RAIRO-Oper. Res. 50 (2016) 891–907
DOI: 10.1051/ro/2015032

RAIRO Operations Research
www.rairo-ro.org

N-POLICY FOR A REPAIRABLE REDUNDANT MACHINING SYSTEM WITH CONTROLLED RATES

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Abstract. The present investigation deals with a multi-component machining system operating under N-policy. There is a provision of k type of mixed standbys units and maintenance crew consisting of permanent repairman as well as removable additional repairmen. The life time and repair time of the units are exponentially distributed with interdependent rates. The permanent repairman can start the repair only when N failed units are accumulated in the system. As soon as a unit fails, it is replaced by an available standby unit. In case when all the standby units are exhausted, the machining system starts to function in a degraded mode due to overload. Markov model is developed by constructing the governing transient state equations which are solved by using Runge−Kutta method. Various performance measures viz. queue size distribution, expected number of failed units, cost function, *etc.* are evaluated. By taking numerical illustration, numerical results are computed and also depicted with the help of graphs and figures.

Mathematics Subject Classification. 90B22, 90B25, 60K25, 60K30, 68M20.

Received February 23, 2015. Accepted September 6, 2015.

1. Introduction

In the modern era of advanced technology, the machining systems have become more complex and are always prone to failure. The spare part support system as well as an efficient repair facility are required for the smooth functioning of such system. In the present investigation we are concerned with the performance analysis of a machine repair problem having mixed standbys, one permanent and r additional removable repairmen to support the functioning of the system in an optimal manner. In order to reduce the cost of maintenance, the repair is initiated by the permanent repairman only when N failed units are accumulated in the system to form the queue.

The machine repair problems (MRPs) with spares have always been dragging the interest of many researchers; for recent survey on the topic MRP we refer the articles by Haque *et al.* [2] and Jain *et al.* [14]. The machine repair problem with k type of warm spares and multiple vacations of the server was studied by Maheshwari

Keywords. Markovian machine repair model, N-policy, mixed standbys, additional repairmen, interdependent rates.

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Article published by EDP Sciences

et al. [24]. Jain [4] studied an embedded machining system having mixed standbys under the priority concept and compared some of the performance measures evaluating by both analytical as well as neuro-fuzzy inference approaches. Sometimes, it is seen that the switch over process fails while standby unit takes the place of a failed operating unit. Thus the incorporation of switching failure has become an important factor to be considered while dealing with the performance modelling of MRP with standby support. Recently, some researchers studied machining systems with standby facility by including the switching failure concept [9,15,22]. Jain and Preeti [8] established the cost function for a redundant machine repair problem in which the server is unreliable and takes working vacation when there is no load of failed units. Very recently, the machine repair problem with multiple vacations has been analysed by Liou [23]. The performance of machine repair system with hot standbys has been studied by Shree *et al.* [33] by incorporating the partial server vacation.

N-policy is the optimal policy to control the repair process. According to N-policy, the repair starts only after the accumulation of N failed units in the system and continues till there is no failed unit in the system. Many researchers have worked on the performance analysis of queueing systems operating under N-policy. Some prominent works on the machine repair problems are present in queueing literature [6,25,29,35]. Jain and Upadhyaya [10] studied a multi-component machine repair system having heterogeneous repairmen working under the N-policy. A machine repair system under N-policy was investigated by Parthasarathy and Sudhesh [28]. Kempa [19,20] gave the transient state solution for a queueing system operating under N-policy by incorporating the busy period and multiple vacations, respectively. Recently, multi-component redundant machine repair problems under N-policy have also been investigated by some researchers [16, 21, 38]. Very recently, Yang [37] studied a queueing system with N-policy and working vacation and developed a cost minimisation model for it. Wu [36] developed an optimisation model for N-policy K-out-of-N: G machine repair system in which the server is prone to breakdown and also goes for multiple vacations in case when the system becomes empty.

