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PRICING AND DETERMINING THE OPTIMAL DISCOUNT TIME OF PERISHABLE GOODS WITH TIME AND PRICE DEPENDENT DEMAND

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Abstract. In the eyes of consumers, the value of perishable goods generally declines during the good's lifetime. In this situation a mechanism to encourage purchases, such as discounts or reduction in price policy can be effective. In this paper, we determine the discount time and the prices for a perishable product with a one period lifetime. Product demand is dependent on price and time. Demand function is different after discount time to an increase in costumer. After modeling, we show that the profit function is concave and optimal price and the discount time is unique. Due to the complexity of the model, using a heuristic algorithm, the near optimal price and the near optimal discount time are calculated.

Mathematics Subject Classification. 45G, 65K.

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1. Introduction

The first paper that combined pricing and capacity decisions is [2], who studied a single period model. They considered the case where capacity is fixed for both products, but the firm can set prices. They obtained the optimal pricing and capacity decisions for two products by assuming demand to be uniformly distributed. Maihami and Kamalabadi [7] and Avinadav *et al.* [1] also considered a single period problem. Maihami and Kamalabadi (2012) assumed that demand is price and time sensitive and developed an optimization model to determine the optimal price, the optimal order quantity and the optimal replenishment schedule for non-instantaneously deteriorating items. Avinadav *et al.* [1] also employed a price and time dependent demand function and developed a mathematical model to calculate the optimal price, the order quantity and the replenishment period for perishable items. Chew *et al.* [4] assumed that demand for perishable products is price sensitive, and developed an optimization model to determine the price and the inventory allocation for a perishable product with predetermined multiple lifetimes. In addition, Chew *et al.* [3] considered a pricing and inventory problem with a perishable product of multiple-periods lifetime. However, the demand for products of different age is assumed to be independent from each other, and they failed to consider demand transfers among products of different ages. We consider a single period problem with a demand function that differs in the time horizon.

Commodities, which lose their value over time are called "perishable products". Medicines, fruits and vegetables, seasonal and fashion goods, electronics, *etc.* are considered as perishable products. Due to technological

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advances, competitive markets, and the value of providing fresh products for customers, sales management and pricing of these goods are important. The significant issues for customers are the lifetime of goods and their expiry date, so marking down the price at the time horizon is an incentive approach to sell more goods. In many businesses, dynamic pricing is considered as a mechanism to attract more customers. Diaz [5] argued that the impact of price on consumer decisions depends on how products are evaluated. Particularly in the case of perishable food products, many consumers believe that new products have a higher value than the products whose dates have expired. Supermarket customers prefer to buy fresh products, not those close to the expiry date. When the prices are the same, they prefer newer products. To encourage customers to buy perishable products close to their expiry date, the use of price discounts is an effective approach. Tajbakhsh *et al.* [8] designed an inventory model with stochastic price discount, and presented a numerical analysis, which showed cost savings through discount offer. Li *et al.* [6] introduced a dynamic pricing approach to optimize profits for supply chain partners, and showed that price change cost and uncertainty in consumer behavior make difficult the implementation of the dynamic pricing model.

In this paper, pricing of perishable goods in extenuating circumstances is considered. Due to the importance of selling those products in their life cycle and their freshness in the customers view, policies that encourage customers to buy more are essential. We propose a rebate policy with synchronizing the demand rate function during the discount interval. The demand rate is a function of the price and time, and due to the discounts offered during the related interval, a different demand rate function is considered based on the discount time. It should be pointed out that a price discount policy effectively produces an initially demand rate increase, but it is reduced over time. The main contribution of the paper is an algorithm for determining the discount time and the prices for a perishable product with a one period lifetime.

The rest of this paper is organized as follows. In Section 2 the model assumptions and notations are defined and the demand rate function depending on the time is introduced. Section 3 presents the demand, revenue and profit functions. In Section 5 numerical examples are considered and the related optimal price and discount time are obtained. Finally, some conclusions and future research are presented in Section 6.

2. Notations and assumptions

The following notations and assumption are used throughout the paper.

2.1. Notation

Demand functions

$D_1(p,t)$ demand rate function before discount time, which depends on the time period and the selling price.
$D_2(p,t)$ demand rate function after discount time, which depends on the time period and the selling price.

