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PRICING DECISIONS IN DUAL-CHANNEL SUPPLY CHAIN WITH ONE MANUFACTURER AND MULTIPLE RETAILERS: A GAME-THEORETIC APPROACH

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Abstract. In this study, to price a product that can be simultaneously sold in the e-tail and retail channels, a dual-channel supply chain is considered containing one manufacturer and multiple retailers. In this setting, the game-theoretic approach is applied to obtain the equilibrium prices To our best knowledge, the game-theoretic frameworks proposed to price the products in the dual-channel supply chain have considered a single retailer in the retail channel, while multiple retailers can exist in the retail channel in practice. It is assumed that the manufacturer and retailers have the same decision powers. First, the Nash game model is established to set the prices in decentralized model A centralized model is presented to maximize the profit of the whole system. Then, a coordination mechanism based on the linear quantity discount schedule is applied to fully coordinate the supply chain. Finally, the Nash bargaining model is used to share the extra profit given by the whole system cooperation among the members

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1. Introduction

Nowadays, E-commerce has considerably changed buying and selling patterns. On one hand, customers prefer the Internet-based sales on account of convenient and secure buying through the Internet. On the other hand, to attract customers who cannot be reached through the traditional retail channel, the manufacturers and suppliers should redesign the traditional selling structures by establishing the Internet-based sales [15]. Regarding the statistical reports, about 42% of the top manufacturers like Dell, Nike, IBM, and Pioneer Electronics use the Internet-based services to sell their products [6]. This growth in the Internet-based sales encourages the manufacturers and suppliers to establish selling directly to customers.

Dual-channel supply chain is one in which a manufacturer or supplier sells his products to consumers simultaneously through the Internet directly (hereafter referred to as "e-tail channel") as well as through a retailer (hereafter referred to as "retail channel"). The e-tail channel attracts customers who prefer to buy the products after viewing them online from the manufacturer's or supplier's website whereas the retail channel captures customers who cannot access the Internet or who prefer to buy the products after viewing them in retail stores.

Keywords. Supply chain management, dual-channel, pricing, economic, game theory.

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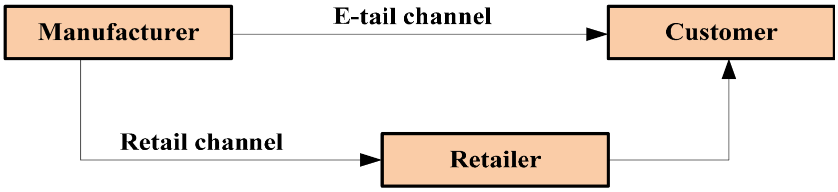


Figure 1. The general structure of the dual-channel supply chain.

The combination of these two distribution channels leads to the channel conflict in the dual-channel structures. Appearance of the competitive pricing policies between the e-tail and retail channels is the most important result of this conflict [7]. Figure 1 depicts the general structure of the dual-channel supply chain.

Game theory is a mathematical instrument for specification of decisions by the rational and intelligent players with conflicting interests [13, 14]. The need to increase our theoretical comprehension of the complicated interdependent relationships in supply chains has motivated a great deal of the study (*e.g.*, [2, 12, 19, 22–24, 45, 46]). There is a considerable body of research on the dualchannel supply chain. Below, we address several studies that have applied the game-theoretic approach in the dual-channel supply chain.

The price is the most important competitive factor for customers [42]. The change of the price directly affects the profit of the members and the whole system. Therefore, the managers' decision about what price to set is one of the most important decisions in the supply chains. Several researchers have studied the pricing policies in the dual-channel supply chain (*e.g.*, see: Yao and Liu [43]; Dai *et al.* [9]; Mukhopadhyay *et al.* [21]; Hua *et al.* [15]; Liu *et al.* [20]; Huang *et al.* [17]; Jafari *et al.* [25]; Huang *et al.* [16]; Cao *et al.* [3]; Chen *et al.* [6]; Jafari *et al.* [27]). Service level influences the purchase option of customers. Some studies have considered the pricing strategies as well as the various types of the services (*e.g.*, see: Dumrongsiri *et al.* [10]; Chen *et al.* [1]; Yan and Pei [44]; Dan *et al.* [8]). Furthermore, coordination (vertical integration) is one of the useful techniques to increase performance of the supply chain management [28]. To coordinate the members, different contracts have been developed in the dual-channel supply chain (*e.g.*, see: Jeuland and Shugan [29]; Cai *et al.* [5]; Yan [30])

In this study, we use the game-theoretic approach to price a single product in a dual-channel supply chain. The considered structure consists of one manufacturer and multiple retailers. To our best knowledge, the game-theoretic models proposed in the dual-channel supply chain have considered a single retailer in the retail channel (*e.g.*, see the researches addressed in the previous paragraph), while multiple retailers can exist in the retail channel in practice. It is assumed that the manufacturer and retailers have the same decision powers. The price competition is analyzed among the players under the different game models. A Nash game is established to set the prices in the decentralized model and the centralized model is presented to maximize the profit of the whole system. Then, the linear quantity discounts schedule is used as the channel coordination mechanism. Finally, the Nash [31] bargaining model is provided to lead to the win–win situation.

The remainder of the paper is organized as follows: the research problem is described in details in Section 2. The Nash and centralized models are developed in Sections 3 and 4, respectively. In Section 5, the linear quantity discounts schedule is presented to coordinate the supply chain. To share the extra profit of the whole system cooperation among the players, the Nash bargaining model is used in Section 6. In Section 7, an instance is presented to well illustrate the research problem and in Section 8, a sensitivity analysis is provided to investigate the effect of the changes of the parameters. Conclusions and directions for future researches are presented in Section 9. Finally, the proofs of all theorems and lemmas are provided in the appendices.

2. Modeling assumptions and notations

In this study, we consider a dual-channel supply chain consisting of one manufacturer and multiple retailers. The manufacturer produces a single product with the unit manufacturing cost C_m and sells it through the e-tail and retail channels, simultaneously. Some customers buy the products through the e-tail channel after viewing them online from the manufacturer's website, whereas others who do not access the Internet or who prefer to purchase the products after viewing them in retail stores buy the products from the retailers. Thus, the scales of the market for the manufacturer and retailers are specified based on the number of potential customers in the channels. Number of the retailers in the retail channel is denoted as K. It is assumed that the players have the same decision powers. Thus, they set the prices simultaneously. The manufacturer determines the e-tail price P_e in the e-tail channel to end customer as well as the wholesale price W in the retail channel to the retailers. Simultaneously, retailers 1, 2, ..., K respectively specify the retail prices P_1, $P_2 \dots P_K$ in the retail channel to end customers. The assumptions are as follows:

(1) The manufacturer has enough production capacity to meet the sum of the demands in the e-tail and retail channels. Moreover, the order quantity of each retailer in the retail channel is exactly equal to his demand (*e.g.*, Ferrer and Ketzenberg [32]; Savaskan *et al.* [35]; Savaskan and VanWassenhove [36]).
(2) The wholesale prices charged by the manufacturer to the retailers are equal (*e.g.*, Yang and Zhou [33]; Wang *et al.* [34]).
(3) The demands are a linear function of the e-tail and retail prices (*e.g.*, Pan *et al.* [39]; Edirisinghe *et al.* [11]).
(4) The self-price sensitivity of the demands is greater than the sum of the cross-price sensitivities β is the self-price sensitivity of the demands whereas θ denotes the cross-price sensitivity indicating the degree of substitutability among the members on the demands. β is the total number of customers who stop buying through a channel as the price in this channel rises by one unit. Some of them switch to K other channels, *i.e.*, $K\theta$ and others give the market up, *i.e.*, $\beta - K\theta$. Obviously, it can be assumed that $\beta > K\theta$. This is a common assumption in the literature (*e.g.*, Dan *et al.* [8]; Wei *et al.* [40]).
(5) The effects of the self-price and cross-prices are similar in all the demand functions. In other words, in each channel if the price rises by one unit, then the number of customers who stop buying through this channel and the number of customers who switch to each of other channels respectively would be β and θ (*e.g.*, Hua *et al.* [15]; Huang *et al.* [17]).
(6) To encourage the retailers to enter into the business, it is assumed that the manufacturer's unit profit margin is not more than the retailers in the retail channel (*e.g.*, Jorgensen and Zaccour [26]; Xie and Neyret [41]; Seyed Esfahani *et al.* [37]).
(7) If the profit the manufacturer or one of the retailers is equal to zero, then he as a rational entity/player will not do business with the others and hence the business/game is not constituted in this situation.