The provision of additional repairmen as and when required is an effort to improve the quality of service of any organization as it will help in reducing the workload of the primary repairman as well as delay experienced by the customers. A few papers have appeared on machine repair problems with additional removable repairmen [3, 7, 32]. Jain *et al.* [13] investigated a multi component repairable system with mixed standbys having state dependent rates and the additional removable repairmen. Recently, Jain [5] developed the transient state Markov model of a machine repair system with standbys and provision of additional repairmen along with permanent repairmen and obtained various performance indices.

In the past, some papers on the performance analysis of queueing systems having controlled rates have appeared [1, 11, 26, 27, 30, 31, 34]. But only a very few papers are related to the queueing models of machine repair problem with controlled rates. A controllable and interdependent MRP with additional repairman and mixed standbys was investigated by Jain *et al.* [12]. Recently, Jain *et al.* [17] studied the M/M/R + r machining system having controlled rates. In this investigation, they incorporated the controlled rates for the Markov modeling of a machine repair system, operating under N-policy and supported by additional repairman apart from regular repairman. Further, Jain *et al.* [18] investigated a multi component machine repair problem with controllable failure and repair rates and provided the comparison of numerical results obtained by product type technique with those results obtained by neuro-fuzzy technique.

In the present investigation, we present the transient analysis of a machine repair system with interdependent rates and supported by a repair facility and k types of mixed standby units. The repair facility consists of one permanent repairman who is made available to the system as per N-policy and multi-additional repairmen who turn on and turn off depending upon workload of failed units in the system. A practical application of the model developed in the present investigation can be found at the billing section of a super market in a shopping mall. There may be M cash counters having cash counting machines which should be kept in working order. As these counting machines are prone to failure, the supermarket organiser has the inventory of two types of standby counting machines each group having S_1 and S_2 units. As soon as a machine fails, the standby machine gets activated and put in place of the failed machine and the failed machine is immediately attended by the repairman who is appointed on permanent basis to take care of failed machines. However to cut down the cost of maintenance, the permanent repairman turns on (off) according to N-policy, *i.e.* start the repair job only when some machines are accumulated in the system. The system manager of the supermarket also has the facility of a few extra (*i.e.* additional repairmen) service engineers on contract basis and those can be called upon one by one to repair the failed units according to threshold policy depending upon the workload of failed machines. In the starting, the permanent repairman is not activated and the failed machines have to wait for its repair till N machines are failed. When the first type of standby machines have utilised, the permanent repairman starts the repair job at a faster rate in the anticipation that the system will not degrade in case when other type of standbys are exhausted in between. To cope up with the increased load, the first additional repairman gets activated at a pre-specified threshold level and starts the repairing work. As soon as the second type of standby machines are exhausted, the second additional repairman is also called upon. The additional removable repairmen are withdrawn at the same threshold level at which they were activated. The failure and repair processes may be interdependent. With the facility of standby machines and provision of additional repairmen apart from permanent repairman, billing section will work smoothly in spite of failure prone cash counting machines. The remaining paper is organized as follows. The mathematical model of the machine repair problem having the provision of mixed standby units and facility of repair, has been described in the next Section 2. Chapman−Kolmogorov equations governing Markov model are given in Section 3. In Section 4, numerical technique based on Runge−Kutta method is employed for the computation of transient state probabilities of the system states, which are further used to evaluate various performance indices. To understand the usefulness and practicality of the model, numerical results and sensitivity analysis of some performance indices have been facilitated in Section 5. The conclusion has been drawn in the last Section 6.

2. The model description

The present study is devoted to the performance modeling of MRP by incorporating many realistic features such as mixed standbys, additional removable repairmen and controllable rates to develop Markov model and solving it for the transient state probabilities. The assumptions used to develop the model are as follows:

▶ The system starts operating in normal mode with M operating units which are required for the system to function smoothly. As soon as an operating unit fails, it is replaced with negligible switching time by the available warm standby units. There are S_i standby units of ith$(1 \leq i \leq k)$ type available in the system. The different types of standby units differ in their failure characteristics. When a spare unit is put into operation in the system to take place of failed operating unit, its failure characteristic is same as that of the operating unit.