Parameters

ED_t total demand quantity at time interval $[0,t]$
c constant purchasing cost per unit
T time horizon for selling product
Q order quantity in the time horizon of selling product
α percentage of discount that is considered $0 < \alpha < 1$
I_0 maximum inventory level in the first of time horizon, it is the order quantity

Variables

p selling price per unit, where $p > c$
t discount time in which the firm announces the purchasing off to the customers
TP total profit per time unit of the inventory system
Note: The optimal value of the variables is denoted with the superindex *.

2.2. Assumptions

Consider an inventory system where a firm purchases a perishable item at initial sales period T, and sells it over that season. Other assumptions are as follows:

(i) A single perishable item is assumed.
(ii) The lead time is zero.
(iii) All of the parameters are deterministic.
(iv) Shortages are not allowed.
(v) The time horizon is finite.

2.3. Demand function

The basic demand rate is a function of time and price during the time horizon of product selling. Due to the importance of time in buying and selling perishable products, the demand change over time is considered exponential. Price is a necessary factor on buying perishable products, so that the demand function is considered as a linear function of price [9]. In this paper, we assumed that the demand rate before discount can be expressed as follows,

$$D_1\,(p,t) = (a - bp)\,\mathrm{e}^{-\lambda t}, \qquad b > 0, a > 0, \lambda > 0, \tag{2.1}$$

where, a and b are constant parameters such that a can guarantee the demand is positive at the beginning and, afterwards, it decreases by changes in p and b is the importance coefficient of p in the demand function. Also, a different λ, can be used in most cases where the demand rate varies over time. The consideration of time and price dependent demand is useful for deteriorate items, such as fashion goods, high-tech product, fruits and vegetables [9].

Let ED_t be the total demand of a product at the time period [0, t] be expressed as follows,

$$ED_t = \int_0^t D\,(p,t)\mathrm{d}t \tag{2.2}$$

The firm can offer a lower price in a mark-down period to attract customers to purchase products approaching their expiry date. As a result, the two following different prices are set in the time horizon for product selling,

$$p\,(t) = \begin{cases} p & [0, t_1] \\ p\,(1-\alpha) & [t_1, T] \end{cases} \tag{2.3}$$

where, t_1 is the price mark-down time after which a discount price $\alpha, (0 < \alpha < 1)$ is deployed for a given product.

During the discount interval, the demand rate has an ascending trend and gradually starts to descend. This function can be expressed as follows,

$$D_2\,(p,t) = (a - bp(1-\alpha))\,t^{\beta}\mathrm{e}^{-\lambda t}, \qquad b > 0, a > 0, > 0,\ \beta > 1. \tag{2.4}$$

Note: After interviewing several salesmen who had discount policies in their stores, we found that after the discount announcement, the demand rate, starting with a positive jump, increased in a short time and then decreased. We asked them about of the quantification of the demand increase and it was estimated a suitable curve for demand rate by using simulation. It is the approximation that we use and it is represented as follows, $D_2\,(p,t)$ is used in the example worked out in Section 5.

$$D_2\,(p,t) = (a - bp(1-\alpha))\,t^3\mathrm{e}^{-\lambda t}, \qquad b > 0, a > 0, \lambda > 0.$$

3. Demand, revenue and profit functions

It is assumed that the maximum inventory in the first period (I_0) is the order quantity, and its decreasing is only affected by demand. As a price mark-down should always be applied before the expiry date of the product, the time horizon for product selling can be divided into two intervals: $[0, t]$ and $[t, T]$.

Notice that due to the discount after the price mark-down, the demand rate function during the time intervals $[0, t]$ and $[t, T]$ is different.

Observe that in the interval the inventory descends over time, and the product is sold out at the price p.

On the other hand, in the interval $[t, T]$, the product is sold out at the discount price $p\,(1-\alpha)$. Due to the discount, a moderate growth in the demand initially occurs; however, it reduces gradually (see Fig. 1).