Regarding assumption (2.3), the demand functions D_e and D_i $(i = 1, 2, \dots K)$ respectively faced by the manufacturer in the e-tail channel and by the retailers in the retail channel are:

$$\begin{cases} D_e = a_e - \beta P_e + \theta \displaystyle\sum_{j=1}^{K} P_j & (2.1) \\ D_i = a_i - \beta P_i + \theta \left(P_e + \displaystyle\sum_{j=1 (j \neq i)}^{K} P_j \right) \qquad i = 1, 2, \dots, K & (2.2) \end{cases}$$

where a_e and a_i $(i = 1, 2, \dots K)$ respectively are the scales of the market for the manufacturer in the e-tail channel and for the retailer i in the retail channel. In relation (2.1), βP_e is the number of customers who quit buying through the e-tail channel when the price in this channel is equal to P_e, while $\theta \sum_{j=1}^{K} P_j$ is the number of customers who switch to the e-tail channel from the retail channel when the prices set by the retailers in the retail channel are equal to P_1, $P_2 \dots P_K$. The demands D_i $(i = 1, 2, \dots K)$ are similarly formulated in relation (2.2).

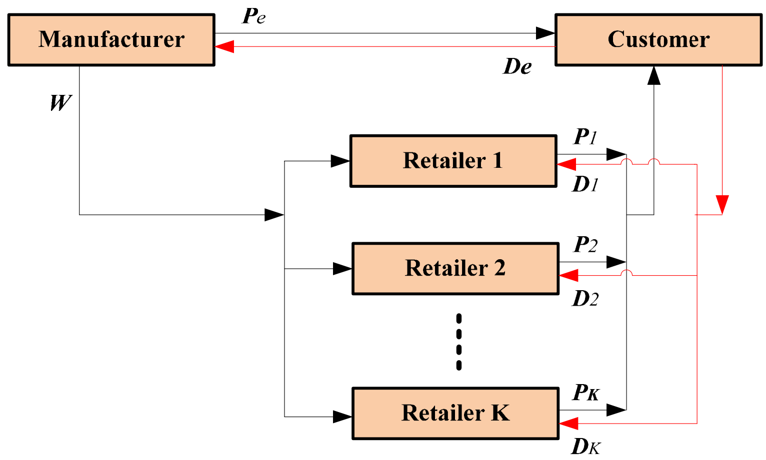


FIGURE 2. The structure of the dual-channel supply chain considered.

The profit functions π_m, π_i $(i = 1, 2, \dots K)$, and π_{SC} for the manufacturer, retailer i, and the whole system respectively are formulated as follows:

$$\begin{cases} \pi_m(W, P_e) = (P_e - C_m)\, D_e + (W - C_m) \sum_{i=1}^{K} D_i & (2.3) \\ \pi_i\,(P_i) = (P_i - W)\, D_i \qquad i = 1, 2, \dots, K & (2.4) \\ \pi_{SC}\,(P_e, P_1, P_2, \dots, P_K) = \pi_m + \sum_{i=1}^{K} \pi_i = (P_e - C_m)\, D_e + \sum_{i=1}^{K} (P_i - C_m)\, D_i & (2.5) \end{cases}$$

We consider the following constraints into the model to ensure that the given prices are reasonable:

$$W,\, P_e \geqslant C_m \tag{2.6}$$

$$W \leqslant P_i \quad (i = 1,\, 2, \dots K) \tag{2.7}$$

The structure of the dual-channel supply chain considered is depicted in Figure 2.

In what follows, we will discuss the equilibrium pricing strategies that the manufacturer and retailers should set in the e-tail and retail channels. The game-theoretic approach is developed to obtain the optimal prices under the decentralized and centralized models. Furthermore, Karush−Kuhn−Tucker (KKT) conditions [38] are applied to give the reasonable pricing policies that satisfy relations (2.6) and (2.7). Note that throughout the paper, symbols N, C, and D denote the Nash, centralized, and linear quantity discounts models, respectively.

3. Nash game model

In Section 2, it was assumed that the manufacturer and retailers have the same decision powers and set the prices independently and simultaneously. This situation is called a Nash game and the optimal pricing strategy obtained from this game is the Nash equilibrium. In the Nash game model, the manufacturer and retailers' decision problems are separately solved. The Nash game model is formulated as follows:

$$\begin{cases} \pi_m(W, P_e) = (P_e - C_m)\, D_e + (W - C_m) \sum_{i=1}^{K} D_i \quad \text{s.t.} \quad W, P_e \geqslant C_m \\ \pi_i\,(P_i) = (P_i - W)\, D_i \quad \text{s.t.} \quad P_i \geqslant W \qquad i = 1, 2, \dots, K \end{cases} \qquad \text{(Problem A)}$$

Proofs of the lemmas and theorems in this section appear in Appendix A (in electronic companion).

Lemma 3.1 (The manufacturer's profit function π_m is not jointly concave in W and P_e)**.** *But it is increasing in line with W.*

In the manufacturer's point of view, from Lemma 3.1 and relation $W \leqslant P_i$ $(i = 1, 2, \ldots K)$, the optimal value of W is $\min P_1, P_2 \ldots P_K$. However, if there is i in $1, 2, \ldots K$ subject to $W = P_i$, then $\pi_i = 0$ and therefore regarding Assumption (2.7), retailer i will not do business with the other players. We use a similar approach as applied by Jorgensen and Zaccour [26], Xie and Neyret [41], and Seyed Esfahani *et al.* [37] to tackle the research problem Regarding Assumption (2.6), the manufacturer's unit profit margin is not higher than the retailers in the retail channel, *i.e.*, $W - C_m \leqslant P_i - W$ $(i = 1, 2, \ldots K)$. Therefore, we have:

$$W \leqslant \frac{P_i + C_m}{2} \qquad i = 1, 2, \ldots K \tag{3.1}$$

The manufacturer's profit function is increasing in line with W As a result, the optimal value of W is equal to $(\min\{P_1, P_2, \ldots, P_K\} + C_m)/2$

Lemma 3.2. *The retailer i's profit function π_i is concave in $P_i (i = 1, 2, \ldots K)$.*

Theorem 3.3. *If the wholesale price W and the e-tail price P_e are determined by the manufacturer, then the optimal retail prices are:*

$$P_i(W, P_e) = \frac{(2\beta + \theta)(\beta W + \theta P_e) + [2\beta - (K-1)\theta]a_i + \theta \sum_{j=1}^{K} a_j}{(2\beta + \theta)[2\beta - (K-1)\theta]} \qquad i = 1, 2, \ldots K \tag{3.2}$$

Without the loss of generality, it is assumed that $a_r = \min a_1 a_2 \ldots a_K$. Thus, from assumption $\beta > K\theta$ we have:

$$[2\beta - (K-1)\theta]a_r \leqslant [2\beta - (K-1)\theta]a_i \qquad i = 1, 2, \ldots K \tag{3.3}$$

Clearly, one can derive that $P_r(W, P_e) = \min\{P_1(W, P_e), P_2(W, P_e), \ldots, P_K(W, P_e)\}$ and thus the optimal value of W is equal to $(P_r + C_m)/2$.