▶ The operating units and ith$(1 \leq i \leq k)$ type of standby units may fail according to a Poisson process with rate λ and $\alpha_i(1 \leq i \leq k)$. After all the spare units are being used, due to increased load on each operating units, it fails with a degraded failure rate λ_d.

▶ If a unit fails, it is immediately sent for repair; the repair times of the operating as well as the standby units are assumed to be exponentially distributed.

▶ If the number of operating units become lesser than M but more than m $(m < M)$ then the system will start working in short mode following the (m, M) policy. As soon as there is less than m operating units in the system, the system breaks down and becomes inoperable till again after repairing, there are m operating units in the system.

▶ After repairing, the failed unit is as good as a new unit. The repaired unit is put into operation if there are less than M operating units; otherwise the repaired unit is kept in the inventory with the other spare units.

▶ Unless and until N operating units are failed and accumulated in the system to form the queue, no repair is provided. When the queue size of the failed units reaches to N, the repair process is started by the

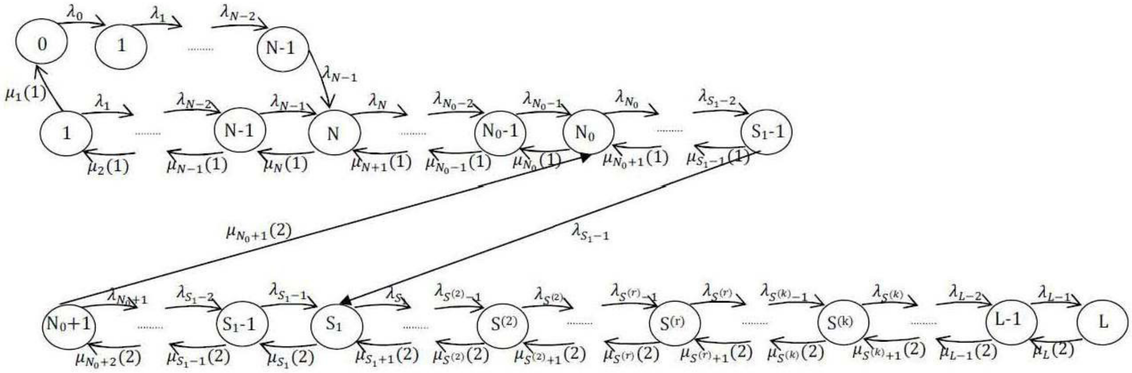

Figure 1. State transition diagram.

permanent repairman.

► The permanent repairman repairs the failed units according to N-policy *i.e.* starts the repair job with rate μ in normal mode when N units are accumulated in the system and continues the repair jobs till when there are no failed units in the system. When the first type of standbys (S_1) are exhausted, to cope up with the workload, the permanent repairman starts to repair the failed units at a faster rate μ_f which is continued till the system queue size ceases to threshold level N_0. At this stage, the first additional removable repairman also gets activated and starts the repairing of failed units waiting in the queue with the rate μ_1. As soon as, the first type of standby units are exhausted and some more operating units fail, the second type of standby units are used to replace the failed units in the system and at this stage second additional repairman gets activated. This process goes on till all ($r \le k$) repairmen become busy. Thus, jth$(2 \le j \le r)$ additional removable repairman will be activated as soon as standby units of ith type $(2 \le i \le r)$ are exhausted. Then after, all r additional repairmen remain activated till the system fails in case when the number of operating units drops below m. The jth$(1 \le j \le r)$ additional repairman is removed at the level of workload of failed machine where it was introduced.

► The permanent as well as additional removable repairmen take the failed units for repair according to *First Come First Served* (*FCFS*) rule.

► The mean dependence rate between the failure and the repair rates is ε which is the covariance between the failure and repair processes. The failure and repair processes are assumed to be inter-dependent and are governed by the bi-variate Poisson distribution.