There is no shortage, nor surplus in the end of the time horizon, *i.e.*, period T, so the inventory level is the demand in that period. On the other hand, the demand in the time interval $[0, t]$ can be expressed as follows,

$$ED_t = \int_0^t D\,(p,t)\mathrm{d}t = \int_0^t (a-bp)\,\mathrm{e}^{-\lambda t}\mathrm{d}t = \frac{(a-bp)}{\lambda}\left[1-\mathrm{e}^{-\lambda t}\right] \tag{3.1}$$

and the demand quantity in time interval $[t, T]$ can be expressed,

$$\begin{aligned} ED_T = \int_t^T D\,(p,t)\,\mathrm{d}t &= \int_t^T (a-bp)\,t^3\mathrm{e}^{-\lambda t}\mathrm{d}t \\ &= \frac{1}{\lambda^4}(a+bp(-1+\alpha))(\mathrm{e}^{-t\lambda}(6+t\lambda(6+t\lambda(3+t\lambda)))+\mathrm{e}^{-T\lambda}(-6-T\lambda(6+T\lambda(3+T\lambda))). \end{aligned} \tag{3.2}$$

The revenue by selling up with price p in the interval [0, t] and with price $p(1-\alpha)$ in the interval $[t, T]$ can be expressed as follows,

$$\begin{aligned} SR &= p\int_0^t D\,(p,t)\,\mathrm{d}t + p\,(1-\alpha)\int_t^T D\,(p,t)\mathrm{d}t \\ &= p\left[\frac{(a-bp)}{\lambda}\left(1-\mathrm{e}^{-\lambda t}\right)\right] + p\,(1-\alpha)\left[\frac{1}{\lambda^4}(a-bp(1-\alpha))(\mathrm{e}^{-t\lambda}(6+t\lambda(6+t\lambda(3+t\lambda)))\right. \\ &\quad \left.+\,\mathrm{e}^{-T\lambda}(-6-T\lambda(6+T\lambda(3+T\lambda))))\right], \end{aligned} \tag{3.3}$$

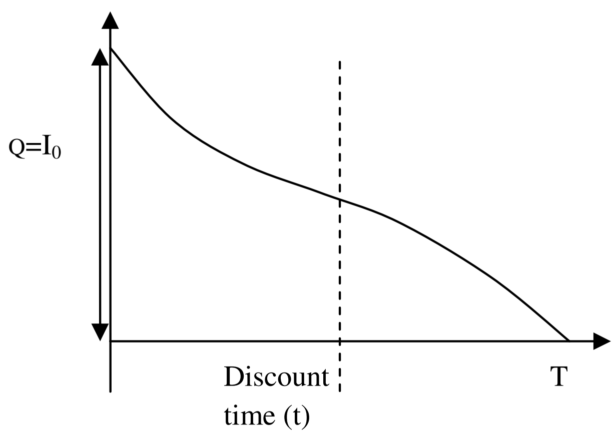

Figure 1. Inventory level in period T.

and the profit can be expressed,

$$\begin{aligned} TP &= (p-c)\int_0^t D(p,t)\,\mathrm{d}t + (p(1-\alpha)-c)\int_t^T D(p,t)\,\mathrm{d}t \\ &= (p-c)\left[\frac{(a-bp)}{\lambda}\left(1-e^{-\lambda t}\right)\right] + (p(1-\alpha)-c) \\ &\times \left[\frac{1}{\lambda^4}(a-bp(1-\alpha))(\mathrm{e}^{-t\lambda}(6+t\lambda(6+t\lambda(3+t\lambda)))+\mathrm{e}^{-T\lambda}(-6-T\lambda(6+T\lambda(3+T\lambda))))\right]. \end{aligned} \tag{3.4}$$

4. Optimal price p^* and discount time t^*

We consider a problem with retailers making important decision related to pricing for maximizing their own profit. Optimally, partial derivatives of the profit function with respect to p and t must be equal to zero (Eqs. (9) and (10)). For solving the problem, the following equations must be simultaneously considered:

$$\begin{aligned} \frac{\partial TP}{\partial p} &= -\frac{b\left(1-\mathrm{e}^{-t\lambda}\right)(-c+p)}{\lambda} + \frac{\left(1-\mathrm{e}^{-t\lambda}\right)(a-bp)}{\lambda} \\ &- \frac{b(c+p(-1+\alpha))(-1+\alpha)\left(\mathrm{e}^{-t\lambda}(6+t\lambda(6+t\lambda(3+t\lambda)))+\mathrm{e}^{-T\lambda}(-6-T\lambda(6+T\lambda(3+T\lambda)))\right)}{\lambda^4} \\ &- \frac{(a+bp(-1+\alpha))(-1+\alpha)(\mathrm{e}^{-t\lambda}(6+t\lambda(6+t\lambda(3+t\lambda)))+\mathrm{e}^{-T\lambda}(-6-T\lambda(6+T\lambda(3+T\lambda))))}{\lambda^4} = 0. \end{aligned} \tag{4.1}$$

