*, as are v and e. For a vertex $u \in V(G)$, the *neighborhood* $N_G(u)$ is $\{v : uv \in E(G)\}$. The *degree of a vertex* u is the cardinality of $N_G(u)$, that is $\deg_G(u) = |N_G(u)|$. Let $\delta(G)$ be the minimum degree over all vertices of G. The *degree of an edge* e, denoted by $\xi_G(e)$, is $\deg_G(u) + \deg_G(v) - 2$, where $e = uv$. The complete graph and the star graph on n vertices are denoted by K_n and $K_{1,n-1}$, respectively. For two disjoint non–empty sets A and B of vertices of G, let $[A, B]$ denote the set of edges with one end-vertex in A and the other in B.

A graph G is *connected* if there is a path between any two vertices of G; otherwise G is *disconnected*. A connected subgraph of a graph G is a *component* of G if it is not a proper subgraph of a connected subgraph of G. For an arbitrary subset $S \subseteq E(G)$, we use $G - S$ to denote the graph obtained by removing all edges in S from G. For any connected graph G, if $G - S$ is disconnected, then S is an *edge-cut*. The *edge–connectivity* of a graph G, denoted by $\lambda(G)$, is the minimum cardinality of an edge–cut of G. A connected graph G is *super edge–connected*, or simply *super–*λ, if every edge–cut of size λ isolates a vertex. A graph G is *maximally edge–connected* if $\lambda(G) = \delta(G)$. Analogous definitions exist for vertex–connectivity denoted by $\kappa(G)$.

The edge–connectivity is an important measure of the fault tolerance of a network and gives the minimum cost to disrupt the network. It is known that the most reliable networks are those having the largest edge–connectivity. Harary [12] generalized the notion of connectivity by imposing conditions on the components of $G - S$ and proposed the concept of conditional connectivity. The *conditional connectivity* of G with respect to some graph-theoretic property P is the smallest cardinality of a set S of edges (vertices), if such a set exists,

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such that $G - S$ is disconnected and every remaining component has property Fiol *et al.* [10] introduced the super edge–connectivity. An edge–cut S is called a *super edge–cut of* G, if $G - S$ contains no isolated vertices. In general, super edge–cuts do not always exist. The *super edge–connectivity* $\lambda'(G)$ is the minimum cardinality over all super edge–cuts, if any, that is,

$$\lambda'(G) = \min\{|S| : S \subseteq E(G) \text{ is a super edge cut of } G\}.$$

If the super edge–connectivity does not exist, then we write $\lambda'(G) = +\infty$. Esfahanian and Hakimi [9] showed that if G is neither $K_{1,n-1}$ nor K_3, then $\lambda'(G)$ exists and satisfies $\lambda(G) \leq \lambda'(G) \leq \xi(G)$, where $\xi(G)$ denotes the *minimum edge–degree of* G defined as $\xi(G) = \min_{e\in E(G)}\{\xi_G(e)\}$. It is easy to see that $\lambda'(G) > \lambda(G)$ is a necessary and sufficient condition for G to be super–λ. For notation and terminology not defined here we follow West [21].

Given any two graphs and the Cartesian product of their vertex sets, four standard graph products are the Cartesian product, the Kronecker product, the strong product and the lexicographic product. The Kronecker product $G_1 \times G_2$ of two graphs G_1 and G_2 is the graph having $V(G_1 \times G_2) = V(G_1) \times V(G_2)$ and $E(G_1 \times G_2) = \{(u_1, v_1)(u_2, v_2) : u_1u_2 \in E(G_1) \text{ and } v_1v_2 \in E(G_2)\}$.

The Kronecker product of graphs has been investigated in areas such as graph colorings, graph recognition and decomposition, graph embeddings, matching theory and graph stability (see, for example, [1, 4], and the references therein). This product has generated a lot of interest mainly due to its various applications. For instance, it is used in complex networks to generate realistic networks [14], in multiprocessor systems to model the concurrency [13], and in automata theory [11]. Moreover, it is known that every graph is an induced subgraph of a suitable Kronecker product of complete graphs [16].