Let the random variable representing the different system states at time t be defined as follows:

$$\xi(t) = \begin{cases} l = 0; \text{ Permanent repairman is idle} \\ l = 1; \text{ Permanent repairman is repairing the failed units} \\ l = 2; \text{ Permanent repairman is working in faster mode and additional repairman is activated} \\ \qquad \text{as per the threshold policy.} \end{cases}$$

Let $'n'$ denotes the number of failed units in the system; the transition rates λ_n and μ_n corresponding to failure and repair processes respectively, are depicted in Figure 1. Now, the state dependent transition rates are defined as

$$\lambda_n = \begin{cases} M(\lambda-\varepsilon)+(S_1-n)(\alpha_1-\varepsilon)+\sum_{j=2}^{k} S_j(\alpha_j-\varepsilon); & 0 \le n < S_1 \\ M(\lambda-\varepsilon)+(S^{(i+1)}-n)(\alpha_{(i+1)}-\varepsilon)+\sum_{j=i+2}^{k} S_j(\alpha_j-\varepsilon); & S^{(i)} \le n < S^{(i+1)},\ \ 1 \le i < k \\ (M+S^{(k)}-n)(\lambda_d-\varepsilon); & S^{(k)} \le n < L \end{cases}$$

$$\mu_n(l) = \begin{cases} 0; & 0 \le n < N, l=0 \\ (\mu-\varepsilon); & 1 \le n \le S_1-1, l=1 \\ (\mu_f-\varepsilon)+(\mu_1-\varepsilon); & N_0+1 \le n \le S^{(2)}-1, l=2 \\ (\mu_f-\varepsilon)+\sum_{j=1}^{i}(\mu_j-\varepsilon); & S^{(i)} \le n \le S^{(i+1)}-1, 2 \le i < r, l=2 \\ (\mu_f-\varepsilon)+\sum_{j=1}^{r}(\mu_j-\varepsilon); & S^{(r)} \le n \le L, l=2 \end{cases}$$

where $S^{(i)} = \sum_{(j=1)}^{i} S_j$ and $L = M + S^{(k)} - m + 1$.

The bi-variate Poisson process of the failure and repair processes has the joint probability mass function of the form: $P(X_1 = x_1, X_2 = x_2; t) = \mathrm{e}^{-(\lambda_n+\mu_n-\varepsilon)t} \sum_{j=0}^{\min(x_1,x_2)} \frac{(\varepsilon t)^j [(\lambda_n-\varepsilon)t]^{(x_1-j)} [(\mu_n-\varepsilon)t]^{(x_2-j)}}{j!(x_1-\varepsilon)!(x_2-\varepsilon)!}$ such that $x_1, x_2 = 0, 1, 2, \ldots; 0 < \lambda_n, \mu_n; \lambda_n > 0$ & $\mu_n > 0$; and $0 < \varepsilon < \min(\lambda_n, \mu_n)$ where $\lambda_n (n = 0, 1, 2, \ldots, L)$ and $\mu_n (n = 1, 2, 3, \ldots, L)$ are the inter-dependent failure and repair rates, respectively and ε is mean dependence rate between failure rate and repair rate.

Let $N(t)$ be number of failed units in the system at time t. Then the ordered pair $\{N(t), \xi(t)\}$ describes the Markov process in continous time t. Now, we define the transient probabilities $P_{i,n}(t) = Prob\{\xi(t) = i, N(t) = n\}$ for various states of Markov chain.

3. Transient state equations

In this section, we formulate Chapman Kolmogorov equations for the system states by using the appropriate transition rates as shown in Figure 1.

