RAIRO-Oper. Res. 50 (2016) 1027–1041
DOI: 10.1051/ro/2015058

RAIRO Operations Research
www.rairo-ro.org

A GENETIC ALGORITHM FOR AN INVENTORY SYSTEM UNDER BELIEF STRUCTURE INFLATIONARY CONDITIONS

S. Nodoust[1], A. Mirzazadeh[1] and M. Mohammadi[1]

Abstract. The literature review on the inflationary inventory systems shows that a lot of researches have been made with considering the inflation as: (1) deterministic and constant; (2) deterministic and variable (time varying); (3) stochastic or (4) fuzzy. However, no attempt has been made to address the issue of how to deal with incomplete, imprecise and missing (ignorance) information in inflation, which is essentially inherent and sometimes inevitable in human being's subjective judgments. The purpose of this paper is to develop a new method, on the basis of the evidential reasoning (ER) approach in order to handle various types of possible uncertainties that may occur in the determining of the inflation rate in the inventory decision making. It is capable of modeling various types of uncertainties using a unified belief structure in a pragmatic, rigorous, reliable, systematic, transparent and repeatable way. The evidential reasoning approach uses a systematic way to accumulate the incomplete data about inflation, which have been gathered from different decision makers. This approach causes interval inflation by accumulating information of all decision makers. Representing the inflation by an interval number and using the interval arithmetic, the objective function for cost is changed to corresponding multi objective functions. These functions are minimized and solved by NSGA- II approach of Multi-objective Genetic Algorithm. The algorithm parameters are tuned by Taguchi method and the mentioned parameter-tuned algorithm has been validated using several numerical examples by comparison with the optimal solution. The results show that the proposed GA takes less time than the classical model in solving the problem. This difference of times is more significant when we want to do a sensitivity analysis in a wide range of parameters.

Mathematics Subject Classification. 90B05, 90C27.

Received 16 June, 2014. Accepted 29 September, 2015.

1. Introduction

Since 1975 many researchers considered the effects of inflation and time value of money on the inventory systems. There are several papers which considered the rate of inflation as a constant and well-known value. Buzacott [3] was the first one which considered time value of money in an economic order quantity model with inflation. Misra [27] considered a discounted cost model, included internal and external inflation for various costs. Datta and Pal [6] considered an inflationary inventory system which demand is an increasing function of time. Sarker and Pan [32] surveyed the effects of inflation and the time value of money on the order quantity with finite replenishment rate. Hariga [14] extended Datta and Pal's work considering demand with a decreasing function

Keywords. Inventory system, genetic algorithm, inflation, Dempster–Shafer theory, evidential reasoning, belief structure.

[1] Department of Industrial Engineering, Kharazmi University, Tehran, Iran. a.mirzazadeh@aut.ac.ir

of time and replenishment cycles that are not equal with each other. Hariga and Ben-Daya [15] presented time-varying lot-sizing models with a time-varying demand pattern and taking into account the effects of inflation and time value of money.

In the above-mentioned researches, deterioration of the goods is not considered. But, there are also many certain inflationary inventory models which have been developed for deteriorating items.

The first attempt to describe optimal ordering policies for deteriorating items was made by Ghare and Schrader [10]. Later, Covert and Philip [5] derived the model with variable deteriorating rate of two-parameter Weibull distribution. Su *et al.* [34] developed an inventory model under inflation with stock dependent consumption rate and exponential rate of deterioration. Sarker *et al.* [33] considered an optimal ordering policy of goods with deterioration and permitted delay in payment and shortages with a sensitivity analysis on different values of inflation and showed that the optimal order quantity and maximum allowable shortage vary with the difference between inflation and time discount. A deteriorating inventory model taking into account the time-value of money is developed for a deterministic inventory system with price-dependent demand by Wee and Law [39]. This study applies the discounted cash flows (DCF) approach for problem analysis. A heuristic approach is presented to derive the near optimal replenishment and pricing policy that tries to maximize the total net present-value profit.

Goyal and Giri [12] presented a review of the advances of deteriorating inventory literature since the early 1990s. Chung and Tsai [4] presented an inventory model for deteriorating items with the demand of linear trend and shortages during the finite planning horizon considering the time value of money. A simple solution algorithm using a line search is presented to determine the optimal interval which has positive inventories. A sensitivity analysis is performed to study the effect of changes in the system parameters. A two-warehouse inventory problem for deteriorating items with a constant demand rate and shortages is developed by Yang [36] which in contrast to the traditional deterministic two-warehouse inventory model with shortages at the end of each replenishment cycle, an alternative model in which each cycle begins with shortages and ends without shortages is proposed in it

Moon *et al.* [20] developed a model for ameliorating/deteriorating items with time varying demand pattern over a finite planning horizon, taking into account the effects of inflation and time value of money.

Optimal solutions are derived and the effects of amelioration/deterioration on the inventory replenishment policies are studied with the help of numerical examples.

Lo *et al.* [17] developed an integrated production and inventory model from the perspectives of both the manufacturer and the retailer. The model assumes a varying rate of deterioration, partial backordering, inflation, imperfect production processes and multiple deliveries. The elapsed time until the production process shift is assumed to be arbitrarily distributed. The discounted cash flow and classical optimization technique are used to derive the optimal solution. Dey *et al.* [8] studied a finite time horizon inventory problem for a deteriorating item having two separate warehouses, one is an own warehouse (OW) of finite dimension and other a rented warehouse (RW), is developed with interval-valued lead-time under inflation and time value of money. Representing the lead-time by an interval number and using the interval arithmetic, the single objective function for profit is changed to corresponding multi-objective functions. These functions are maximized and solved by Fast and Elitist Multi-objective Genetic Algorithm (FEMGA).

Sana [31] developed a model for deteriorating and ameliorating items with capacity constraint for storage facility. The effect of inflation and time value of money in the profit and cost parameters is also considered in the model and the associated profit function is maximized by Euler–Lagrange's method, and it is illustrated by various time varying demands like quadratic, linear and exponential demand functions.

Maiti [18] proposed an inventory model with credit-linked promotional demand in an imprecise planning horizon. In this model a genetic algorithm (GA) with varying population size is developed and It is assumed that lifetime of the product is finite and imprecise (fuzzy) in nature.