$$\begin{aligned} \frac{\partial TP}{\partial t} &= \mathrm{e}^{-t\lambda}(-c+p)(a-bp) - \frac{1}{\lambda^4}(c+p(-1+\alpha))(a+bp(-1+\alpha)) \\ &\times (\mathrm{e}^{-t\lambda}(t\lambda(t\lambda^2+\lambda(3+t\lambda))+\lambda(6+t\lambda(3+t\lambda))) - \mathrm{e}^{-t\lambda}\lambda(6+t\lambda(6+t\lambda(3+t\lambda)))) = 0. \end{aligned} \tag{4.2}$$

As a result the discount time t^* and selling price p^* are:

$$t^* = \frac{(-1)^{2/3}(c-p)^{1/3}(a-bp)^{1/3}}{((c+p(-1+\alpha))(a+bp(-1+\alpha)))^{1/3}}, \tag{4.3}$$

and,

$$\begin{aligned} p^* = \Bigg(&-\frac{a\left(1-\mathrm{e}^{-t\lambda}\right)}{\lambda} - \frac{bc\left(1-\mathrm{e}^{-t\lambda}\right)}{\lambda} \\ &+ \frac{a(-1+\alpha)\left(\mathrm{e}^{-t\lambda}(6+t\lambda(6+t\lambda(3+t\lambda)))+\mathrm{e}^{-T\lambda}(-6-T\lambda(6+T\lambda(3+T\lambda)))\right)}{\lambda^4} \\ &+ \frac{bc(-1+\alpha)\left(\mathrm{e}^{-t\lambda}(6+t\lambda(6+t\lambda(3+t\lambda)))+\mathrm{e}^{-T\lambda}(-6-T\lambda(6+T\lambda(3+T\lambda)))\right)}{\lambda^4}\Bigg) \Bigg/ \Bigg(-\frac{2b\left(1-\mathrm{e}^{-t\lambda}\right)}{\lambda} \\ &- \frac{2b(-1+\alpha)^2\left(\mathrm{e}^{-t\lambda}(6+t\lambda(6+t\lambda(3+t\lambda)))+\mathrm{e}^{-T\lambda}(-6-T\lambda(6+T\lambda(3+T\lambda)))\right)}{\lambda^4}\Bigg). \end{aligned} \tag{4.4}$$

Theorem 4.1. *The price p^* and discount time t^* obtained from expressions* (11) *and* ((12) *have absolute second order conditions for profit objective maximization* (*TP*).

Proof. Let us compute the Hessian matrix of the profit function with respect to p^* and t^*. If the determinant of the Hessian is positive, the proof is completed, such that

$$\frac{\partial^2 TP}{\partial p^2} = -\frac{2b(1-\mathrm{e}^{-t\lambda})}{\lambda} - \frac{1}{\lambda^4}2b\left(-1+\alpha\right)^2\left(\mathrm{e}^{-t\lambda}\left(6+t\lambda\left(6+t\lambda\left(3+t\lambda\right)\right)\right)+\mathrm{e}^{-T\lambda}\left(-6-T\lambda\left(6+T\lambda\left(3+T\lambda\right)\right)\right)\right) \tag{4.5}$$

$$\begin{aligned}\frac{\partial^2 TP}{\partial t^2} = &-\mathrm{e}^{-t\lambda}\left(-c+p\right)\left(-bp+a\right)\lambda - \frac{1}{\lambda^4}\left(c+p\left(-1+\alpha\right)\right)\left(a+bp\left(-1+\alpha\right)\right)\left(\mathrm{e}^{-t\lambda}\left(2t\lambda^3+2\lambda\left(t\lambda^2+\lambda\left(3+t\lambda\right)\right)\right)\right.\\ &\left.-2\mathrm{e}^{-t\lambda}\lambda\left(t\lambda\left(t\lambda^2+\lambda\left(3+t\lambda\right)\right)+\lambda\left(6+t\lambda\left(3+t\lambda\right)\right)\right)+\mathrm{e}^{-t\lambda}\lambda^2\left(6+t\lambda\left(6+t\lambda\left(3+t\lambda\right)\right)\right)\right).\end{aligned} \tag{4.6}$$