(i) ***Level l = 0: When the permanent repairman is idle:***
At this level, the failed units are not repaired as the permanent repairman is in the accumulation (*i.e.* idle) state, and remains idle till the number of failed units becomes N. By equating the out-flow from the state $(0, n), 0 \le n \le N-1$ having probability $P_{0,n}(t)$ with the in-flow from the neighbouring state, the equations are constructed as:

$$\frac{\mathrm{d}}{\mathrm{d}t} P_{0,0}(t) = -\left\{ M(\lambda-\varepsilon) + \sum_{j=1}^{k} S_j(\alpha_j-\varepsilon) \right\} P_{0,0}(t) + (\mu-\varepsilon) P_{1,1}(t) \tag{3.1}$$

$$\begin{aligned} \frac{\mathrm{d}}{\mathrm{d}t} P_{0,n}(t) = &-\left\{ M(\lambda-\varepsilon) + (S_1-n)(\alpha_1-\varepsilon) + \sum_{j=2}^{k} S_j(\alpha_j-\varepsilon) \right\} P_{0,n}(t) \\ &+\left\{ M(\lambda-\varepsilon) + (S_1-n+1)(\alpha-\varepsilon) + \sum_{j=2}^{k} S_j(\alpha_j-\varepsilon) \right\} P_{0,n-1}(t); 1 \le n \le N-1. \end{aligned} \tag{3.2}$$

(ii) ***Level l = 1: When the permanent repairman gets activated and is busy in the repair job:***
As soon as the number of failed units in the queue reaches N, the permanent repairman starts repair job with normal rate and continues till the first type of standbys are finished, then after permanent repairman switches

over to faster rate. In this case, we construct the transient state equations for states (1, n) by balancing the in-flows and out-flows between the states (0, n), (1, n) and (2, n) as follows:

$$\frac{\mathrm{d}}{\mathrm{d}t}P_{1,1}(t) = -\left\{M(\lambda-\varepsilon)+(S_1-1)(\alpha_1-\varepsilon)+\sum_{j=2}^{k}S_j(\alpha_j-\varepsilon)+(\mu-\varepsilon)\right\}P_{1,1}(t)+(\mu-\varepsilon)P_{1,2}(t) \tag{3.3}$$

$$\begin{aligned}\frac{\mathrm{d}}{\mathrm{d}t}P_{1,n}(t) = &-\left\{M(\lambda-\varepsilon)+(S_1-n)(\alpha_1-\varepsilon)+\sum_{j=2}^{k}S_j(\alpha_j-\varepsilon)+(\mu-\varepsilon)\right\}P_{1,n}(t)+(\mu-\varepsilon)P_{1,n+1}(t)\\ &+\left\{M(\lambda-\varepsilon)+(S_1-n+1)(\alpha_1-\varepsilon)+\sum_{j=2}^{k}S_j(\alpha_j-\varepsilon)\right\}P_{1,n-1}(t);2\leq n\leq N-1\end{aligned} \tag{3.4}$$

$$\begin{aligned}\frac{\mathrm{d}}{\mathrm{d}t}P_{1,N}(t) = &-\left\{M(\lambda-\varepsilon)+(S_1-N)(\alpha_1-\varepsilon)+\sum_{j=2}^{k}S_j(\alpha_j-\varepsilon)+(\mu-\varepsilon)\right\}P_{1,N}(t)+(\mu-\varepsilon)P_{1,N+1}(t)\\ &+\left\{M(\lambda-\varepsilon)+(S_1-N+1)(\alpha_1-\varepsilon)+\sum_{j=2}^{k}S_j(\alpha_j-\varepsilon)\right\}P_{1,N-1}(t)\\ &+\left\{M(\lambda-\varepsilon)+(S_1-N+1)(\alpha_1-\varepsilon)+\sum_{j=2}^{k}S_j(\alpha_j-\varepsilon)\right\}P_{0,N-1}(t)\end{aligned} \tag{3.5}$$

$$\begin{aligned}\frac{\mathrm{d}}{\mathrm{d}t}P_{1,n}(t) = &-\left\{M(\lambda-\varepsilon)+(S_1-n)(\alpha_1-\varepsilon)+\sum_{j=2}^{k}S_j(\alpha_j-\varepsilon)+(\mu-\varepsilon)\right\}P_{1,n}(t)+(\mu-\varepsilon)P_{1,n+1}(t)\\ &+\left\{M(\lambda-\varepsilon)+(S_1-n+1)(\alpha_1-\varepsilon)+\sum_{j=2}^{k}S_j(\alpha_j-\varepsilon)\right\}P_{1,n-1}(t);\qquad N+1\leq n\leq N_0-1\end{aligned} \tag{3.6}$$