π_m^N and π_i^N $(i = 1, 2, \ldots K)$ are respectively calculated substituting $W = (P_r + C_m)/2$ into π_m and π_i $(i = 1, 2, \ldots K)$. Moreover, the players' decision problems are equivalent to:

$$\begin{cases} \pi_m^N(P_e) = (P_e - C_m) D_e + \left(\dfrac{P_r - C_m}{2}\right) \displaystyle\sum_{i=1}^{K} D_i \qquad \text{s.t.} \quad P_e \geqslant C_m & (3.4) \\ \pi_r^N(P_r) = \left(\dfrac{P_r - C_m}{2}\right) D_r \quad \text{s.t.} \quad P_r \geqslant C_m & (3.5) \\ \pi_i^N(P_i) = \left(\dfrac{2P_i - P_r - C_m}{2}\right) D_i \quad \text{s.t.} \quad P_i \geqslant \dfrac{(P_r + C_m)}{2} i = 1, 2, \ldots, K (i \neq r) & (3.6) \end{cases}$$

Lemma 3.4. *The new profit functions π_m^N and ($\pi_i^N (i = 1, 2, \ldots K)$ are concave in P_e and P_i, respectively*

Theorem 3.5. *Assuming $a_r = min a_1 a_2 \ldots a_K$ the optimal prices made by the manufacturer and retailers in the Nash game model can be given as follows*:

$$\begin{cases} W^N = \dfrac{E_1 C_m + \theta a_e + \theta \sum\limits_{j=1}^{K} a_j + (2\beta - K\theta) a_r}{E_2} \\ P_e^N = \dfrac{(2\beta+\theta) E_3 C_m + (2\beta+\theta) E_4 a_e + \theta E_5 \sum\limits_{j=1}^{K} a_j + K\theta E_6 a_r}{(2\beta+\theta) E_2} \\ P_r^N = \dfrac{E_7 C_m + 2\theta a_e + 2\theta \sum\limits_{j=1}^{K} a_j + 2(2\beta - K\theta) a_r}{E_2} \\ P_i^N = \dfrac{(2\beta+\theta)\left[E_7 C_m + 2\theta a_e + 2\theta \sum\limits_{j=1}^{K} a_j\right] + (2\beta^2 + K\theta^2) a_r + E_2 a_i}{(2\beta+\theta) E_2} & i = 1, 2, \ldots, K (i \neq r) \end{cases} \tag{3.7}$$

$$\begin{cases} W^N = \dfrac{E_8 C_m + \theta \sum\limits_{j=1}^{K} a_j + [2\beta - (K-1)\theta] a_r}{E_9} \\ P_e^N = C_m \\ P_r^N = \dfrac{E_{10} C_m + 2\theta \sum\limits_{j=1}^{K} a_j + 2[2\beta - (K-1)\theta] a_r}{E_9} \\ P_i^N = \dfrac{(\beta+2\theta)(2\beta+\theta)[(3\beta+\theta) C_m + a_r] + \theta(5\beta+2\theta) \sum\limits_{j=1}^{K} a_j + E_9 a_i}{(2\beta+\theta) E_9} & i = 1, 2, \ldots, K (i \neq r) \end{cases} \tag{3.8}$$

- *If $C_m \leqslant \min\{A_e, A_i \, (i = 1, 2, \ldots, K)$, then*:
  - *If $C_m \leqslant \min\{A'_i \, (i = 1, 2, \ldots, K)$ and $C_m > A'_e$, then*:
  - *Otherwise, the business/game is not constituted or there is no equilibrium solution to Problem A*

where, sets $\{A_e, A'_e, A_i, A'_i \, (i = 1, 2, \ldots, K)\}$ and $\{E_1, E_2, \ldots, E_{10}\}$ appear in Appendix A

4. Centralized model

In this section, the relationship among the manufacturer and retailers is modeled as a cooperative game in which all the members agree to cooperate and maximize the profit of the whole system. The centralized model is as follows:

$$\begin{cases} \pi_{SC}(P_e, P_1, P_2, \ldots, P_K) = \pi_m + \sum\limits_{i=1}^{K} \pi_i = (P_e - C_m) D_e + \sum\limits_{i=1}^{K} (P_i - C_m) D_i \\ \quad \text{s.t.} \quad P_e, P_i \geqslant C_m \qquad\qquad i = 1, 2, \ldots, K \end{cases} \tag{Problem B}$$

Proofs of the lemmas and theorems in this section are presented in Appendix B (in electronic companion).

Lemma 4.1. *The profit function of the whole system π_{SC} is jointly concave in $P_e P_1 P_2 \ldots P_K$*

Theorem 4.2. *The optimal prices in the centralized model are given as follows*:

$$\begin{cases} P_e^C = \dfrac{(\beta+\theta)(\beta-K\theta)C_m + [\beta-(K-1)\theta]a_e + \theta\sum\limits_{j=1}^{K} a_j}{2(\beta+\theta)(\beta-K\theta)} & \\ P_i^C = \dfrac{(\beta+\theta)(\beta-K\theta)C_m + \theta a_e + \theta\sum\limits_{j=1}^{K} a_j + (\beta-K\theta)a_i}{2(\beta+\theta)(\beta-K\theta)} & i=1,2,\ldots,K \end{cases} \quad (4.1)$$

- *If* $C_m \leqslant \min\{B_e,\, B_i (i=1,2,\ldots K)\}$:
- *If* $C_m \leqslant \min\{B_i^{'} (i=1,2,\ldots K)\}$ *and* $C_m > B_e$:

$$\begin{cases} P_e^C = C_m & \\ P_i^C = \dfrac{(\beta+\theta)[\beta-(K-2)\theta]C_m + \theta\sum\limits_{j=1}^{K} a_j + [\beta-(K-1)\theta]a_i}{2(\beta+\theta)[\beta-(K-1)\theta]} & i=1,2,\ldots,K \end{cases} \quad (4.2)$$

- *Otherwise, the business/game is not constituted or there exists no equilibrium solution to Problem B.*

where, set ?B_e, B_i, $B'_i (i=1,2,\ldots K)\}$ *is shown in Appendix B*

5. Linear quantity discounts schedule

In this section, a linear quantity discounts schedule introduced by Ingene and Parry [18] is applied to coordinate the considered supply chain. The quantity discounts schedule is permitted under the Federal Robinson-Patman Act that does not discriminate among the retailers [4]. In fact, to discriminate among the same retailers by offering them different the wholesale prices is unlawful Thus, it is assumed that the discount slope of per-unit wholesale pricing schedule is the same for all the retailers.