Balkhi [2] solved a general finite horizon trade credit economic ordering policy for an inventory model with deteriorating items under inflation and time value of money when shortages are not allowed. The time horizon is divided into different cycles each of which has its own demand rate and its own trade credit period offered from the supplier to his retailer so that the retailer should pay his supplier before or after the end of the permissible trade credit of that cycle. Yang [37] developed the two-warehouse partial backlogging inventory model to incorporate three-parameter Weibull deterioration distribution. The objective is to derive the optimal replenishment policy that minimizes the net present value of total relevant cost per unit time. Two alternative models are compared based on the minimum cost approach. The study shows that the optimal solution not only exists but also is unique. Ghoreishi *et al.* [11] developed an Inventory model which studies the effect of inflation and customer returns on joint pricing and inventory control for deteriorating items. The main objective is determining the optimal selling price, the optimal replenishment cycles, and the order quantity simultaneously such that the present value of total profit in a finite time horizon is maximized. An algorithm has been presented to find the optimal solution.

A mixed binary integer mathematical programming model is developed by Mousavi *et al.* [21] for ordering items in multi-item multi-period inventory control systems, in which unit and incremental quantity discounts as well as interest and inflation factors are considered. The goal is to find optimal order quantities of the products so that the net present value of total system cost over a finite planning horizon is minimized. A genetic algorithm (GA) is presented to solve the proposed mathematical problem. Further, since no benchmarks can be found in the literature to assess the performance of the proposed algorithm, a branch and bound and a simulated annealing (SA) algorithm are employed to solve the problem as well. In addition, to make the algorithms more effective, the Taguchi method is utilized to tune different parameters of GA and SA algorithms.

A two-warehouse partial backlogging inventory model for deteriorating items is studied by Yang and Chang [38]. The objective of the study is to derive the retailer's optimal replenishment policy that maximizes the net present value of the profit per unit time. The necessary and sufficient conditions for an optimal solution are characterized and an algorithm is developed to find the optimal solution.

Mondala *et al.* [19] presented a production-repairing inventory model in fuzzy rough environment incorporating inflationary effects where a part of the produced defective units are repaired and sold as fresh units. Production and repairing rates are assumed as dynamic control variables and due to complexity of environment, different costs and coefficients are considered as fuzzy rough type and these are reduced to crisp ones using fuzzy rough expectation. Here production cost is production rate dependent, repairing cost is repairing rate dependent and demand of the item is stock-dependent. Goal of the research work is to find decisions for the decision maker (DM) who likes to maximize the total profit from the above system for a finite time horizon.

In all of the mentioned works, rate of inflation is constant and well-known during the time horizon. There are also some efforts with uncertain inflation rate. Mirzazadeh and Sarfaraz [26] developed a stochastic mathematical inventory model for multiple items with the internal and external inflation rates.

A production inventory model for a newly launched product is developed by Roy *et al.* [30] incorporating inflation and time value of money. It is assumed that demand of the item is displayed stock dependent and lifetime of the product is random in nature and follows exponential distribution with a known mean. The model is formulated to maximize the expected profit from the whole planning horizon. A genetic algorithm (GA) with varying population size is used to solve the model and is obtained using a fuzzy rule base and possibility theory. This GA is named fuzzy genetic algorithm (FGA) and is used to make decision for above production inventory model in different cases. A sensitivity analysis on expected profit function is also presented.

Mirzazadeh [22] discussed a model with probabilistic internal and external inflation rates and constant demand with complete backlogging shortage for the deteriorating items. Mirzazadeh [23], also, developed another research with time-varying inflation-dependent demand rate for deteriorating items with complete backlogging shortage. Mirzazadeh [24] considered an inventory system with inflation and time value of money under uncertain conditions, shortages and the effects of deterioration. The mentioned system is formulated with two methods, which are derived under some assumptions that the objective of inventory management is to minimize the average annual cost and the discounted cost. These methods are compared to each other carefully. The results reveal that the mentioned methods (the average annual cost and the discounted cost) have a negligible different to each other. Neetu and Tomer [28] prepared an inventory with variable inflation and delay in payment without shortages. The supplier provides a permissible delay of payments for a large order that is greater than or equal to the pre-determined quantity. The demand rate is taken as function of selling price. The deterioration rate follows the two parameter Weibull distribution. The objective is to minimize the total cost in the planning horizon.

An entropic economic order quantity model for items with imperfect quality considering constant rate of deterioration under fuzzy inflationary conditions is developed by Ameli *et al.* [1]. A mathematical model is developed to determine the number of cycles that maximizes the present value of total revenue in a finite planning horizon. The fuzzified model for inflation and discount rate is formulated and solved by two methods: signed distance and fuzzy numbers ranking. Results show that the number of cycles decreases in fuzzy inflationary conditions. Jana *et al.* [16] established a model with fuzzy inflation and stock dependent demand rate for deteriorating items with partial backlogging shortage over a random planning horizon. The model is described in two different environments: random and fuzzy random. The proposed model allows stock-dependent consumption rate and shortages with partial backlogging. In the fuzzy stochastic model, possibility chance constraints are used for defuzzification of imprecise expected total profit. Finally, genetic algorithm (GA) and fuzzy simulation-based genetic algorithm (FSGA) are used to make decisions for the above inventory models. A complex inventory system has been analyzed under uncertain situations for deteriorating items by Mirzazadeh [25] with considering probabilistic inflation rate. Shortages are allowable and the objectives of the problem are: (1) Minimization of the total present value of costs over time horizon and (3.2) Decreasing the total quantity of goods in the warehouse over time horizon. Furthermore, a new mathematical model for the optimal inventory is formulated and the solution procedure has been prepared with using the ideal point approach.

The above literature review clearly shows that quite efforts have been made to deal with the inventory systems with considering the inflation as: (1) Deterministic and constant, (3.2) Deterministic and variable (time varying), (3) Stochastic or (3.4) Fuzzy. However, no attempt has been made to address the issue of how to deal with incomplete, imprecise and missing (ignorance) information in inflation. In real situations, it is often too restrictive and difficult for decision makers to give precise (crisp) assessments for alternatives such as inflation. Stochastic consider random behavior and fuzzy logic based approaches have been extensively used to model vagueness and ambiguity, but they cannot deal with such uncertainties as incomplete assessments of several decision makers.

In this paper we have assumed inflation to be dependent on decision maker's (or some expert team) information. In this case, all experts say their opinion about inflation as a degree with a percent of probability. In reality in most of countries or occasions it is possible to face with this kind of uncertainty, and this paper gives a solution for such these problems. After gathering all information of decision makers, it is necessary to aggregate them all to attain a considerable and countable rate of inflation. The ER (Evidential Reasoning) approach is used for this purpose. The ER approach models both quantitative and qualitative attributes using a distributed modeling framework, in which each attribute is characterized by a set of collectively exhaustive assessment grades, probabilistic uncertainty including incomplete information and complete ignorance by a belief structure, or fuzzy uncertainty by fuzzy linguistic variables. In certain decision situations such as group decision making, however, a new type of interval uncertainty is likely encountered. For example, quantitative data may not be known precisely but may be estimated to belong to intervals with certain confidence levels. A decision maker (DM) may be unable to give precise judgment. In group decision analysis, different DMs may assign different degrees of belief to the same judgment.