$$\begin{aligned}\frac{\mathrm{d}}{\mathrm{d}t}P_{1,N_0}(t) = &-\left\{M(\lambda-\varepsilon)+(S_1-N_0)(\alpha_1-\varepsilon)+\sum_{j=2}^{k}S_j(\alpha_j-\varepsilon)+(\mu-\varepsilon)\right\}P_{1,N_0}(t)+(\mu-\varepsilon)P_{1,N_0+1}(t)\\ &+\left\{M(\lambda-\varepsilon)+(S_1-N_0+1)(\alpha_1-\varepsilon)+\sum_{j=2}^{k}S_j(\alpha_j-\varepsilon)\right\}P_{1,N_0-1}(t)\\ &+\{(\mu_f-\varepsilon)+(\mu_1-\varepsilon)\}P_{2,N_0+1}(t)\end{aligned} \tag{3.7}$$

$$\begin{aligned}\frac{\mathrm{d}}{\mathrm{d}t}P_{1,n}(t) = &-\left\{M(\lambda-\varepsilon)+(S_1-n)(\alpha_1-\varepsilon)+\sum_{j=2}^{k}S_j(\alpha_j-\varepsilon)+(\mu-\varepsilon)\right\}P_{1,n}(t)+(\mu-\varepsilon)P_{1,n+1}(t)\\ &+\left\{M(\lambda-\varepsilon)+(S_1-n+1)(\alpha_1-\varepsilon)+\sum_{j=2}^{k}S_j(\alpha_j-\varepsilon)\right\}P_{1,n-1}(t);\qquad N_0+1\leq n\leq S_1-2\end{aligned} \tag{3.8}$$

$$\frac{\mathrm{d}}{\mathrm{d}t}P_{1,S_1-1}(t) = -\left\{M(\lambda-\varepsilon)+(\alpha_1-\varepsilon)+\sum_{j=2}^{k}S_j(\alpha_j-\varepsilon)+(\mu-\varepsilon)\right\}P_{1,S_1-1}(t)$$
$$+\left\{M(\lambda-\varepsilon)+2(\alpha_1-\varepsilon)+\sum_{j=2}^{k}S_j(\alpha_j-\varepsilon)\right\}P_{1,S_1-2}(t). \tag{3.9}$$

(iii) ***Level $l=2$: When the permanent repairman is working at a faster rate and additional repairmen are also activated as per threshold policy:*** In this case, the standbys of first type are already used and the other types of standbys are being deployed in place of more failed units in the system. Due to overload the additional removable repairmen are introduced one by one according to threshold policy in the system to provide the repair of failed units. By balancing the in-flows and out-flows of the states (2, n) with their respective neighbouring states, we obtain the balance equations for the state probabilities for this case as follows:

$$\frac{\mathrm{d}}{\mathrm{d}t}P_{2,N_0+1}(t) = -\left\{M(\lambda-\varepsilon)+(S_1-N_0-1)(\alpha_1-\varepsilon)+\sum_{j=2}^{k}S_j(\alpha_j-\varepsilon)+(\mu_f-\varepsilon)+(\mu_1-\varepsilon)\right\}P_{2,N_0+1}(t)$$
$$+\{(\mu_f-\varepsilon)+(\mu_1-\varepsilon)\}P_{2,N_0+2}(t) \tag{3.10}$$

$$\frac{\mathrm{d}}{\mathrm{d}t}P_{2,n}(t) = -\left\{M(\lambda-\varepsilon)+(S_1-n)(\alpha_1-\varepsilon)+\sum_{j=2}^{k}S_j(\alpha_j-\varepsilon)+(\mu_f-\varepsilon)+(\mu_1-\varepsilon)\right\}P_{2,n}(t)$$
$$+\left\{M(\lambda-\varepsilon)+(S_1-n+1)(\alpha_1-\varepsilon)+\sum_{j=2}^{k}S_j(\alpha_j-\varepsilon)\right\}P_{2,n-1}(t)$$
$$+\{(\mu_f-\varepsilon)+(\mu_1-\varepsilon)\}P_{2,n+1}(t); \qquad N_0+2\le n\le S_1-1 \tag{3.11}$$