Under the linear quantity discounts schedule, the wholesale prices charged by the manufacturer to the retailers depend on their order quantities, *i.e.*, their demands in the retail channel. If the order quantity of retailer i ($i = 1,2,\ldots K$) is equal to D_i, then he is charged by the manufacturer with $(W-\varphi D_i)D_i$, where W and φ respectively are the maximum variable wholesale price and the discount slope of per-unit wholesale pricing schedule. In other words, if the order quantity of retailer $i$is D_i, then the discount quantity of per-unit wholesale pricing charged by the manufacturer to retailer i is equal to φD_i. The linear quantity discounts model is formulated as follows:

$$\begin{cases} \pi_m^D(W,P_e,\varphi) = (P_e - C_m)D_e + \sum\limits_{i=1}^{K}(W-\varphi D_i - C_m)D_i & \\ \text{s.t.} \quad P_e \geqslant C_m, W-\varphi D_i \geqslant C_m, \quad \text{and} \quad \varphi \geqslant 0 & i=1,2,\ldots,K \\ \pi_i^D(P_i) = (P_i - W + \varphi D_i)D_i & \\ \text{s.t.} \quad P_i \geqslant W - \varphi D_i & i=1,2,\ldots,K \end{cases} \quad \text{(Problem C)}$$

Based on the vertical integration mechanism proposed by Ingene and Parry [18], as the prices in the linear quantity discounts model are equal to the centralized model the supply chain is fully coordinated under the linear quantity discounts schedule. Thus, the considered supply chain is fully coordinated when $P_e^D = P_e^C$and $P_i^D = P_i^C (i=1,2,\ldots K)$ where symbols C and D respectively denote the centralized and linear quantity discounts models. Proofs of theorems in this section appear in Appendix C (in electronic companion).

Theorem 5.1. *By setting the wholesale price W, e-tail price P_e, and discount slope φ by the manufacturer and assuming $\varphi\beta < 1\pi_i^D (i = 1, 2 \dots K)$ is concave with respect to P_i. Furthermore, the optimal retail prices are:*

$$P_i^D (W, P_e, \varphi) = \frac{F_1 W + F_2 P_e + \theta(1 - 2\varphi\beta)^2 \sum\limits_{j=1}^{K} a_j + F_3 a_i}{F_4} \qquad i = 1, 2, \dots K. \tag{5.1}$$

where, set $\{F_1, F_2, F_3, F_4\}$ is defined in Appendix C.

By solving equations $P_e^D = P_e^C$ and $P_i^D (W, P_e, \varphi) = P_i^C (i = 1, 2, \dots K)$ simultaneously, the following theorem can be given:

Theorem 5.2. *The dual-channel supply chain considered is fully coordinated by a linear quantity discounts schedule as follows*:

$$\begin{cases} W^D = \dfrac{[2\beta - (K-1)\theta]\,(\beta - K\theta)\,C_m + \theta\left(a_e + \sum\limits_{j=1}^{K} a_j\right)}{2\,(\beta+\theta)\,(\beta - K\theta)} \\[2ex] P_e^D = \dfrac{(\beta+\theta)\,(\beta - K\theta)\,C_m + [\beta - (K-1)\,\theta]a_e + \theta \sum\limits_{j=1}^{K} a_j}{2\,(\beta+\theta)\,(\beta - K\theta)} \\[2ex] P_i^D = \dfrac{(\beta+\theta)\,(\beta - K\theta)\,C_m + \theta a_e + \theta \sum\limits_{j=1}^{K} a_j + (\beta - K\theta)a_i}{2\,(\beta+\theta)\,(\beta - K\theta)} \qquad i = 1, 2, \dots, K \\[2ex] \varphi^D = \dfrac{\theta}{2\beta\,(\beta+\theta)} \end{cases} \tag{5.2}$$

$$\begin{cases} W^D = \dfrac{2\left[\beta^2 - (K-1)\beta\theta + \theta^2\right] C_m + \theta \sum\limits_{j=1}^{K} a_j}{2\,(\beta+\theta)\,[\beta - (K-1)\,\theta]} \\[2ex] P_e^D = C_m \\[2ex] P_i^D = \dfrac{(\beta+\theta)\,[\beta - (K-2)\,\theta]C_m + \theta \sum\limits_{j=1}^{K} a_j + [\beta - (K-1)\,\theta]a_i}{2\,(\beta+\theta)\,[\beta - (K-1)\,\theta]} \qquad i = 1, 2, \dots, K \\[2ex] \varphi^D = \dfrac{\theta}{2\beta\,(\beta+\theta)} \end{cases} \tag{5.3}$$

- *If $C_m \leqslant \min\{B_e,\, B_i,\, H_i,\, G_i\,(i = 1, 2, \dots K\,)\}$:*
  - *If $C_m \leqslant \min\{\,B'_i,\, H'_i,\, G_i\,(i = 1, 2, \dots K\,)\}$ and $C_m > B_e$:*
  - *Otherwise, the business/game is not constituted or there is no equilibrium solution to Problem C*

where, sets $\{B_e,\, B_i, B'_i\,(i = 1, 2, \dots, K)\}$ and $\{H_i,\, H'_i, G'_i\,(i = 1, 2, \dots, K)\}$ respectively are included in Appendices B and C

6. Bargaining model

In Section 4, the optimal prices that maximize the profit of the whole system are obtained under the centralized model. Due to the maximization of the profit of the whole system in this model, the profit of the whole system in the centralized model is higher than in the Nash game model, *i.e.*, $\pi_{SC}^{C} \geqslant \pi_{SC}^{N} = \pi_m^N + \sum\limits_{i=1}^{K} \pi_i^N$, where symbols C and N denote the centralized and Nash game models, respectively Moreover, since the wholesale price W is neglected in the centralized model, the profits of the members cannot be specified. Now, the Nash [31] bargaining model is applied to obtain the following values in the centralized model based on the bargaining powers of the members:

- The share of increased profit obtained from the whole system cooperation for the members.
- The profit of the members.
- The wholesale price.

Assume that the manufacturer and retailer i ($i = 1, 2, \ldots K$) respectively receive the shares $\Delta\pi_m$ and $\Delta\pi_i$ of the extra profit given by the whole system cooperation. In fact, $\Delta\pi_m$ and $\Delta\pi_i (i = 1, 2, \ldots K\,)$ are the increased profit of the members in the centralized model compared to the Nash model in which they set the decisions independently. Thus, we have:

$$\Delta \begin{cases} \pi_m = \pi_m^C - \pi_m^N & (6.1) \\ \Delta\pi_i = \pi_i^C - \pi_i^N \qquad i = 1, 2, \ldots, K & (6.2) \\ \Delta\pi_{SC} = \pi_{SC}^C - \pi_{SC}^N = \pi_{SC}^C - \left(\pi_m^N + \sum\limits_{i=1}^{K} \pi_i^N\right) & (6.3) \end{cases}$$

Based on the Nash [31] bargaining model, the bargaining profit scheme is shown as follows:

$$\begin{cases} \max\ Z = (\Delta\pi_m)^{\gamma_m} \prod\limits_{i=1}^{K} (\Delta\pi_i)^{\gamma_i} \\ \text{s.t.} \quad \Delta\pi_m + \sum\limits_{i=1}^{K} \Delta\pi_i = \Delta\pi_{SC} \end{cases} \qquad \text{(Problem D)}$$

where, the positive parameters γ_m and $\gamma_i (i = 1, 2, \ldots K\,)$ respectively are the bargaining powers of the manufacturer and retailer i Proofs of the theorems appearing in this section are included in Appendix D (in electronic companion).

Theorem 6.1. *The extra profits of the members obtained from the whole system cooperation in the centralized model are:*

$$\Delta\pi_m \begin{cases} {}^{B} = \dfrac{\gamma_m}{\gamma_m + \sum\limits_{i=1}^{K} \gamma_i} \Delta\pi_{SC} & (6.4) \\ \Delta{\pi_i}^B = \dfrac{\gamma_i}{\gamma_m + \sum\limits_{i=1}^{K} \gamma_i} \Delta\pi_{SC} \qquad i = 1, 2, \ldots K & (6.5) \end{cases}$$

As a result, based on the Nash bargaining model, the profit of the members in the centralized model is not less than in the Nash game model, *i.e.*, $\pi_m^C \geqslant \pi_m^N$ and $\pi_i^C \geqslant \pi_i^N (i = 1, 2, \ldots K)$ Furthermore, from relations (6.1), (6.2), (6.4), and (6.5), the profit of the members in the centralized model can be determined, easily.