Gathering information through ER approach causes an interval inflation number. According to Dey *et al.* [8] representing the inflation by an interval number, the objective function for cost is changed to corresponding multi objective functions. These functions are minimized and solved by NSGA-II approach of Multi-objective Genetic Algorithm. The algorithm parameters are tuned by Taguchi method and the mentioned parameter-tuned algorithm has been validated using several numerical examples by comparison with the optimal solution.

The rest of the paper is organized as follows. The assumptions and notations of the inventory model considered, is described in Section 2. The concepts of belief structure and ER approach and the inflation component are presented in Section 3. Section 4 represents the formulation and description of the proposed inventory model.

In Section 5 the proposed GA is defined. The results of parameter tuning and numerical examples are presented then in Section 6, in order to show the validity of the algorithm.

2. Assumptions and notations

2.1. Assumptions

The mathematical models in this paper are developed based on the following assumptions:

1. The inflation rate is uncertain as a belief structure.
2. The demand rate is time-dependent.
3. The initial inventory level is zero.
4. Shortages are allowed and fully backlogged.
5. The replenishment is instantaneous and the replenishment cycle is the same for each period.
6. The time horizon is infinite.
7. A constant fraction of the on-hand inventory deteriorates per unit time, as soon as the item is received into inventory.

2.2. Notations

Also, the following notations are used:

u	The inflation rate per unit time.
r	The interest rate per unit time.
$f(t)$	The time-dependent demand rate
θ	The constant deterioration rate, where $0 < \theta < 1$.
c_1	The inventory holding cost per unit per unit time at time zero.
c_2	The inventory shortage cost per unit per unit time at time zero.
c	The purchase cost per unit of the item at time zero.
A	The ordering cost per order at time zero.
T	The replenishment time interval.
k	The proportion of time in any given inventory cycle which orders can be filled from the existing stock.
j	A variable where $j = 0, 1, 2, \ldots$
$DC(kT)$	The expected value of costs.

Additional notations will be introduced later.

3. Inflation assessment and evidential reasoning approach

3.1. Inflation assessment

As the mentioned in the literature, in real situations, it is often too restrictive and difficult for decision makers to give precise (crisp) assessments for alternatives such as inflation. In this paper we study the case of having assessments of several decision makers about inflation rate. Each member gives an idea with a belief degree. For better understanding define a set of M decision makers asfollows:

$$D = \{D_1 D_2 \ldots D_n \ldots D_M\}. \tag{3.1}$$

Estimate the relative weights of the decision makers where ω_i is the relative weight for D_i and is normalized so that $0 < \omega_i < 1$ and equation (3.2) is fulfilled.

$$\sum_{i=1}^{M} \omega_i = 1. \tag{3.2}$$

TABLE 1. Belief structure.

	D_M	...	D_i	...	D_1
H_1	$\beta_{1,1}$	...	$\beta_{1,i}$	...	$\beta_{1,M}$
...	...	...	...	...	...
H_n	$\beta_{n,1}$	...	$\beta_{n,i}$	...	$\beta_{n,M}$
...	...	...	...	...	...
H_N	$\beta_{N,1}$	...	$\beta_{N,i}$	...	$\beta_{N,M}$

Define N distinctive values H_i as probable values of inflation, presented by decision makers.

$$H = \{H_1 H_2 \dots H_i \dots H_N\} \tag{3.3}$$

$\beta_{n,i}$ denotes the probable degree that nth decision maker allocates to the ith value of inflation. So $\beta_{n,i}$ is a belief degree that:

$$\sum_{n=1}^{N} \beta_{n,i} \leqslant 1, \quad \beta_{n,i} \geqslant 0. \tag{3.4}$$

The evaluation is called to be complete if:

$$\sum_{n=1}^{N} \beta_{n,i} = 1. \tag{3.5}$$

And it is called to be incomplete if:

$$\sum_{n=1}^{N} \beta_{n,i} < 1. \tag{3.6}$$

For example, decision maker D_n say that the rate of inflation is H_i by the belief degree of $\beta_{n,i}$ and so on each member gives a belief degree for each value of set H. Table 1 has been structured to demonstrate this fact.

The elements of the first column are probable values of inflation and the first row contains the members of decision maker team. $\beta_{n,i}$ denotes the probable degree that nth decision maker allocates to the ith value of inflation. Each member can express his numerical idea in any way, complete or incomplete. The team members can also say that they have no idea about someone. This condition causes an incomplete assessment, because in this condition the sum of the belief degrees for the member is not equal to 1. In cases which the sum of all columns is equal to 1, we have a complete assessment. Making decision in this condition is with a high uncertainty because it is difficult to use inflation in solving model with this structure. To solve this problem, a method based on Dempster-Shafer evidence theory [13] is proposed called Evidential Reasoning (ER) approach with aggregating these belief degrees with consideration the relative weights of the decision makers and the probable values of inflation The method is presented in the next section.

3.2. Evidential Reasoning method

Let $m_{n,i}$ be a basic probability mass, calculated as follows

$$m_{n,i} = \omega_i \beta_{n,i}. \tag{3.7}$$

Let $m_{H,i}$ be the remaining probability mass unassigned to each decision maker D_i, $m_{H,i}$ is calculated as follows:

$$m_{H,i} = 1 - \sum_{n=1}^{N} m_{n,i} = 1 - \omega_i \sum_{n=1}^{N} \beta_{n,i}. \tag{3.8}$$