$$\frac{\mathrm{d}}{\mathrm{d}t}P_{2,S_1}(t) = -\left\{M(\lambda-\varepsilon)+\sum_{j=2}^{k}S_j(\alpha_j-\varepsilon)+(\mu_f-\varepsilon)+(\mu_1-\varepsilon)\right\}P_{2,S_1}(t)$$
$$+\left\{M(\lambda-\varepsilon)+(\alpha_1-\varepsilon)+\sum_{j=2}^{k}S_j(\alpha_j-\varepsilon)\right\}P_{1,S_1-1}(t)+\{(\mu_f-\varepsilon)+(\mu_1-\varepsilon)\}P_{2,S_1+1}(t)$$
$$+\left\{M(\lambda-\varepsilon)+(\alpha_1-\varepsilon)+\sum_{j=2}^{k}S_j(\alpha_j-\varepsilon)\right\}P_{2,S_1-1}(t) \tag{3.12}$$

$$\frac{\mathrm{d}}{\mathrm{d}t}P_{2,n}(t) = -\left\{M(\lambda-\varepsilon)+(S_1+S_2-n)(\alpha_2-\varepsilon)+\sum_{j=3}^{k}S_j(\alpha_j-\varepsilon)+(\mu_f-\varepsilon)+(\mu_1-\varepsilon)\right\}P_{2,n}(t)$$
$$+\{(\mu_f-\varepsilon)+(\mu_1-\varepsilon)\}P_{2,n+1}(t)$$
$$+\left\{M(\lambda-\varepsilon)+(S_1+S_2-n+1)(\alpha_2-\varepsilon)+\sum_{j=3}^{k}S_j(\alpha_j-\varepsilon)\right\}P_{2,n-1}(t); \quad S_1+1\le n\le S^{(2)}-2 \tag{3.13}$$

$$\frac{\mathrm{d}}{\mathrm{d}t}P_{2,S^{(i)}-1}(t) = -\left\{M(\lambda-\varepsilon)+(\alpha_i-\varepsilon)+\sum_{j=i+1}^{k}S_j(\alpha_j-\varepsilon)+(\mu_f-\varepsilon)+\sum_{j=1}^{i-1}(\mu_j-\varepsilon)\right\}P_{2,S^{(i)}-1}(t)$$
$$+\left\{(\mu_f-\varepsilon)+\sum_{j=1}^{i}(\mu_j-\varepsilon)\right\}P_{2,S^{(i)}}(t)$$
$$+\left\{M(\lambda-\varepsilon)+2(\alpha_i-\varepsilon)+\sum_{j=i+1}^{k}S_j(\alpha_j-\varepsilon)\right\}P_{2,S^{(i)}-2}(t);\quad 2\le i\le r-1 \tag{3.14}$$

$$\frac{\mathrm{d}}{\mathrm{d}t}P_{2,S^{(r)}-1}(t) = -\left\{M(\lambda-\varepsilon)+(\alpha_r-\varepsilon)+\sum_{j=r+1}^{k}S_j(\alpha_j-\varepsilon)+(\mu_f-\varepsilon)+\sum_{j=1}^{r-1}(\mu_j-\varepsilon)\right\}P_{2,S^{(r)}-1}(t)$$
$$+\left\{(\mu_f-\varepsilon)+\sum_{j=1}^{r}(\mu_j-\varepsilon)\right\}P_{2,S^{(r)}}(t)$$
$$+\left\{M(\lambda-\varepsilon)+2(\alpha_r-\varepsilon)+\sum_{j=r+1}^{k}S_j(\alpha_j-\varepsilon)\right\}P_{2,S^{(r)}-2}(t) \tag{3.15}$$