TABLE 1. The value of the parameters in the illustrative instance.

Parameters	N	C_m	β	θ	a_e	a_1	a_2	a_3	a_4	a_5
Values	5	80	1.8	0.3	300	150	130	170	200	120

Insight 1. *Regarding relations* (6.4) *and* (6.5)*, we obviously derive that*:

- *When all the members have the same bargaining powers, they will share the extra profit, equally.*
- *The more the bargaining power of a member the more the share of the increased profit for his/her.*
- *When the bargaining power of a member is significantly higher than the others, he receives approximately the total value of the extra profit, and vice versa.*

Now, the Nash bargaining model is used to set wholesale price W in the centralized model.

Theorem 6.2. *The wholesale price W^B in the centralized model is*:

$$W^B = \frac{1}{\sum_{i=1}^{K} D_i^C}\left[\frac{\gamma_m}{\gamma_m + \sum_{i=1}^{K}\gamma_i}\Delta\pi_{SC} + \pi_m^N - (P_e^C - C_m)D_e^C\right] + C_m \tag{6.6}$$

where, D_e^C and $D_i^C (i = 1, 2, \ldots K)$ respectively are the optimal values of the e-tail and retail demands in the centralized model.

7. Illustrative instance

In this section, an instance is presented to well illustrate the characteristics of the studied models. Consider a manufacturer who produces one product that can be sold both through the Internet in the e-tail channel and through the retailers in the retail channel. The manufacturer delivers the products to end customers in the retail channel through five retailers. Hence, a dual-channel structure is considered consisting of one manufacturer and five retailers. The value of the parameters has been shown in Table 1 and the results given by the different models have been summarized in Table 2 In the linear quantity discounts model, regarding Theorem 5.2, the optimal value of the discount slope φ is equal to 0.04

In the Nash (centralized) model, the players determine the prices independently (jointly) to maximize their own profits (the profit of the whole system). Thus, due to the competition among the players in the Nash game, the prices specified in this game are less than in the centralized game. Customers are assumed to be price sensitive and therefore higher prices lead to lower demands. Consequently, the demands in the Nash game are more than in the centralized game. Regarding the applied vertical integration mechanism, the e-tail and retail prices, demands, and profits of the whole system are exactly equal in both the centralized and linear quantity discounts models. Obviously, the competition among the players in the Nash game leads to lower profits for them than in the linear quantity discounts schedule in which they made the decisions jointly. Thus, the profits of the players and the whole system in the linear quantity discounts schedule are higher than in the Nash game model

Now, to share the increased profit obtained from the cooperation in the centralized model among the members, the Nash bargaining model presented in Section 6 is used. The increased profit is shared among the members under the different bargaining powers using relations (6.4) and (6.5). Moreover, the wholesale price in the centralized model is given using relation (6.6). The results are provided in Table 3. Obviously, more bargaining powers lead to higher shares of the extra profit for the members. Furthermore, the more the manufacturer's bargaining power, the more the given wholesale price.

TABLE 2. The results of the investigated models for the illustrative instance.

models	Prices						
	W	P_e	P_1	P_2	P_3	P_4	P_5
Nash	130.73	223.71	189.15	184.02	194.28	201.97	181.46
Centralized	–	366.19	330.48	325.71	335.24	342.38	323.33
Coordination	300.48	366.19	330.48	325.71	335.24	342.38	323.33
	Demands						
	D_e	D_1	D_2	D_3	D_4	D_5	
Nash	183	105	96	114	128	91	
Centralized	138	63	53	73	88	48	
Coordination	138	63	53	73	88	48	
	Profits						
	π_m	π_1	π_2	π_3	π_4	π_5	π_{SC}
Nash	53382.09	6143.73	5112.50	7269.64	9136.01	4632.39	85676.36
Centralized	–	–	–	–	–	–	12 169 889
Coordination	110271.05	2047.50	1448.66	2748.95	3994.50	1188.23	12 169 889

TABLE 3. The results of the Nash bargaining model for the illustrated instance.

Bargaining powers						Extra profit	Shares of the extra profit						Wholesale price
γ_m	γ_1	γ_2	γ_3	γ_4	γ_5	$\Delta\pi_{SC}$	$\Delta\pi_m{}^B$	$\Delta\pi_1{}^B$	$\Delta\pi_2{}^B$	$\Delta\pi_3{}^B$	$\Delta\pi_4{}^B$	$\Delta\pi_5^B$	W^B
0.5	0.1	0.1	0.1	0.1	0.1	36022.5	18011.3	3602.3	3602.3	3602.3	3602.3	3602.3	178.2
0.4	0.2	0.1	0.1	0.1	0.1	36022.5	14409.0	7204.5	3602.3	3602.3	3602.3	3602.3	167.1
0.3	0.2	0.1	0.2	0.1	0.1	36022.5	10806.8	7204.5	3602.3	7204.5	3602.3	3602.3	156.0
0.2	0.2	0.2	0.2	0.1	0.1	36022.5	7204.5	7204.5	7204.5	7204.5	3602.3	3602.3	144.9
0.1	0.3	0.2	0.2	0.1	0.1	36022.5	3602.3	10806.8	7204.5	7204.5	3602.3	3602.3	133.8

8. SENSITIVITY ANALYSIS

In this section, the effects of the changes of the self-price and cross-price sensitivities of the demands and the number of the retailers in the retail channel (*i.e.*, β, θ, and K) are investigated on the optimal strategies obtained from the models.

Theorem 8.1. *In the linear quantity discounts model, the more the self-price/cross-price sensitivity of the demands leads to the less/more the discount slope of per-unit wholesale pricing schedule while the number of the retailers in the retail channel has no effect on it.*

Proof. From Theorem 5.1, the discount slope of per-unit wholesale pricing schedule, *i.e.*, φ is equal to $\theta/\left[2\beta\left(\beta+\theta\right)\right]$. We have: $\partial\varphi/\partial\beta = -(2\beta+\theta)/\left[2\beta^2\left(\beta+\theta\right)^2\right] < 0$, $\partial\varphi/\partial\theta = 1/\left[2\left(\beta+\theta\right)^2\right] > 0$, and $\partial\varphi/\partial K = 0$. This completes the proof of Theorem 8.

Due to complexity of the given relations in general case, one cannot derive such explicit results for other decisions. Thus, the numerical sensitivity analysis is implemented to investigate the effects of the changes of β, θ, and K on the optimal decisions. Consider a case in which the total number of potential customers for the product is $a = 500$. Assume that the number of the retailers in the retail channel is $K = 3$ and the default values of the parameters are: $C_m = 10$, $\beta = 1.7$, $\theta = 0.2$, $a_e = 0.4a = 200$, and $a_1 = a_2 = a_3 = 0.6a/K = 100$. In fact, the scales of the market for the retailers are assumed to be similar. Consequently, under all the studied models, the prices, demands, and profits in the retail channel are equal, *i.e.*, $P_1 = P_2 = P_3 = P$, $D_1 = D_2 = D_3 = D$,