From where,

$$\begin{aligned}H = &\begin{bmatrix}\frac{\partial^2 TP}{\partial p^2} & \frac{\partial^2 TP}{\partial p\partial t}\\ \frac{\partial^2 TP}{\partial t\partial p} & \frac{\partial^2 TP}{\partial t^2}\end{bmatrix} \to \mathrm{Det}\left(H\right) = \frac{\partial^2 TP}{\partial p^2} * \frac{\partial^2 TP}{\partial t^2} - \left[\frac{\partial^2 TP}{\partial p\partial t}\right]^2 > 0 \to \mathrm{e}^{-2t\lambda}\left(-\left(b\left(c\left(1+t^3\left(-1+\alpha\right)\right)\right.\right.\right.\\ &\left.\left.+2p\left(-1+t^3\left(-1+\alpha\right)^2\right)\right)+s\left(1+t^3\left(-1+\alpha\right)\right)\right)^2+\mathrm{e}^{t\lambda}\\ &\times\left(a\left(c\left(3t^2+\lambda-t^3\lambda\right)-p\left(-3t^2\left(-1+\alpha\right)+\lambda+t^3\left(-1+\alpha\right)\lambda\right)\right)-bp\left(c\left(-3t^2\left(-1+\alpha\right)+\lambda+t^3\left(-1+\alpha\right)\lambda\right)\right.\right.\\ &\left.\left.+p\left(-3t^2\left(-1+\alpha\right)^2-\lambda+t^3\left(-1+\alpha\right)^2\lambda\right)\right)\right)\left(-\frac{2b\left(1-\mathrm{e}^{-t\lambda}\right)}{\lambda}-\frac{1}{\lambda^4}2b\left(-1+\alpha\right)^2\right.\\ &\left.\left.\left(\mathrm{e}^{-t\lambda}\left(6+t\lambda\left(6+t\lambda\left(3+t\lambda\right)\right)\right)+\mathrm{e}^{-T\lambda}\left(-6-T\lambda\left(6+T\lambda\left(3+T\lambda\right)\right)\right)\right)\right)\right)>0.\end{aligned} \tag{4.7}$$

And, by using Lemma 4.2 (see below), the proof of the theorem is made. □

Lemma 4.2. *If the second derivative of function* Det (H) *is negative (and, so, it is concave) and if the function* Det (H) *has two answers for each variable* p *and* t*, then* Det (H) *is positive in the interval between two responses.*

Proof. The second derivative of Det (H) with respect to variables p computed, see (16). By inspection, it becomes clear that each statement of (16) is negative, so the sum of the statements is negative.

$$\begin{aligned}\frac{\partial^2 D(H)}{\partial p^2} = &4b^2\left(-2\left(-1+t^3\left(-1+\alpha\right)^2\right)^2-\frac{1}{\lambda^4}\mathrm{e}^{t\lambda}\left(-\lambda+t^2\left(-1+\alpha\right)^2\left(-3+t\lambda\right)\right)\right.\\ &\left.\times\left(\left(-1+\mathrm{e}^{-t\lambda}\right)\lambda^3-\left(-1+\alpha\right)^2\left(\mathrm{e}^{-t\lambda}\left(6+t\lambda\left(6+t\lambda\left(3+t\lambda\right)\right)\right)+\mathrm{e}^{-T\lambda}\left(-6-T\lambda\left(6+T\lambda\left(3+T\lambda\right)\right)\right)\right)\right)\right)<0\end{aligned} \tag{4.8}$$

By solving Det $(H) = 0$ only two values are obtained for p (see below), and, since the second derivative of Det (H) is negative thus the function Det (H) is positive. Notice that Det (H) is located at the top of the vertical axis between the roots (two values) as shown in Figure 2. The two values of variable p mentioned above are as follows:

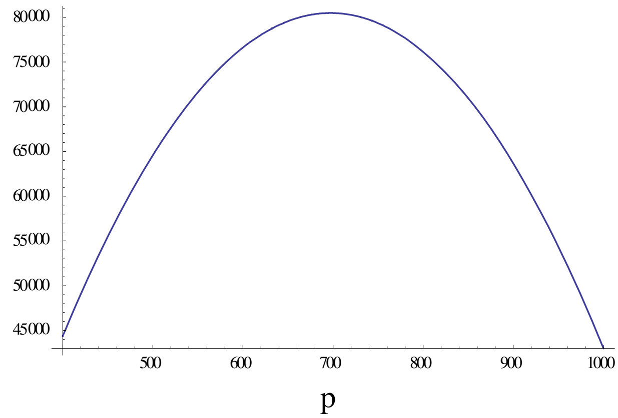

FIGURE 2. Graphical representation of Det (H) with respect to p.