Decompose $m_{H,i}$ into $\tilde{m}_{H,i}$ and $\bar{m}_{H,i}$ as follows:

$$\bar{m}_{H,i} = 1 - \omega_i \tag{3.9}$$

$$\tilde{m}_{H,i} = \omega_i \left(1 - \sum_{n=1}^{N} \beta_{n,i}\right). \tag{3.10}$$

with

$$m_{H,i} = \bar{m}_{H,i} + \tilde{m}_{H,i}. \tag{3.11}$$

The assessments of the decision makers that constitute the general property are aggregated to form a single assessment of the general property. The probability masses assigned to the various assessments as well as the probability mass left unassigned are denoted by:

$$m_{n,Q(L)}, (n = 1, \ldots, N), \bar{m}_{H,Q(L)}, \tilde{m}_{H,Q(L)}$$

and

$$m_{H,Q(L)}.$$

Let $Q(1) = 1$, then we get

$$m_{n,Q(1)} = m_{n,1}, (n = 1, \ldots, N), \bar{m}_{H,Q(1)} = \bar{m}_{H,1}, \tilde{m}_{H,Q(1)} = \tilde{m}_{H,1} \tag{3.12}$$

$$m_{H,Q(1)} = m_{H,1}. \tag{3.13}$$

The combined probability masses can be generated by aggregating all the probability assignments using the following recursive ER algorithm:

$$\begin{aligned} &\{H_n\}: \quad (n = 1, \ldots, N) \\ &m_{n,Q(i+1)} = v_{Q(i+1)} \left[m_{n,Q(i)} m_{n,i+1} + m_{H,Q(i)} m_{n,i+1} + m_{n,Q(i)} m_{H,i+1}\right], \\ &\quad n = 1, \ldots, N. \end{aligned} \tag{3.14}$$

We continue to let $i = 1$, which leads to that in (14)

$$\{H\}:$$

$$m_{H,Q(i+1)} = \bar{m}_{H,Q(i+1)} + \tilde{m}_{H,Q(i+1)} \tag{3.15}$$

$$\tilde{m}_{H,Q(i+1)} = v_{Q(i+1)} \left[\tilde{m}_{H,Q(i)} \tilde{m}_{H,i+1} + \bar{m}_{H,Q(i)} \tilde{m}_{H,i+1} + \tilde{m}_{H,Q(i)} \bar{m}_{H,i+1}\right] \tag{3.16}$$

$$\bar{m}_{H,Q(i+1)} = v_{Q(i+1)} \left[\bar{m}_{H,Q(i)} \bar{m}_{H,i+1}\right] \tag{3.17}$$

$$v_{Q(i+1)} = \left[1 - \sum_{s=1}^{N} \sum_{j=1, j\neq s}^{N} m_{s,I(i)} m_{j,i+1}\right]^{-1}, \quad i = \{1, 2, \ldots, M-1\} \tag{3.18}$$

$v_{Q(2)}$ as calculated by (3.18) is used to normalize $m_{n,Q(2)}, m_{H,Q(2)}$ so that

$$\sum_{n=1}^{N} m_{n,Q(2)} + m_{H,Q(2)} = 1. \tag{3.19}$$

Let β_n denote the combined degree of belief assessed to H_n which is generated by combining the assessments for all decision makers. β_n is then calculated by:

$$\{H_n\}:$$

$$\beta_n = \frac{m_{n,Q(M)}}{1 - \bar{m}_{H,Q(M)}}, (n = 1, \ldots, N) \tag{3.20}$$

$$\{H\}:$$

$$\beta_H = \frac{\tilde{m}_{H,Q(M)}}{1 - \bar{m}_{H,Q(M)}}. \tag{3.21}$$

Suppose some relative importance for the probable values of inflations called utility functions. For example, $u(H_n)$ $(n = 1, \ldots, N)$ is the relative importance of H_n which is a number between 0 and 1.

Now, the utilities of H_n $(n = 1, \ldots, N)$ are estimated *via* utility functions $u(H_n)$ Provided we have some belief β_H left unassigned in the assessments we can somewhat arbitrarily calculate an *utility interval* for the values of H_n being assessed. This interval is calculated as follows:

$$u_{\max} = (\beta_N + \beta_H)\, u\,(H_N) + \sum_{n=1}^{N-1} \beta_n u\,(H_n), \tag{3.22}$$

$$u_{\min} = (\beta_1 + \beta_H)\, u\,(H_1) + \sum_{n=2}^{N} \beta_n u\,(H_n)\,, \tag{3.23}$$

$$u_{\text{avg}} = \frac{u_{max} + u_{\min}}{2}\cdot \tag{3.24}$$

The inflation value after aggregating the decision makers' opinions by ER approach is conducted to become an interval number called $[u_{\min},\, u_{\max}]$.

Representing the inflation by an interval number and using the interval arithmetic, the objective function for profit is changed to corresponding multi objective functions. These functions are minimized and solved by NSGA-II Multi-objective Genetic Algorithm which is represented in Section 5.

Under the assumption of complete assessment, the utility of the values of H_n is calculated as a single point according to

$$u = \sum_{n=1}^{N} \beta_n u\,(H_n)\,. \tag{3.25}$$

But the purpose in this research is the incomplete assessment which deals with the interval inflation value.

The inventory model, considered in this paper is represented in the next section.

4. Model description and formulation

4.1. Description of the model

k $(0 < k < 1)$ is the proportion of time in any given inventory cycle which orders can be filled from the existing stock. The inventory level leads to zero from jT until $(j+1)T$ and we have shortages at time $(j+k)T$. During $[(j+k)T,\, (j+1)T]$, shortages level linearly increases by the demand rate and we do not have any deterioration. Shortages are accumulated until $(j+1)T$ before they are backordered. The replenishment cycle is the same for each period but because the demand is time dependent, the replenishment height is increasing by the time through the cycles. The optimal inventory policy minimizes the expected inventory cost. The time horizon is infinite.

4.2. The model formulation

The amount of deteriorated units during a given time interval depends on the on-hand inventory and the elapsed time in the system. Therefore, if $I_1\,(t)$ denotes the inventory level at any time t in $[0, kT]$ during the period of positive inventory, then the instantaneous state of $I_1(t)$ can be described by the following differential equation:

$$\frac{\mathrm{d}I_1\,(t)}{\mathrm{d}t} = \theta I_1\,(t) - f(t), \quad 0 \leqslant t \leqslant kT. \tag{4.1}$$

The differential equation governing the system during the period of shortage in $[kT, T]$ is given by:

$$\frac{\mathrm{d}I_2\,(t)}{dt} = -f(t), \quad 0 \leqslant t \leqslant (1-k)T. \tag{4.2}$$