The connectivity of Kronecker products of two connected graphs has been investigated by Miller [15] and Weichsel [20]. Brešar an Špacapan [5] obtained some bounds on the vertex–connectivity and edge–connectivity of the Kronecker product of graphs with some exceptions. Wang and Xue [18] and Wang and Wu [19] independently showed that $\kappa(G \times K_n) = \min\{n\kappa(G), (n-1)\delta(G)\}$ for $n \geq 3$ for any graph G. Wang *et al.* [17] proved that $G \times K_n$ $(n \geq 3)$ is super$-\kappa$ for any maximally connected graph, except when $n = 3$ and $G = K_{m,m}$ for $m \geq 1$. Zhou [22] proved that $G \times K_n$ is not super$-\kappa$ if and only if either $\kappa(G \times K_n) = n\kappa(G)$ or $G \times K_n \cong K_{\ell,\ell} \times K_3 (\ell > 0)$, where $n \geq 3$. The authors [3] established the super–connectivity and the h–extra–connectivity of the graphs $K_{m,r} \times K_n$ and $K_m \times K_n$, where $K_{m,r}$ is the complete bipartite graph. Recently, the authors [2] determined the super–connectivity of $G \times K_n$, for any connected graph G satisfying some given conditions.

Cao *et al.* [6] obtained the following result for the edge–connectivity of the Kronecker product of a graph and a complete graph.

Theorem 1.1 [6]**.** *For any graph G and $n \geq 3$, $\lambda(G \times K_n) = \min\{n(n-1)\lambda(G), (n-1)\delta(G)\}$.*

The Kronecker product of a maximally edge–connected graph G and a complete graph K_n, for $n \geq 3$, was shown to be super edge–connected by Cao and Vumar [7]. It is thus natural to ask what is the super edge–connectivity of the Kronecker product of a maximally edge–connected graph G and a complete graph K_n. Moreover, Harary [12] enquired about the value of the conditional connectivity of $G_1 \circ G_2$ in terms of the conditional connectivities of G_1 and G_2, where $\circ$ is any binary operation on graphs. Motivated by the above, in this paper we establish the super edge–connectivity of $G \times K_n$ for any graph G.

2. Main result

We follow the notation used in [17]. For a graph G and a complete graph K_n $(n \geq 3)$, we let $V(G) = \{u_1, u_2, \ldots, u_m\}$ and $V(K_n) = \{v_1, v_2, \ldots, v_n\}$. A vertex (u_i, v_j) is abbreviated as ω_{ij}, where $i \in \{1, 2, \ldots, m\}$ and $j \in \{1, 2, \ldots, n\}$. For any vertex $u_i \in V(G)$, we let $K_n^{u_i} = \{(u_i, v_j) \in V(G) \times V(K_n) : v_j \in V(K_n)\}$ and call it the K_n–*layer* of $G \times K_n$ with respect to u_i. For all $i \in \{1, 2, \ldots, m\}$, the set $K_n^{u_i} = \{\omega_{i1}, \omega_{i2}, \ldots, \omega_{in}\}$ is an independent set in $G \times K_n$.

Lemma 2.1. *For any graph G,*

$$\lambda'(G \times K_n) \leq \min\{n(n-1)\lambda'(G), \min_{xy \in E(G)}\{\deg_G(x) + \deg_G(y)\}(n-1) - 2\},$$

where $n \geq 3$.

Proof. Consider a minimum super edge–cut $T \subset E(G)$ of the graph G. The resulting graph $G - T$ has exactly two components each having more than one vertex, say X_1 and X_2. Let $Y_1 = V(X_1) \times V(K_n)$ and $Y_2 = V(X_2) \times V(K_n)$. Since the subgraphs of $G \times K_n$ induced by Y_1 and Y_2 are both connected and do not contain an isolated vertex, the edge set $[Y_1, Y_2]$ forms a super edge–cut of the graph $G \times K_n$. Thus,

$$\lambda'(G \times K_n) \leq |[Y_1, Y_2]| = n(n-1)\lambda'(G).$$

On the other hand, since $\xi(G \times K_n) = \min_{xy \in E(G)}\{\deg_G(x) + \deg_G(y)\}(n-1) - 2$ and $\lambda'(G \times K_n) \leq \xi(G \times K_n)$, the result follows. □