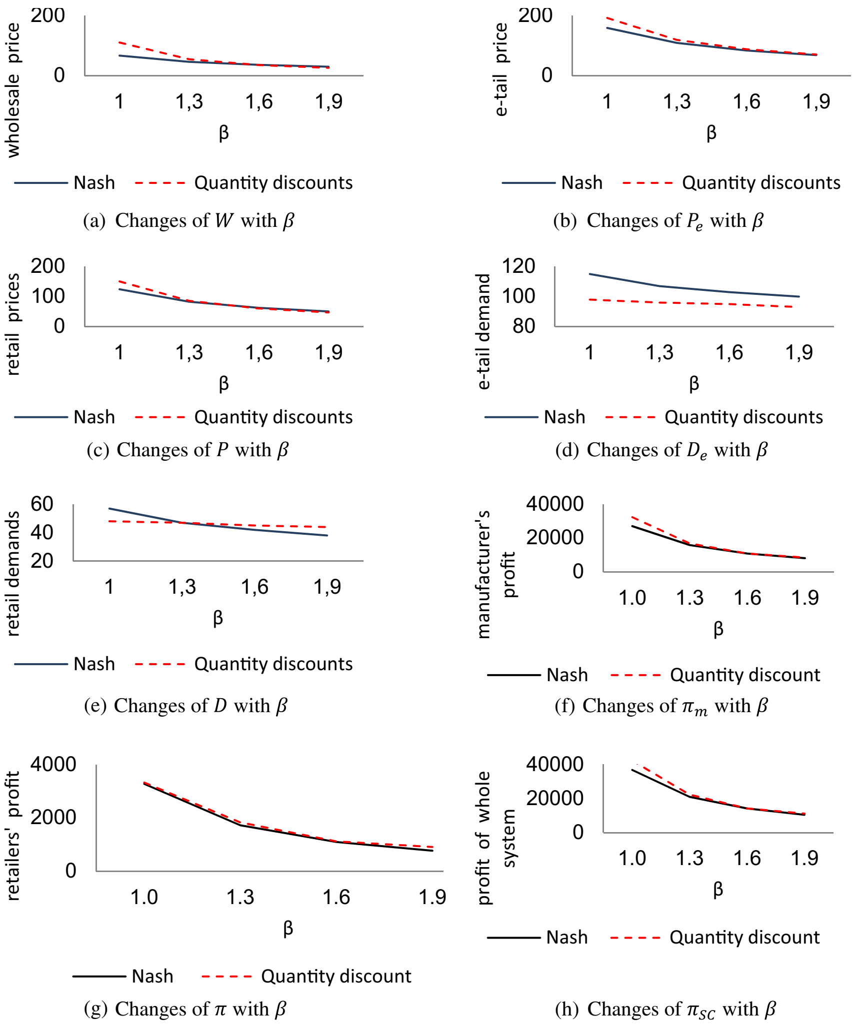


FIGURE 3. Changes of the optimal strategies with β.

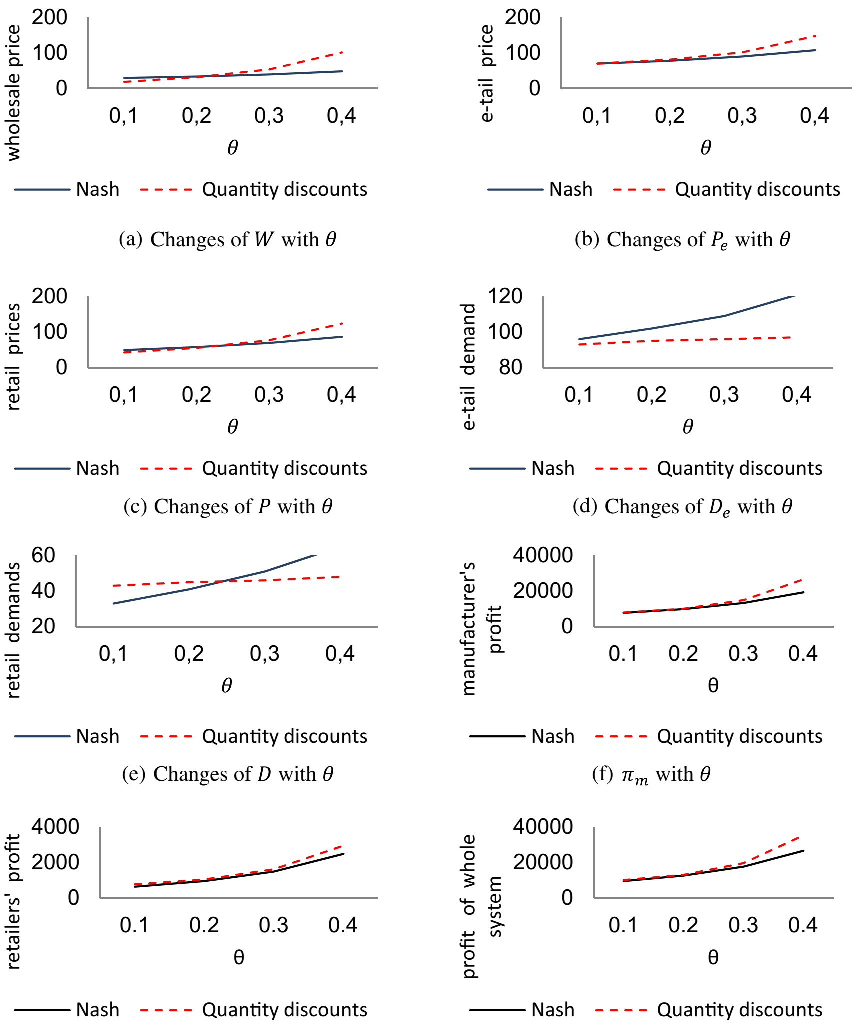
and $\pi_1 = \pi_2 = \pi_3 = \pi$. Therefore, in the retail channel, the effects of the changes of β, θ, and K are only investigated on P, D, and π Moreover, regarding the developed coordination mechanism, the e-tail and retail prices, demands, and profit of the whole system are equal in both the centralized and linear quantity discounts models. Thus, the effects of the changes of the parameters are only investigated on the decisions obtained from the Nash and linear quantity discount models.

8.1. Investigating the effects of the self-price sensitivity of the demands, *i.e.*, β

We change parameter β from 1. to 1.9 in step sizes of 0.3. The changes of the optimal strategies with respect to β are shown in Figure 3

Insight 2. *The higher the self-price sensitivity of the demands, i.e., β, the lower the values of all the optimal strategies obtained from all the models, i.e., if β increases, then the optimal values of the prices, demands, and profits decrease under all the models.*

Interpretation: β is the number of customers who quit buying through a channel when the price in this channel increases by one unit. A portion of these customers switch to the other channels and others give buying



(a) Changes of W with θ (b) Changes of P_e with θ

(c) Changes of P with θ (d) Changes of D_e with θ

(e) Changes of D with θ (f) π_m with θ

(g) Changes of π with θ (h) Changes of π_{SC} with θ

FIGURE 4. Changes of the optimal strategies with θ.