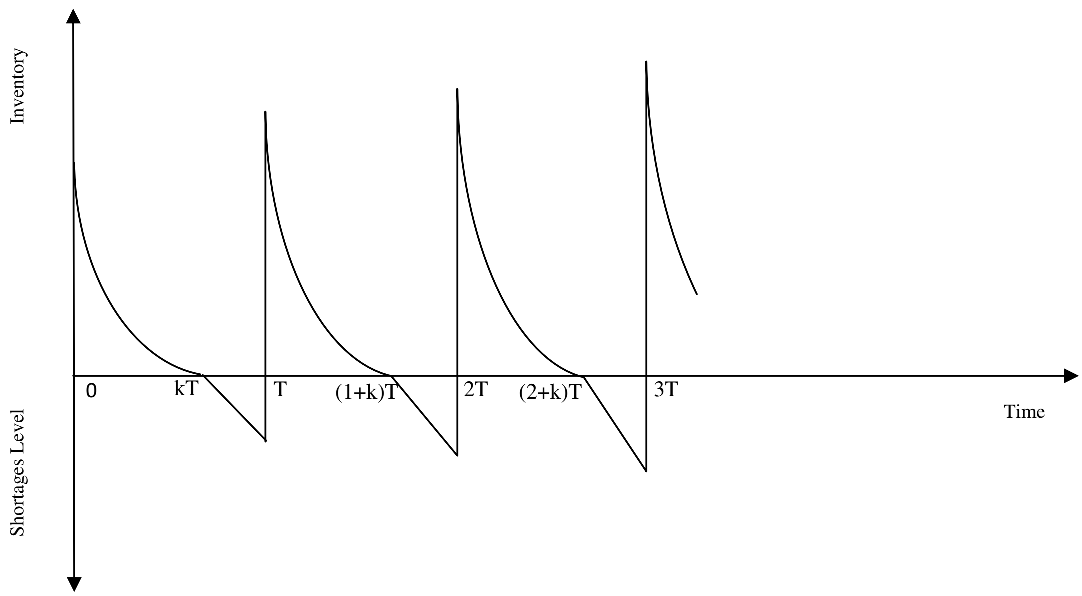

FIGURE 1. Graphical representation of the inventory system.

with the conditions $I_1(kT) = 0$ and $I_2(0) = 0$, The solutions of the differential equations (4.1) and (4.2) are given by:

$$I_1(t) = \int_t^{kT} \mathrm{e}^{\theta(w-t)} f(w)\mathrm{d}w \quad 0 \leqslant t \leqslant kT$$
$$I_2(t) = \int_0^{t} -f(w)\mathrm{d}w \quad 0 \leqslant t \leqslant (1-k)T.$$

The total inventory cost, consists of replenishment or ordering cost, holding cost, shortage cost, and purchase cost.

The present value of the ordering cost for the $(j+1)$th cycle $(j = 0, 1, 2, \ldots)$, is

$$DCR_j = A\mathrm{e}^{-j(r-u)T}. \tag{4.3}$$

Therefore, the present value of the total ordering cost is obtained by

$$DCR = A\sum_{j=0}^{\infty} \mathrm{e}^{-j(r-u)T} = A\frac{1}{1-\mathrm{e}^{-(r-u)T}}. \tag{4.4}$$

And in similar way we obtain the present value of the other costs for one cycle first, and subsequently the present value of the total costs.

If we call DCH_1 and DCS_1 as the present value of the holding cost and shortage cost for the first cycle, for all cycles we have

$$DCH = \sum_{j=0}^{\infty} DCH_1\mathrm{e}^{-j(r-u)T} = \frac{c_1\left[\int_0^{kT} I_1(t)\,\mathrm{e}^{-(r-u)t}dt\right]}{\left(1-\mathrm{e}^{-(r-u)T}\right)}, \tag{4.5}$$

$$DCS = \sum_{j=0}^{\infty} DCS_1\mathrm{e}^{-j(r-u)T} = \frac{-c_2\left[\int_0^{(1-k)T} I_2(t)\,\mathrm{e}^{-(r-u)t}dt\right]\mathrm{e}^{-(r-u)kT}}{\left(1-\mathrm{e}^{-(r-u)T}\right)}. \tag{4.6}$$

And similarly the present value of the purchase cost for all the cycles is

$$DCP = \frac{c\left(\int_{t}^{kT} \mathrm{e}^{\theta t} f\left(t\right) \mathrm{d}t + \left(\int_{kT}^{T} f\left(t\right)\mathrm{d}t\right) \mathrm{e}^{-(r-u)T}\right)}{1-\mathrm{e}^{-(r-u)T}}. \tag{4.7}$$

Hence, the present worth of the total variable cost of the inventory system is given by:

$$DC\left(k,T\right) = DCR\left(k,T\right) + DCP\left(k,T\right) + DCH\left(k,T\right) + DCS\left(k,T\right). \tag{4.8}$$

The objective is to determine the optimal values of T and k to minimize $DC\left(k,T\right)$

From the above formulations and the inflation assessment which was represented as an interval number in Section 3, the lower and the upper bounds called as $u_{\min}$ and $u_{\min}$ lead to better using NSGA-II Multi-objective Genetic Algorithm in order to better handling the lower and upper bounds of the cost function caused by the interval inflation [8].

5. Multi-objective genetic algorithm

In order to use NSGA-II Multi-objective Genetic Algorithm, two approaches are used called fast non-dominated sorting and crowding distance which are comprehensively and well described by Deb *et al.* [7]. The subsequent operators are as follows.

5.1. New population, parent selection and stopping criteria

After fast non-dominated sorting and crowding distance, we chose *(1-re)*N_{pop} members which are better than the others. *re* is the replacement rate which is one of the GA parameters that will be tuned later. These members are moved to new population. The remained members of population generate with selecting parents from this population, doing crossover and mutation on them. For selecting parents, Tournament Selection mechanism is used. Stopping criteria is maximum number of generation which is tuned later. In other word when the generation reaches to the maximum, the algorithm stops. For more information see Fattahi [9].

5.2. The chromosome presentation, crossover and mutation operators

A chromosome is a 2 dimensional vector containing the proportion of time (k) and the replenishment time interval (T) (see Fig. 2).

The crossover operator is presented in Figure 3.

For each one of the solutions according to the mutation probability, P_m, if the mutation is done, the new chromosome is replaced by the last one generated by making a random number in the limited bounds of the chromosome.

K	T

Figure 2. Chromosome presentation.

Parent1	K1	T1
Parent2	K2	T2

Child1	K1	T2
Child2	K2	T1

Figure 3. Crossover operator.

6. Tuning Parameters and numerical examples

In a meta-heuristic algorithm (a process thereafter), the parameters are controllable factors, the problem being solved is the process input, and the fitness function is the process output. Therefore, instead of employing suggested values by other researchers or using a trial and error procedure, it seems reasonable to adjust the parameters using statistical methods based on a set of experiments defined in the next subsection. The Taguchi method is employed to tune the parameters.