Lemma 2.2. *For any connected graph G, let $S \subset E(G \times K_n)$ be a super edge–cut of $G \times K_n$ and let $C_1, C_2, \ldots C_r$ be the components of $(G \times K_n) - S$, where $r \geq 2$. For every vertex $u_i \in V(G)$, if there exists a component C_f, for $f \in \{1, 2, \ldots, r\}$, such that $K_n^{u_i} \subseteq C_f$, then $|S| \geq n(n-1)\lambda'(G)$.*

Proof. Suppose that one of the components, say C_1, of $(G \times K_n) - S$ contains only one intact K_n–layer. Since $K_n^{u_i}$ is an independent set in $G \times K_n$ for any $u_i \in V(G)$, the component C_1 is composed of isolated vertices, a contradiction. Thus, every component of $(G \times K_n) - S$ contains at least two intact K_n–layers. This implies that the set of vertices of G corresponding to the first index of the vertices of each component has two or more vertices. The super edge–connectivity of G, $\lambda'(G)$, gives the minimum number of edges that need to be removed from G to get a disconnected graph that contains no isolated vertex, that is, the minimum number of adjacent vertices in G which are in different components when the edges of a super edge–cut of G are removed. Hence, there are at least $\lambda'(G)$ pairs of vertices such that $K_n^{u_i} \subseteq C_t$ and $K_n^{u_j} \subseteq C_k$, where $t \neq k$ and $u_i u_j \in E(G)$. The result follows since $|[K_n^{u_i}, K_n^{u_j}]| = n(n-1)$ for any edge $u_i u_j \in E(G)$. □

Theorem 2.3. *For any graph G, and $n \geq 3$*

$$\lambda'(G \times K_n) = \min\{n(n-1)\lambda'(G), \min_{xy \in E(G)}\{\deg_G(x) + \deg_G(y)\}(n-1) - 2\}.$$

Proof. From Lemma 2.1, we have

$$\lambda'(G \times K_n) \leq \min\{n(n-1)\lambda'(G), \min_{xy \in E(G)}\{\deg_G(x) + \deg_G(y)\}(n-1) - 2\},$$

where $n \geq 3$. In order to prove the theorem, it is enough to show that $\lambda'(G) \geq \min\{n(n-1)\lambda'(G), \min_{xy \in E(G)}\{\deg_G(x) + \deg_G(y)\}(n-1) - 2\}$. Suppose, to the contrary, that S is a minimum super edge–cut of $G \times K_n$ such that $|S| < n(n-1)\lambda'(G)$ and $|S| < \min_{xy \in E(G)}\{\deg_G(x) + \deg_G(y)\}(n-1) - 2$. The resulting graph $(G \times K_n) - S$ has exactly two components C_1 and C_2 each having at least two vertices.

If each K_n–layer of $G \times K_n$ is contained in one of the two components, then by Lemma 2.2, we get $|S| \geq n(n-1)\lambda'(G)$, a contradiction. Thus there is at least one K_n–layer $K_n^{u_i}$, where $u_i \in V(G)$, which has vertices in both C_1 and C_2, that is $K_n^{u_i} \cap C_1 \neq \emptyset$ and $K_n^{u_i} \cap C_2 \neq \emptyset$. Without loss of generality, assume that $\omega_{ip} \in K_n^{u_i} \cap C_1$ and $\omega_{iq} \in K_n^{u_i} \cap C_2$, where $p \neq q$ and $v_p, v_q \in V(K_n)$. Since S is a super edge–cut, the vertex ω_{ip} is not an isolated vertex in C_1. Thus, $N_{G \times K_n}(\omega_{ip}) \cap C_1 \neq \emptyset$. Similarly, $N_{G \times K_n}(\omega_{iq}) \cap C_2 \neq \emptyset$.

Harary [12] stated that for any graph theoretical property P and a graph containing two disjoint subgraphs F and H having the property P, Dirac's form [8] of Menger's theorem says that the minimum number of edges which separate F and H equals the maximum number of edge disjoint paths between F and H. Using this statement, $|S|$ is not less than the maximum number of edge–disjoint paths connecting the vertices of C_1 and C_2. We consider the following two cases:

(1) There is a vertex $u_j \in N_G(u_i)$ such that the layer $K_n^{u_j}$ has vertices in both C_1 and C_2,
(2) For each $u_j \in N_G(u_i)$, the layer $K_n^{u_j}$ is contained completely in either C_1 or C_2.