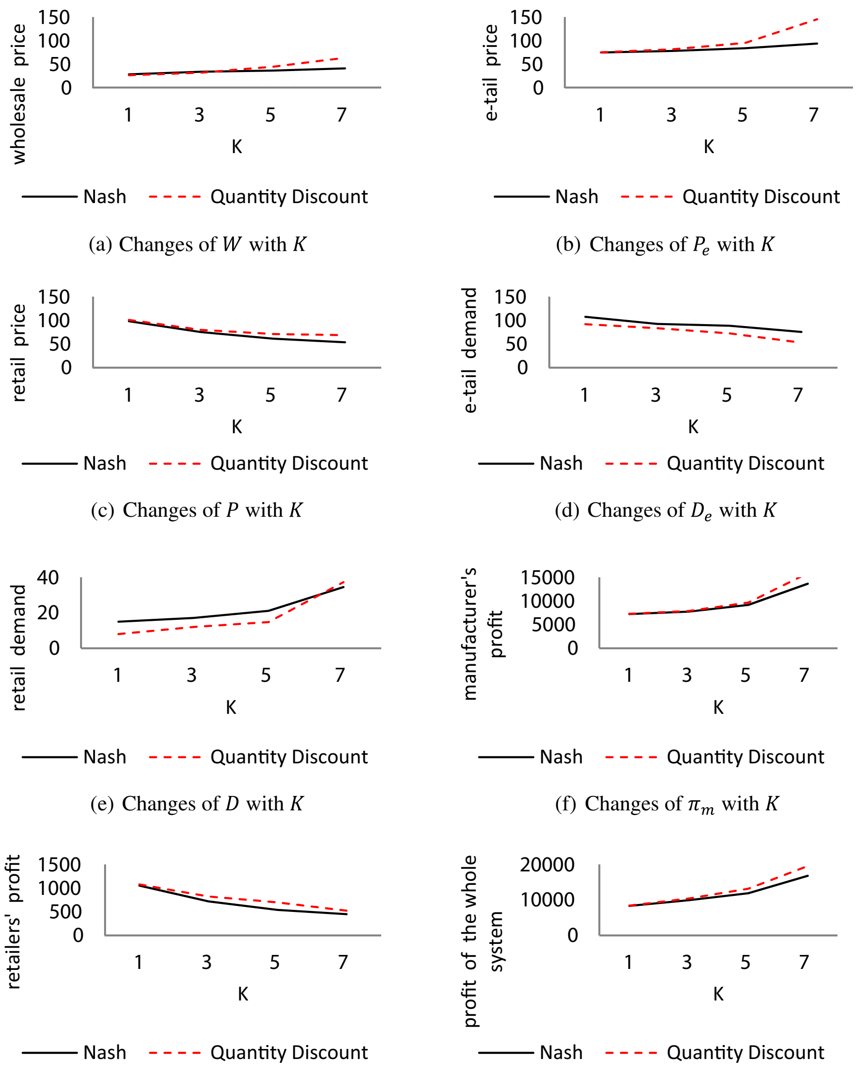
up. Consequently, as β increases, each player decreases his price to reduce the number of customers who quit buying through his channel. On the other hand, the number of customers who give up buying increases by increasing β. Hence, the demands in the channels are reduced as β increases. Clearly, the profits of the players and the whole system decrease by reducing all the prices and demands.

8.2. Investigating the effects of the cross-price sensitivity of the demands, *i.e.*, θ

Parameter θ is changed from 0.1 to 0.4 in step sizes of 0.1. The changes of the optimal decisions with respect to θ are shown in Figure 4.

Insight 3. *The higher the cross-price sensitivity of the demands, i.e., θ, the higher the values of all the optimal strategies given by all the models, i.e., if θ increases, then the optimal values of the prices, demands, and profits increase under all the models.*

Interpretation: θ is the number of customers who switch from a channel to another channel when the price in the first channel increases by one unit. When θ increases, the number of customers who switch to a channel increases while the number of customers who give up buying is reduced and hence all the demands increase.



(a) Changes of W with K (b) Changes of P_e with K

(c) Changes of P with K (d) Changes of D_e with K

(e) Changes of D with K (f) Changes of π_m with K

(g) Changes of π with K (h) Changes of π_{SC} with K

FIGURE 5. Changes of the optimal decisions with K.

By increasing θ, the players set the prices with a lower risk and consequently the prices increase. Obviously, the profits of the players and the whole system increase by increasing all the prices and demands

In this study, the dual-channel structure has been investigated with multiple retailers in the retail channel in comparison with the previous studies that have considered only one retailer in the retail channel. In what follows, the effects of considering more than one retailer in the retail channel are discussed.

8.3. Investigating the number of the retailers in the retail channel, *i.e.*, K

To discuss the effects of the number of the retailers in the retail channel on the optimal decisions, K is changed from 1 to 7 in step sizes of 2. The changes are shown in Figure 5.

Insight 4. *Larger the number of the retailers in the retail channel, i.e., K, leads to lower the optimal values of the retail prices, retail demands, and retailers' profits, while the optimal values of the other strategies increase*

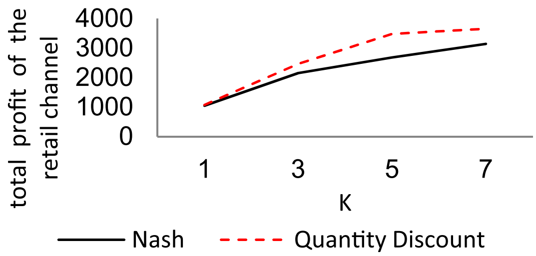


FIGURE 6. Changes of the optimal decisions with K.

Interpretation: As K increases, the scales of the market for the retailers and consequently for the retail demands decrease. Therefore, the retailers reduce the retail prices to increase their demands. As a result, the retailers gain less profit by decreasing the retail prices and the retail demands. By increasing K, the manufacturer sets the wholesale and e-tail prices with a lower risk and consequently the optimal values of these prices increase. Obviously, the e-tail demand can decrease by increasing the e-tail price and decreasing the retail prices. When the number of the retailers increases, the competition among them increases and as the result the manufacturer gains more profit as K becomes larger. Regarding Figure 6, the total profit of the retail channel (*i.e.*, sum of the retailers' profits) can increase as K increases Clearly, the profit of the whole system increases by increasing the profits in the e-tail and retail channels

Insight 5. *When there are more than one retailer in the retail channel (i.e., $K > 1$), one can observe that the total profit of the retail channel, manufacturer's profit, and profit of the whole system become higher than the situation in which there is a single retailer in the retail channel.*

Interpretation: Although, as $K > 1$, the retailers gain less profits but the total profit of the retail channel in this case is higher than in case $K = 1$. Moreover, the manufacturer can benefit by increasing the competition among the retailers in the retail channel. As a result, the total profit of the whole system increases as $K > 1$.

Insight 6. *From coordination mechanism investigated in Section* 5 *and regarding Figure* 3–5*, the profits of all the players and the whole system given by the centralized model and the linear quantity discounts schedule are higher than by the Nash game.*

Interpretation: In the centralized and linear quantity discounts models, the players cooperate to maximize the total profit of the whole system while in the Nash game model the players set the prices independently to maximize their own profits. Thus, the profits of the players and the whole system obtained from the centralized model and linear quantity discounts schedule cannot be lower than from the Nash game.

9. CONCLUSIONS

In this study, to price a single product that can be sold simultaneously in the e-tail and retail channels, a dual-channel structure was considered consisting of one manufacturer and multiple retailers. The manufacturer produces the product and sells it through the e-tail and retail channels, simultaneously. A portion of customers purchase the products through the e-tail channel after viewing them online from the manufacturer's website, while others who do not access the Internet or who prefer to buy the products after viewing them in retail stores purchase the products from the retailers. It was assumed that the manufacturer and retailers have the same decision powers

The game-theoretic approach was used to analyze the equilibrium pricing strategies that the manufacturer and retailers should set in the e-tail and retail channels. First, the Nash game model was established to set the prices in the decentralized model. The centralized model was developed to maximize the profit of the whole system. Then, a linear quantity discounts model was presented to coordinate the supply chain. Ultimately, the Nash bargaining model was applied to share the extra profit obtained from the centralized model among the players. The results indicated that more bargaining powers lead to higher shares of the extra profit for the players.