In the Taguchi method [29], the factors affecting the performance (response) of a process are divided into two categories: noise factors N such as weather condition that are not controlled, and controllable factors S like the parameters of a meta-heuristic algorithm. Taguchi first designed experiments using orthogonal arrays, and then presented a procedure to control N in order to reduce the variation or scatter around the target; in other words, the design that is impressed less by N is a robust design. There are two different ways of analyzing the results obtained by the Taguchi method. First, the standard approach (using analysis of variance) for experiments with only one replication. Second, the approach uses signal to noise ratio (S/N) for experiments with more than one replications. Since meta-heuristic algorithms do not present a unique result in different runs, more than one replication is required and hence the second approach has to be employed. In the S/N analysis, a condition is good if the quality characteristic (the fitness value in this research) impression of S is more than N, and the aim is to find a condition that optimizes S/N. The quality characteristics are basically classified in three groups. First, smaller is better for which the objective function is of a minimization type. Second, nominal is the best; for which the objective function has modest variance around its target. Third, bigger is better, where the objective function is of a maximization type.

Due to the minimization objective function in this research the S/N is defined respectively as [29]:

$$S/N = -10 \log \left(1/n \sum_{i=1}^{n} y_i^2 \right), \tag{6.1}$$

where n is the number of replications, y_i is the response in ith replication and $\bar{y}$ is the average response. For a good overview of the Taguchi method, interested readers are referred to [35].

6.1. Taguchi method implementation

In the Taguchi method, the parameters that may have significant effects on the process output are first selected for calibration. Then, using a trial and error procedure, the values that present good fitness function are selected to implement the experiments, mainly because the Taguchi compelled to utilize quality design at the beginning of production and not during it [29]. At the end, four levels are selected in this research for the values of parameters to be considered in the experiments. In other words, five parameters each with four levels are considered in the calibration process of GA. The parameters are shown in Table 2. The L^{16} from of the orthogonal arrays of the Taguchi method is utilized for experiments. Besides, since the problem is of a minimization type, the values for S/N s are calculated using equation (36) and are presented in the last column of Table 3.

Suppose the rate of parameters of the model for parameter tuning and the numerical examples as below:

$$A = 500, \quad c = 20, \quad r = 0.15, \quad c_1 = 3, \quad c_2 = 80, \theta = 0.3. \tag{6.2}$$

Table 2. Levels of GA parameters.

	Npop	*Pc*	*Pm*	*Gen*	*Re*
Level 1	30	0.65	0.05	100	0.5
Level 2	40	0.7	0.1	450	0.6
Level 3	50	0.8	0.2	500	0.7
Level 4	100	0.9	0.25	700	0.8

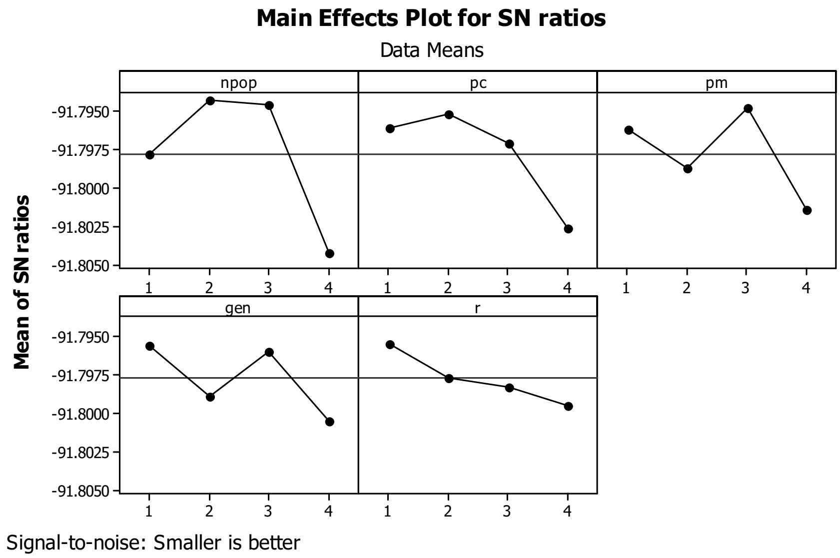

FIGURE 4. The mean S/N plot for different levels of the GA parameters.

TABLE 3. Tuning the parameters.

Number	*Npop*	*Pc*	*Pm*	*Gen*	Re	Average fitness of GA			S/N
						DC_1	DC_2	DC_3	
1	1	1	1	1	1	19419	28958.6	57574	−91.7902
2	1	2	2	2	2	19434	28979	57623	−91.7973
3	1	3	3	3	3	19421	28964.3	57595.6	−91.793
4	1	4	4	4	4	19423	28964.6	57756.6	−91.8108
5	2	1	2	3	4	19423.1	28966.8	57599.3	−91.7936
6	2	2	1	4	3	19425.1	28967.5	57597.1	−91.7935
7	2	3	4	1	2	19414	28976	57610.8	−91.7951
8	2	4	3	2	1	19428.6	28972.8	57608.3	−91.7952
9	3	1	3	4	2	19420.1	28962.3	57593.5	−91.7926
10	3	2	4	3	1	19449.4	28951.6	57581.3	−91.7918
11	3	3	1	2	4	19426.8	28983.1	57605.5	−91.7953
12	3	4	2	1	3	19444.5	28977.8	57633.1	−91.7988
13	4	1	4	2	3	19496.5	28991.1	57687.1	−91.8078
14	4	2	3	1	4	19465.5	28977.3	57662.6	−91.7983
15	4	3	2	4	1	19491	28991.5	57666.3	−91.8049
16	4	4	1	3	2	19497.6	28968.3	57683.6	−91.8058

And suppose the inflation rate after using ER approach be [0.07, 0.08] for the parameter tuning example. According to the Taguchi method we have 16 orthogonal arrays for the replacements of the GA parameters levels here. For each array which are constructed the rows of Table 3, the algorithm is run 6 times and the average of them is written. Note that three values of costs are obtained called DC_1, DC_2, DC_3 because of the multi objective cost function. The numbers from the second to the sixth columns of Table 3 are the parameters levels shown in Table 2. This construction of the level numbers is L^{16} of the orthogonal arrays of the Taguchi method. The average six runs of genetic algorithm considering the mentioned parameters, is represented in Table 3 in order to choose the optimal values of GA parameters from S/N ratios. These values are then depicted in Figure 4, based on which the best levels of all the parameters are then chosen.

Table 4. Experimental results of numerical examples considering inflation rate interval with little distance.