Case 1. Suppose that there is a vertex $u_j \in N_G(u_i)$, such that $K_n^{u_j} \cap C_1 \neq \emptyset$ and $K_n^{u_j} \cap C_2 \neq \emptyset$. If the vertices ω_{jp} and ω_{jq} are in different components, then we let ω_{jr} be the one in $K_n^{u_j} \cap C_1$ and ω_{js} be the one in $K_n^{u_j} \cap C_2$, that is, $\omega_{jr} \in \{\omega_{jp}, \omega_{jq}\} \cap C_1$ and $\omega_{js} \in \{\omega_{jp}, \omega_{jq}\} \cap C_2$. If both of the vertices ω_{jp} and ω_{jq} are in the same component, say $\{\omega_{jp}, \omega_{jq}\} \subseteq C_1$, then without loss of generality we let ω_{jr} be the vertex ω_{jp} and let $\omega_{js} \in K_n^{u_j} \cap C_2$ such that $s \notin \{p, q\}$.

Let $N_G(u_i) = \{u_j\ (= u_{h_1}), u_{h_2}, \ldots, u_{h_k}\}$ and $N_G(u_j) = \{u_i (= u_{g_1}), u_{g_2}, \ldots, u_{g_\ell}\}$, where $k = \deg_G(u_i)$ and $\ell = \deg_G(u_j)$. Consider the following paths in the Kronecker product graph $G \times K_n$.

- For each $t \in \{2, \ldots, k\}$ and each $f \in \{1, 2, \ldots, n\} \setminus \{p, q\}$, there exists a path P_1 defined in the following way:
  $P_1 := \omega_{ip} \to \omega_{h_t f} \to \omega_{iq}$.
  The number of the paths with this structure is $(k-1)(n-2)$.
- For each $t \in \{2, \ldots, \ell\}$ and each $f \in \{1, 2, \ldots, n\} \setminus \{r, s\}$, there exists a path P_2 defined in the following way:
  $P_2 := \omega_{jr} \to \omega_{g_t f} \to \omega_{js}$.
  The number of the paths with this structure is $(\ell-1)(n-2)$.
- For each $t \in \{2, \ldots, k\}$, there exists a path P_3 defined in the following way:
  $P_3 := \omega_{ip} \to \omega_{h_t q} \to \omega_{if} \to \omega_{h_t p} \to \omega_{iq}$,
  where $f \in \{1, 2, \ldots, n\} \setminus \{p, q\}$. The number of the paths with this structure is $(k-1)$.
- For each $t \in \{2, \ldots, \ell\}$, there exists a path P_4 defined in the following way:
  $P_4 := \omega_{jr} \to \omega_{g_t s} \to \omega_{jf} \to \omega_{g_t r} \to \omega_{js}$,
  where $f \in \{1, 2, \ldots, n\} \setminus \{r, s\}$. The number of the paths with this structure is $(\ell-1)$.
- For each $f \in \{1, 2, \ldots, n\} \setminus \{r, s\}$, there exists a path P_5 defined in the following way:
  $P_5 := \omega_{jr} \to \omega_{if} \to \omega_{js}$.
  The number of the paths with this structure is $(n-2)$.
- For each $f \in \{1, 2, \ldots, n\} \setminus \{\{p, q\} \cup \{r, s\}\}$, there exists a path P_6 defined in the following way:
  $P_6 := \omega_{ip} \to \omega_{jf} \to \omega_{iq}$.
  The number of the paths with this structure is $(n - |\{\{p, q\} \cup \{r, s\}\}|) = n-2$, except when $|\{p, q\} \cup \{r, s\}| = 3$, in which case there exists a path P_7 defined in the following way:
  $P_7 := \omega_{ip} \to \omega_{js}$,
  Thus the total number of paths with the structure P_6 or P_7 is $(n-2)$.

Note that the above paths are all edge–disjoint paths connecting a vertex of C_1 and a vertex of C_2. It follows that

$$
\begin{aligned}
|S| &\geq (k-1)(n-2) + (\ell-1)(n-2) + (n-2) + (k-1) + (\ell-1) + (n-2) \\
&= (n-1)(\deg_G(u_i) + \deg_G(u_j)) - 2 \\
&\geq \min_{xy \in E(G)} \{\deg_G(x) + \deg_G(y)\}(n-1) - 2,
\end{aligned}
$$

a contradiction.