An instance was presented to well-illustrate the research problem. Then, the effects of the changes of the self-price and cross-price sensitivities of the demands and the number of the retailers in the retail channel were investigated on the optimal strategies. The following managerial insights were derived:

In the Nash (centralized) model, the players set the prices independently (jointly) to maximize their profits (the profit of the whole system). Hence, due to the competition among them in the Nash game, the prices given by this game are less than by the centralized game. It was assumed that customers are price sensitive and as a result more prices lead to lower demands. Consequently, the demands in the Nash game are more than in the centralized game. From the vertical integration mechanism, the final prices, demands, and total profits of the whole system are exactly equal in the centralized and linear quantity discounts models. Moreover, the competition among the players in the Nash game leads to lower profits for the players than in the centralized model and linear quantity discounts schedule in which they made the decisions jointly. Thus, profits of the players and whole system given by the centralized model and linear quantity discounts schedule are higher than by the Nash game.

The more the self-price/cross-price sensitivity of the demands leads to the less/more the values of all the optimal policies given by all the studied models. Moreover, in the linear quantity discounts schedule, the more the self-price/cross-price sensitivity of the demands, the less/more the discount slope of per-unit wholesale pricing schedule whereas the number of the retailers has no effect on it.

In this study, a multi-retailer dual-channel supply chain was considered in comparison with the previous studies that have investigated one retailer in the retail channel. In this setting, the effects of considering more than one retailer in the retail channel were investigated on the optimal decisions. One can derive that the larger the number of the retailers in the retail channel, the lower the optimal values of the retail prices, retail demands, and retailers' profits, whereas the optimal values of the other decisions increase. Moreover, when there is more than one retailer in the retail channel, one can observe that the total profit of the retail channel, manufacturer's profit, and profit of the whole system become higher than the situation in which there is a single retailer in the retail channel.

There are several directions for future studies: In this study it was assumed that the demands are the linear function of the e-tail and retail prices. A different form of the demand function could be adopted. Moreover, all the models presented in this study were established under a deterministic environment. One can investigate the effects of the demand disruption in the stochastic environments on the optimal policies

Appendix A. Notations and proofs in the Nash game model

$$E_1 = 4\beta^2 - (2K-3)\beta\theta - 2K\theta^2$$
$$E_2 = 6\beta^2 - 4(K-1)\beta\theta - 3K\theta^2$$
$$E_3 = 3\beta^2 - 2(K-1)\beta\theta + K(K-1)\theta^2$$
$$E_4 = 3\beta - 2(K-1)\theta$$
$$E_5 = 3\beta + 2(K+2)\theta$$
$$E_6 = 3\beta - (K-1)\theta$$
$$E_7 = 2\beta^2 + 2\beta\theta - K\theta^2$$
$$E_8 = 6\beta^2 - (3K-8)\,\beta\theta + (K-2)\theta^2$$
$$E_9 = 8\beta^2 - (5K-9)\beta\theta - 2(K-1)\theta^2$$
$$E_{10} = 4\beta^2 - (K-7)\,\beta\theta + 2\theta^2$$

$$E_{11} = 2\beta^2 - 2(K+1)\beta\theta - (K+1)\theta^2$$

$$E_{12} = 8\beta^2 - (5K-9)\,\beta\theta - 2(K-1)\theta^2$$

$$E_{13} = (2\beta+\theta)(\beta-K\theta)\,[4\beta+(K+2)\theta]$$

$$A_e = \frac{K\theta E_6 a_r + \theta\,[3\beta+(K+2)\,\theta]\sum_{j=1}^{K} a_j + (2\beta+\theta)E_4 a_e}{(2\beta+\theta)(\beta-K\theta)[3\beta+(K+2)\theta]}$$

$$A'_e = \frac{\theta\,[(3K-1)\beta - K(K-1)\,\theta]\,a_r + \theta\,[4\beta+(K+2)\theta]\sum_{j=1}^{K} a_j + E_{12}a_e}{E_{13}}$$

$$A_r = \frac{(2\beta-K\theta)a_r + \theta\sum_{j=1}^{K} a_j + \theta a_e}{(\beta-K\theta)(2\beta+\theta)}$$

$$A'_r = \frac{[2\beta-(K-1)\theta]a_r + \theta\sum_{j=1}^{K} a_j}{(\beta-K\theta)(2\beta+\theta)}$$

$$A_i = \frac{-2\,(\beta+\theta)\,(\beta-K\theta)\,a_r + \theta\,(2\beta+\theta)\left[a_e + \sum_{j=1}^{K} a_j\right] + E_2 a_i}{(2\beta+\theta)(\beta-K\theta)} \qquad i = 1,2,\ldots K\,(i \neq r)$$

$$A'_i = \frac{-E_{11}a_r + \theta\,(3\beta+\theta)\sum_{j=1}^{K} a_j + E_{12}a_i}{(2\beta+\theta)(\beta-K\theta)(3\beta+\theta)} \qquad i = 1,2,\ldots K\,(i \neq r)$$

Supplementary materials associated with the proofs of Lemmas 3.1−3 and Theorems 3.3 and 3.5 can be found in the online version.

Appendix B. Notations and proofs in the centralized model

$$B_e = \frac{[\beta-(K-1)\theta]\,a_e + \theta\sum_{j=1}^{K} a_j}{(\beta+\theta)(\beta-K\theta)}$$

$$B_i = \frac{\theta a_e + (\beta-K\theta)a_i + \theta\sum_{j=1}^{K} a_j}{(\beta+\theta)(\beta-K\theta)} \qquad i = 1,2,\ldots K$$

$$B'_i = \frac{[\beta-(K-1)\theta]\,a_i + \theta\sum_{j=1}^{K} a_j}{(\beta+\theta)(\beta-K\theta)} \qquad i = 1,2,\ldots K$$

Supplementary materials associated with the proofs of Lemma 4.1 and Theorem 4.2 can be found in the online version.

Appendix C. Notations and proofs in the linear quantity discount model

$$
\begin{aligned}
F_1 &= \beta\left(2\beta+\theta\right)-2\varphi\beta^2(\beta+\theta)\\
F_2 &= 4\varphi\beta\theta\left(\varphi\beta^2+\varphi\beta\theta-\theta\right)+2\beta\theta\left(1-3\varphi\beta\right)+\theta^2\\
F_3 &= 4K\varphi\beta\theta\left(1-\varphi\beta\right)+\left[2\beta-\left(K-1\right)\theta\right]+4\varphi\beta\theta\left(K-1\right)\left(1-\varphi\beta\right)+2\varphi\beta^2(2\varphi\beta-3)\\
F_4 &= \left[\left[\beta-\left(K-1\right)\theta\right]\left(1-2\varphi\beta\right)+\beta\right]\left[\left(2\beta+\theta\right)-2\varphi\beta(\beta+\theta)\right]\\
H_i &= \frac{2\beta\left(a_e+\sum\limits_{j=1}^{K}a_j\right)-(\beta-K\theta)a_i}{\left(\beta-K\theta\right)\left[\left(2K+1\right)\beta+K\theta\right]}\qquad i=1,\,2,\ldots K\\
H'_i &= \frac{2\beta\sum\limits_{j=1}^{K}a_j-\left[\beta-\left(K-1\right)\theta\right]a_i}{\left(\beta-K\theta\right)\left[\left(2K-1\right)\beta+\theta\right]}\qquad i=1,\,2,\ldots K\\
G_i &= \frac{(2\beta+\theta)a_i}{\left(\beta-K\theta\right)\left(2\beta+\theta\right)}\qquad i=1,\,2,\ldots K
\end{aligned}
$$

Supplementary materials associated with the proofs of Theorems 5.1 and 5 can be found in the online version.

Appendix D. Proofs in the bargaining model

Supplementary materials associated with the proofs of Theorems 6.1 and 6.2 can be found in the online version.

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