Number	Inflation	k	T	Average fitness of GA				Optimal solution			
				DC_1	DC_2	DC_3	CPU time(s)	DC_1	DC_2	DC_3	CPU time(s)
1	[0.06,0.07]	0.77	0.84	17809.5	18833	19984.6	122.6	17739.6	18759.3	19906.3	243.1
2	[0.07,0.08]	0.77	0.8	19943.8	21244.6	22733.3	106.5	19906.3	21206.4	22691.9	240
3	[0.08,0.09]	0.79	0.78	22759.5	24463.9	26477.2	107.9	22691.9	24405.7	26405.2	243.2
4	[0.09,0.1]	0.79	0.82	26487.3	28855.8	31710.5	104.2	26405.2	28768.1	31603.5	245
5	[0.1,0.11]	0.79	0.81	31758.2	35239.3	39590.7	101.7	31603.5	35068.8	39400.4	246
6	[0.11,0.12]	0.77	0.81	39490	45081.9	52516.5	99.6	39400.4	44969.4	52394.6	243.1
7	[0.12,0.13]	0.78	0.81	52469.1	62879.6	78495.5	99.5	52394.6	62789.6	78381.9	246.3
8	[0.13,0.14]	0.79	0.8	78906.1	105051.2	157400	113.1	78381.9	104368	156341	243.1

Table 5. Experimental results of numerical examples considering inflation rate interval with large distance.

Number	r	Inflation	k	T	Average fitness of GA				Optimal solution			
					DC_1	DC_2	DC_3	CPU time(s)	DC_1	DC_2	DC_3	CPU time(s)
1	0.15	[0.06,0.12]	0.76	0.84	17778.7	26436.1	52456.2	129	17739.6	26405.2	52394.6	245
2	0.15	[0.01,0.1]	0.77	0.79	11577.3	16847.5	31619.6	122.8	11546.9	16827.2	31604.4	245.1
3	0.3	[0.02,0.2]	0.79	0.78	6015.12	8672.7	16123.9	108.9	5965.4	8611.2	16006	245.1
4	0.4	[0.05,0.3]	0.79	0.82	4883.30	7358.6	16045.1	109.2	4845.4	7332.8	16006	243.1
5	0.2	[0.05,0.15]	0.79	0.8	10834.4	16052.6	31724.4	111.8	10803.4	16006	31604.4	243.8
6	0.5	[0.1,0.45]	0.77	0.81	4398.8	7480.2	31618.1	123.8	4285.1	7331.4	31604.4	243
7	0.16	[0.06,0.14]	0.78	0.81	16021.1	26459.1	78536.1	119.4	16006	26405.6	78388	245.1
8	0.23	[0.05,0.2]	0.79	0.78	9137.6	15367.9	52772.7	134.6	9068.1	15263	52397.8	243.5

Then the optimal values of parameters are:

$$Npop^* = 40, \quad Pc^* = 0.7, \quad Pm^* = 0.2, \quad Gen^* = 100, \quad re^* = 0.5$$

It should be noted that the best levels of the parameters are problem dependent and change when different problems with different sizes are considered.

For running all the test instances, we used MATLAB software version 2009a in a PC with 2.2 GHz Intel Core 5 CPU, and 4 GB of RAM memory. Besides, Figure 4 and results of Table 3 are obtained using Minitab 16.

The experimental results of average of ten runs, for 16 examples with assuming different inflations, are illustrated in Tables 4 and 5. After determining the GA parameters now we run algorithm with these fixed parameters and the other parameters of the model which were defined before, with assuming several values of interval inflation through the examples. The algorithm is run 10 times for each example and the average cost function and the average of k and T are written as the best solution of variables. Furthermore, in order to have the best choice between the classic methods with the meta- heuristic one, the optimal solutions are also given in the tables. The optimal solutions are obtained by using the first, middle and the last point of the inflation interval for each time of running using Maple software version 12. Moreover, the CPU times are represented for both methods.

The numerical examples are categorized into two Tables. For the examples of Table 4, the first and the last values of interval have a little distance and for the other examples distances are large. In all examples in Tables 4 and 5 inflations are different. The other parameters were defined previously.

In examples of Table 5, according to the last value of interval, the rate of r is changed along the examples.

The results of CPU times in Tables 4 and 5 illustrate that GA is better than classic models to use for this kinds of problems. This difference between CPU times becomes very significant in times of doing a sensitive analysis over a wide range of parameters. Furthermore, the fitness comparison between the first and the second method shows the good efficiency of the proposed GA because the solutions are so near to the optimal solutions.

7. Conclusion

In this research inflation is considered as a belief structure in order to its comprehensiveness of uncertainty in times of lacking information. A method based on evidence theory was presented to convert the incomplete data of inflation to an interval and somehow decrease its uncertainty. Because of appearing the interval parameters during the mathematical model development, a meta-heuristic genetic algorithm was developed to solve the model. The experimental results of running the algorithm for numerical examples have been compared with the classical optimal solution. The results show that the proposed GA takes less time than the classical model in solving the problem. This difference of times is more significant when we want to do a sensitivity analysis in a wide range of parameters.

References

[1] M. Ameli, A. Mirzazadeh and M. Akbarpour Shirazi, Entropic Economic Order Quantity Model for Items with Imperfect Quality Considering Constant Rate of Deterioration under Fuzzy Inflationary Conditions. *Int. J. Ind. Eng. Prod. Res.* **24** (2013) 91–99.

[2] Z.T. Balkhi, Optimal economic ordering policy with deteriorating items under different supplier trade credits for finite horizon case. *Int. J. Prod. Econ.* **133** (2011) 216–223.

[3] J.A. Buzacott, Economic Order Quantities with Inflation. *Oper. Res. Quart.* **26** (1975) 553–558.

[4] K.J. Chung and S.F. Tsai, Inventory systems for deteriorating items with shortages and a linear trend in demand-taking account of time value. *Comput. Oper. Res.* **28** (2001) 915–934.

[5] R.P. Covert, G.C. Philip, An EOQ model for items with Weibull distribution deterioration. *AIIE Trans.* **5** (1973) 323–326.

[6] T.K. Datta and A.K. Pal, Effects of inflation and time value of money on an inventory model with linear time dependent demand rate and shortages. *Eur. J. Oper. Res.* **52** (1991) 326–333.

[7] K. Deb, S. Agrawal, A. Pratap and T. Meyarivan, A Fast Elitist Non-Dominated Sorting Genetic Algorithm for Multi-Objective Optimization: NSGA-II. Indian Institute of Technology Kanpur, Kanpur, PIN 208016, India (2001).