$$
\begin{aligned}
&\{\{p \to (-36\,a\,b\,e^{t\lambda}\,t^2 - 36\,b^2\,c\,e^{t\lambda}\,t^2 + \ll 230 \gg)\,/ \\
&\quad (2\,(-36\,b^2\,e^{t\lambda}\,t^2 + 36\,b^2\,e^{T\lambda}\,t^2 + 144\,b^2\,e^{t\lambda}\,t^2\,\alpha - 144\,b^2\,e^{T\lambda}\,t^2\,\alpha - \\
&\qquad 216\,b^2\,e^{t\lambda}\,t^2\,\alpha^2 + 216\,b^2\,e^{T\lambda}\,t^2\,\alpha^2 + 144\,b^2\,e^{t\lambda}\,t^2\,\alpha^3 - 144\,b^2\,e^{T\lambda}\,t^2\,\alpha^3 - \\
&\qquad 36\,b^2\,e^{t\lambda}\,t^2\,\alpha^4 + 36\,b^2\,e^{T\lambda}\,t^2\,\alpha^4 - 12\,b^2\,e^{t\lambda}\,\lambda + 12\,b^2\,e^{T\lambda}\,\lambda + 12\,b^2\,e^{t\lambda}\,t^3\,\lambda + \\
&\qquad \ll 92 \gg + 8\,b^2\,e^{T\lambda}\,t^3\,\alpha\,\lambda^4 + 4\,b^2\,e^{t\lambda+T\lambda}\,t^3\,\alpha\,\lambda^4 - 8\,b^2\,e^{T\lambda}\,t^6\,\alpha\,\lambda^4 + \\
&\qquad 4\,b^2\,e^{t\lambda}\,T^3\,\alpha\,\lambda^4 - 8\,b^2\,e^{t\lambda}\,t^3\,T^3\,\alpha\,\lambda^4 - 4\,b^2\,e^{T\lambda}\,t^3\,\alpha^2\,\lambda^4 - 2\,b^2\,e^{t\lambda+T\lambda}\,t^3\,\alpha^2\,\lambda^4 + \\
&\qquad 12\,b^2\,e^{T\lambda}\,t^6\,\alpha^2\,\lambda^4 - 2\,b^2\,e^{t\lambda}\,T^3\,\alpha^2\,\lambda^4 + 12\,b^2\,e^{t\lambda}\,t^3\,T^3\,\alpha^2\,\lambda^4 - 8\,b^2\,e^{T\lambda}\,t^6\,\alpha^3\,\lambda^4 - \\
&\qquad 8\,b^2\,e^{t\lambda}\,t^3\,T^3\,\alpha^3\,\lambda^4 + 2\,b^2\,e^{T\lambda}\,t^6\,\alpha^4\,\lambda^4 + 2\,b^2\,e^{t\lambda}\,t^3\,T^3\,\alpha^4\,\lambda^4))\}, \\
&\ \{p \to (\ll 231 \gg + \sqrt{}\,((36\,a\,b\,e^{t\lambda}\,t^2 + \ll 227 \gg + 2\,a\,b\,e^{t\lambda}\,t^3\,T^3\,\alpha^3\,\lambda^4 + \\
&\qquad\qquad 2\,b^2\,c\,e^{t\lambda}\,t^3\,T^3\,\alpha^3\,\lambda^4)^2 - 4\,(-36\,a\,b\,c\,e^{t\lambda}\,t^2 + 36\,a\,b\,c\,e^{T\lambda}\,t^2 + \\
&\qquad\qquad \ll 96 \gg + 2\,a\,b\,c\,e^{t\lambda}\,t^3\,T^3\,\alpha^2\,\lambda^4)\,(-36\,b^2\,e^{t\lambda}\,t^2 + 36\,b^2\,e^{T\lambda}\,t^2 + \\
&\qquad\qquad \ll 127 \gg + 2\,b^2\,e^{T\lambda}\,t^6\,\alpha^4\,\lambda^4 + 2\,b^2\,e^{t\lambda}\,t^3\,T^3\,\alpha^4\,\lambda^4)))\,/ \\
&\quad (2\,(-36\,b^2\,e^{t\lambda}\,t^2 + 36\,b^2\,e^{T\lambda}\,t^2 + 144\,b^2\,e^{t\lambda}\,t^2\,\alpha - 144\,b^2\,e^{T\lambda}\,t^2\,\alpha - \\
&\qquad 216\,b^2\,e^{t\lambda}\,t^2\,\alpha^2 + 216\,b^2\,e^{T\lambda}\,t^2\,\alpha^2 + \ll 116 \gg + 12\,b^2\,e^{t\lambda}\,t^3\,T^3\,\alpha^2\,\lambda^4 - 8\,b^2\,e^{T\lambda} \\
&\qquad t^6\,\alpha^3\,\lambda^4 - 8\,b^2\,e^{t\lambda}\,t^3\,T^3\,\alpha^3\,\lambda^4 + 2\,b^2\,e^{T\lambda}\,t^6\,\alpha^4\,\lambda^4 + 2\,b^2\,e^{t\lambda}\,t^3\,T^3\,\alpha^4\,\lambda^4))\}\}
\end{aligned}
$$