Case 2. Suppose that $K_n^{u_j} \subset C_1$ or $K_n^{u_j} \subset C_2$ for every vertex $u_j \in N_G(u_i)$. There is at least one K_n–layer contained in C_1, say $K_n^{u_a}$ where $u_a \in N_G(u_i)$, otherwise ω_{ip} is an isolated vertex in the resulting graph. Using a similar reasoning, let $K_n^{u_b} \subset C_2$, where $u_b \in N_G(u_i)$.

Let $N_G(u_i) = \{u_a(= u_{h_1}), u_b(= u_{h_2}), u_{h_3} \ldots, u_{h_k}\}$, where $k = \deg_G(u_i)$. Consider the following paths in the Kronecker product graph $G \times K_n$.

- For each $f \in \{1, 2, \ldots, n\} \setminus \{p\}$, there exists a path P_1 defined in the following way:
  $P_1 := \omega_{ip} \to \omega_{bf}$
  The number of paths with this structure is $(n-1)$.
- For each $t \in \{3, \ldots, k\}$, there exists a path P_2 defined in the following way:
  $P_2 := \omega_{ip} \to \omega_{h_t q} \to \omega_{if} \to \omega_{h_t p} \to \omega_{iq}$,
  where $f \in \{1, 2, \ldots, n\} \setminus \{p, q\}$. The number of paths with this structure is $(k-2)$.
- For each $t \in \{3, \ldots, k\}$ and each $f \in \{1, 2, \ldots, n\} \setminus \{p, q\}$, there exists a path P_3 defined in the following way:
  $P_3 := \omega_{ip} \to \omega_{h_t f} \to \omega_{iq}$
  The number of paths with this structure is $(k-2)(n-2)$.
- For each $f \in \{1, 2, \ldots, n\} \setminus \{q\}$, there exists a path P_4 defined in the following way:
  $P_4 := \omega_{af} \to \omega_{iq}$
  The number of paths with this structure is $(n-1)$.
- For each $g \in \{1, 2, \ldots, n\} \setminus \{p, q\}$ and each $f \in \{1, 2, \ldots, n\}$, where $f \neq g$, there exists a path P_5 defined in the following way:
  $P_5 := \omega_{af} \to \omega_{ig} \to \omega_{bf}$.
  Since there are $(n-1)$ paths for each $g \in \{1, 2, \ldots, n\} \setminus \{p, q\}$, the total number of paths with this structure is $(n-2)(n-1)$.
- If $u_a \in N_G(u_b)$, then for each $f \in \{2, \ldots, n\}$, there exists a path P_6 defined in the following way:
  $P_6 := \omega_{a1} \to \omega_{bf}$.
  The number of paths with this structure is $(n-1)$.

Note that the paths above are all edge–disjoint paths connecting a vertex of C_1 and a vertex of C_2. If $u_a \in N_G(u_b)$, then it follows that

$$\begin{aligned}
|S| &\geq (n-1) + (k-2) + (k-2)(n-2) + (n-1) + (n-2)(n-1) + (n-1) \\
&= (n-1)(\deg_G(u_i) + (n-1)) \\
&\geq (n-1)(\deg_G(u_a) + \deg_G(u_b)) \\
&\geq \min_{xy \in E(G)} \{\deg_G(x) + \deg_G(y)\}(n-1) \\
&> \min_{xy \in E(G)} \{\deg_G(x) + \deg_G(y)\}(n-1) - 2,
\end{aligned}$$

a contradiction. If $u_a \notin N_G(u_b)$, then we have $\deg_G(u_a) \leq n-2$. Considering the paths P_1–P_5, we have

$$\begin{aligned}
|S| &\geq (n-1) + (k-2) + (k-2)(n-2) + (n-1) + (n-2)(n-1) \\
&= (n-1)(\deg_G(u_i) + (n-2)) \\
&\geq (n-1)(\deg_G(u_i) + \deg_G(u_a)) \\
&\geq (n-1) \min_{xy \in E(G)} \{\deg_G(x) + \deg_G(y)\} \\
&> (n-1) \min_{xy \in E(G)} \{\deg_G(x) + \deg_G(y)\} - 2,
\end{aligned}$$

a contradiction. □

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