[8] J.K. Dey, S.K. Mondal and M. Manoranjan, Two storage inventory problem with dynamic demand and interval valued lead-time over finite time horizon under inflation and time-value of money. *Eur. J. Oper. Res.* **185** (2008) 170–194

[9] P. Fattahi, Meta heuristic algorithms. Hamedan, Bu Ali Sina University published, Iran (2009).

[10] P.M. Ghare, G.H. Schrader, A model for exponentially decaying inventory system. *Int. J. Prod. Res.* **21** (1963) 449–460.

[11] M. Ghoreishi, A. Arshsadi khamseh, and A. Mirzazadeh, Joint Optimal Pricing Inventory Control for Deteriorating Items under Inflation and Customer Returns. *Hindawi Publishing Corporation: J. Indust. Eng.* **2013** (2013) 709083.

[12] S.K. Goyal and B.C. Giri, Recent trends in modeling of deteriorating inventory. *Eur. J. Oper. Res.* **134** (2001) 1–16.

[13] P. Gustafsson, R. Lagerström, P. Nrman and M. Simonsson, The ICS Dempster-Shafer How To. Department of Industrial Information and Control Systems Royal Institute of Technology (KTH), SE-100 44 Stockholm, Sweden.

[14] M. Hariga, Effects of inflation and time value of money on an inventory model with time dependent demand rate and shortages. *Eur. J. Oper. Res.* **81** (1995) 512–520.

[15] M. Hariga and M. Ben-Daya, Optimal time varying lot-sizing models under inflationary conditions. *Eur. J. Oper. Res.* **89** (1996) 313–325.

[16] D.K. Jana, B. Das and T.K. Roy, A Partial Backlogging Inventory Model for Deteriorating Item under Fuzzy Inflation and Discounting over Random Planning Horizon: A Fuzzy Genetic Algorithm Approach. *Hindawi Publishing Corporation: Adv. Oper. Res.* **2013** (2013) 973125.

[17] S.T. Lo, H.M. Wee and W.C. Huang, An Integrated Production-inventory Model with Imperfect Production Processes and Weibull Distribution Deterioration Under Inflation. *Int. J. Prod. Econ.* **106** (2007) 248–260.

[18] M.K. Maiti, A fuzzy genetic algorithm with varying population size to solve an inventory model with credit-linked promotional demand in an imprecise planning horizon. *Eur. J. Oper. Res.* **213** (2011) 96–106.

[19] M. Mondala, A.K. Maity, M.K. Maiti, and M. Maiti, A productionrepairing inventory model with fuzzy rough coefficients under infiation and time value of money. *Appl. Math. Model.* **37** (2013) 3200–3215.

[20] I. Moon, B.C. Giri, B. Ko, Economic order quantity models for ameliorating/deteriorating items under inflation and time discounting. *Eur. J. Oper. Res.* **162** (2005) 773–785.

[21] S.M. Mousavi, V. Hajipour, S.T. Akhavan Niaki and N. Alikar, Optimizing multi-item multi-period inventory control system with discounted cash flow and inflation: Two calibrated meta-heuristic algorithms. *Appl. Math. Model.* **37** (2013) 2241–2256.

[22] A. Mirzazadeh, Effects of Uncertain Inflationary Conditions on an Inventory Model for Deteriorating Items with Shortages. *J. Appl. Sci.* **10** (2010) 2805–2813.

[23] A. Mirzazadeh, Effects of Variable Inflationary Conditions on an Inventory Model with Inflation-Proportional Demand Rate. *J. Appl. Sci.* **10** (2010) 551–557.

[24] A. Mirzazadeh, A Comparison of the Mathematical Modeling Methods in the Inventory Systems under Uncertain Conditions. *Int. J. Eng. Sci. Technol.* **3** (2011) 6131–6142.

[25] A. Mirzazadeh, Inventory management under stochastic conditions with multiple objectives. *Artif. Intell. Res.* **2** (2013) 16–26.

[26] A. Mirzazadeh and A.R. Sarfaraz, Constrained Multiple Items Optimal Order Policy under Stochastic Inflationary Conditions. *Second Annual International Conference on Industrial Engineering Application and Practice*. USA, San Diego (1997) 725730.

[27] R.B. Misra, A note on optimal inventory management under inflation. *Naval Res. Logist. Quart.* **26** (1979) 161–165.

[28] A.K. Neetu and A. Tomer, Deteriorating Inventory Model Under Variable Inflation When Supplier Credits Linked to Order Quantity. *Proc. Eng.* **38** (2012) 1241–1263.

[29] R. Roy, *A Primer on the Taguchi Method*. Society of Manufacturing Engineers, New York, USA (1990).

[30] R. Roy, S. Pal and M.K. Maiti, A production inventory model with stock dependent demand incorporating learning and inflationary effect in a random planning horizon: A fuzzy genetic algorithm with varying population size approach. *Comput. Ind. Eng.* **57** (2009) 1324–1335.

[31] S.S. Sana, Demand influenced by enterprises' initiatives – A multi-item EOQ model of deteriorating and ameliorating items. *Math. Comput. Model.* **52** (2010) 284–302.

[32] B.R. Sarker and H. Pan, Effects of inflation and the time value of money on order quantity and allowable shortages. *Int. J. Prod. Econ.* **34** (1994) 65–72.

[33] B.R. Sarker, A.M.M. Jamal and S. Wang, Supply chain models for perishable products under inflation and permissible delay in payment. *Comput. Oper. Res.* **27** (2000) 59–75.

[34] C.T. Su, L.I. Tong and H.C. Liao, An inventory model under inflation for stock dependent consumption rate and exponential decay. *Oper. Res.* **33** (1996) 71–82.

[35] G. Taguchi, S. Chowdhury and Y. Wu, *Taguchi's Quality Engineering Handbook*. John Wiley & Sons, Inc., New Jersey, USA (2005).

[36] H.L. Yang, Two-warehouse inventory models for deteriorating items with shortages under inflation. *Eur. J. Oper. Res.*. **157** (2004) 344–356.

[37] H.L. Yang, Two-warehouse partial backlogging inventory models with three-parameter Weibull distribution deterioration under inflation. *Int. J. Prod. Econ.* **138** (2012) 107–116.

[38] H.L. Yang and C.T. Chang, A two-warehouse partial backlogging inventory model for deteriorating items with permissible delay in payment under inflation. *Appl. Math. Model.* **37** (2013) 2717–2726.

[39] H.M. Wee and S.T. Law, Replenishment and pricing policy for deteriorating items taking into account the time-value of money. *Int. J. Prod. Econ.* **71** (2001) 213–220.