Similarly, the above steps can be performed for variable t and completed the proof. In the following function Det (H) with respect to p and t is drawn at specified intervals. As shown in Figures 2 and 3 the curve is above the horizontal axis, and is positive. For drawing, the range of p is [400, 1000] and t is [0.5, 2].

Hence, the Hessian matrix H at point (t^*, T^*) is negative definite. Consequently, it can be concluded that the stationary point (t^*, T^*) is a global maximum for the optimization problem.

4.1. Algorithm

As proved above, the profit function is concave and has a unique solution. A simple heuristic algorithm [7] is used for obtaining the initial solution say (p_1, t_1) We point out that for obtaining the values of price and time it is not necessary to go for the exact decimal, so the iterative algorithm can get the optimal value p^* and t^*

Step 1. Let k denote a given iteration. Start by setting $k := 1$ and $p_k := p_1$ and $t_k := t_1$.

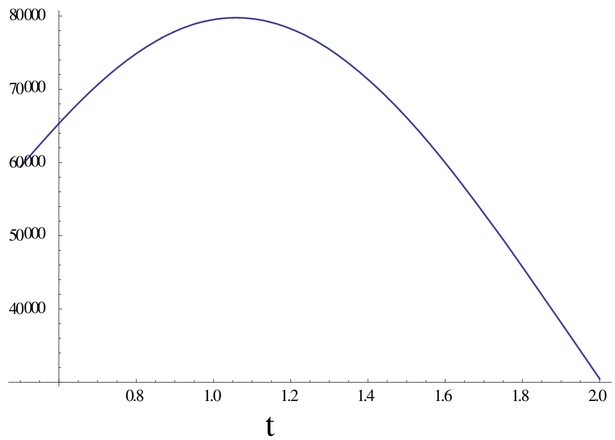

FIGURE 3. raphical representation of Det (H) with respect to t.

TABLE 1. Computational results of Example 5.1.

k	p_k	t_k	TP_k
1	600.000	1.078	100182.512
2	690.310	1.012	104548.806
3	694.597	1.008	104558.573
4	694.813	1.008	104558.611
5	694.825	1.008	104558.612
6	694.826	1.008	104558.612

Step 2. Find the optimal value t_{k+1} by solving (11) for p_k.
Step 3. Find the optimal value p_{k+1} by solving (12) for t_{k+1}.
Step 4. If k>1 and the difference between p_k and p_{k+1} is enough small (*i.e.* $|p_k p_{k+1}| \leqslant \varepsilon$, where ε is the quasi-optimal tolerance), then $p^k := p_{k+1}$, $t^* := t_{k+1}$ is the near optimal solution, and stop. Otherwise, get $k := k+1$ and go back to Step 2.

By using the above algorithm, the near optimal solution (p^*, t^*) is obtained. With (p^*, t^*), TP* can be obtained by (8).

5. Numerical examples

The proposed algorithm is used for solving the following numerical example to illustrate the solution process and results. Mathematica 9 was used.

Example 5.1. The following parameters and functions are used.

$D_1\,(p,t) = (500 - 0.5p)\,\mathrm{e}^{-0.98t}$ $D_2\,(p,t) = (500 - 0.5p)\,t^3\mathrm{e}^{-0.98t}T = 2,\ c = 200, \alpha = 0.3.$

Table 1 show, the convergence of the algorithm, where for the quasi-optimal tolerance ε, it results $p^* = 694.826$, $t^* = 1.008$, $TP^* = 104558.612$, $Q^* = 293.945$, and the numerical results are obtained for the price interval [400, 1000] (see Figs. 5 and 6).

We can observe that the numerical results reveal that TP^* is strictly concave in p, and concave in t (see Fig. 4). Also as shown in Figure 5, TP is concave in t. In the figure the interval of t is [0.5, 2]; the surface has

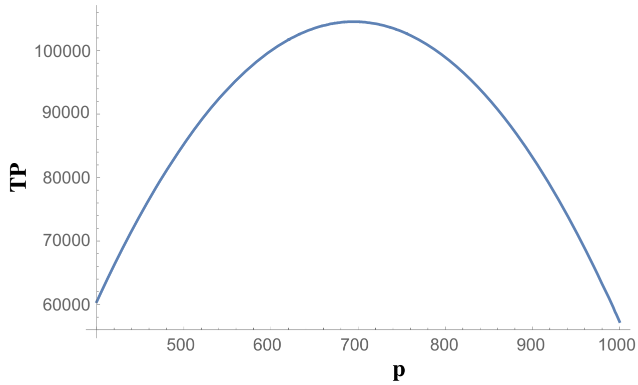

FIGURE 4. Graphical representation of $TP(p|t^*)$.

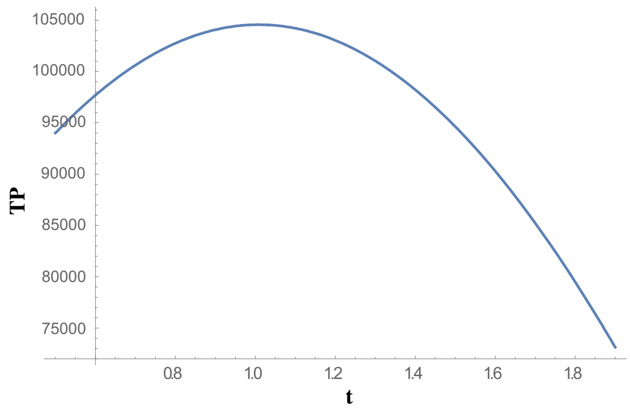

FIGURE 5. Graphical representation of $TP(t|p^*)$.

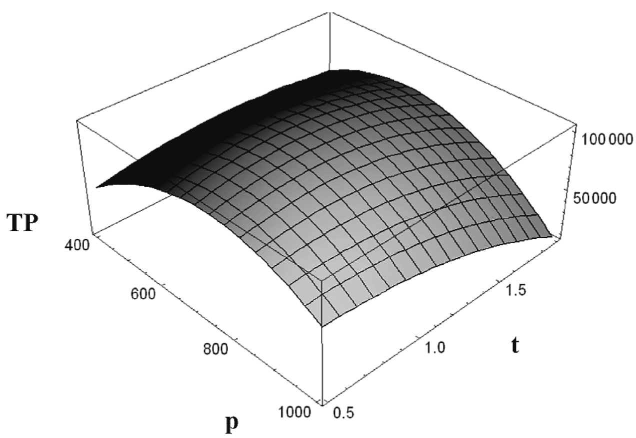

FIGURE 6. Graphical representation of $TP(p,t)$.

been obtained by using p^*. As a result, the local maximum obtained here from the heuristic algorithm is indeed the global maximum solution.

6. Conclusion

In this paper, a model for obtaining the pricing for perishable goods in terms of discounts is presented. It has been proven that the objective function value obtained from the optimal price and discount time is unique and optimal. Finally, a numerical example using the proposed heuristic algorithm illustrate the results, showing that the objective function is concave, and the optimal profit is global. The approach presented in this paper is comprehensive and flexible to different values of the parameters of the demand function. It can be extended in several ways; for instance, by considering a variable percentage discount. Other aspects include advertising policies, delays in payment and models for coordinating in the system (*i.e.*, supply chain).

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