$$\frac{\mathrm{d}}{\mathrm{d}t}P_{2,S^{(i)}-1}(t) = -\left\{M(\lambda-\varepsilon)+(\alpha_i-\varepsilon)+\sum_{j=i+1}^{k}S_j(\alpha_j-\varepsilon)+(\mu_f-\varepsilon)+\sum_{j=1}^{r}(\mu_j-\varepsilon)\right\}P_{2,S^{(i)}-1}(t)$$
$$+\left\{(\mu_f-\varepsilon)+\sum_{j=1}^{r}(\mu_j-\varepsilon)\right\}P_{2,S^{(i)}}(t)$$
$$+\left\{M(\lambda-\varepsilon)+2(\alpha_i-\varepsilon)+\sum_{j=i+1}^{k}S_j(\alpha_j-\varepsilon)\right\}P_{2,S^{(i)}-2}(t);\quad r+1\le i\le k \tag{3.16}$$

$$\frac{\mathrm{d}}{\mathrm{d}t}P_{2,S^{(i)}}(t) = -\left\{M(\lambda-\varepsilon)+\sum_{j=i+1}^{k}S_j(\alpha_j-\varepsilon)+(\mu_f-\varepsilon)+\sum_{j=1}^{i}(\mu_j-\varepsilon)\right\}P_{2,S^{(i)}}(t)$$
$$+\left\{(\mu_f-\varepsilon)+\sum_{j=1}^{i}(\mu_j-\varepsilon)\right\}P_{2,S^{(i)}+1}(t)$$
$$+\left\{M(\lambda-\varepsilon)+(\alpha_i-\varepsilon)+\sum_{j=i+1}^{k}S_j(\alpha_j-\varepsilon)\right\}P_{2,S^{(i)}-1}(t);\quad 2\le i\le r-1 \tag{3.17}$$

$$\begin{aligned}\frac{\mathrm{d}}{\mathrm{d}t}P_{2,S^{(i)}}(t) = &-\left\{M(\lambda-\varepsilon)+\sum_{j=i+1}^{k}S_j(\alpha_j-\varepsilon)+(\mu_f-\varepsilon)+\sum_{j=1}^{r}(\mu_j-\varepsilon)\right\}P_{2,S^{(i)}}(t)\\ &+\left\{(\mu_f-\varepsilon)+\sum_{j=1}^{r}(\mu_j-\varepsilon)\right\}P_{2,S^{(i)}+1}(t)\\ &+\left\{M(\lambda-\varepsilon)+(\alpha_i-\varepsilon)+\sum_{j=i+1}^{k}S_j(\alpha_j-\varepsilon)\right\}P_{2,S^{(i)}-1}(t);\quad r\le i\le k-1\end{aligned} \tag{3.18}$$

$$\begin{aligned}\frac{\mathrm{d}}{\mathrm{d}t}P_{2,n}(t) = &-\left\{M(\lambda-\varepsilon)+\left(S^{(i+1)}-n\right)(\alpha_{i+1}-\varepsilon)+\sum_{j=i+2}^{k}S_j(\alpha_j-\varepsilon)+(\mu_f-\varepsilon)+\sum_{j=1}^{i}(\mu_j-\varepsilon)\right\}P_{2,n}(t)\\ &+\left\{(\mu_f-\varepsilon)+\sum_{j=1}^{i}(\mu_j-\varepsilon)\right\}P_{2,n+1}(t)\\ &+\left\{M(\lambda-\varepsilon)+\left(S^{(i+1)}-n+1\right)(\alpha_{i+1}-\varepsilon)+\sum_{j=i+2}^{k}S_j(\alpha_j-\varepsilon)\right\}\\ &\times P_{2,n-1}(t);\quad S^{(i)}+1\le n\le S^{(i+1)}-2, 2\le i\le r-1\end{aligned} \tag{3.19}$$

$$\begin{aligned}\frac{\mathrm{d}}{\mathrm{d}t}P_{2,n}(t) = &-\left\{M(\lambda-\varepsilon)+\left(S^{(i+1)}-n\right)(\alpha_{i+1}-\varepsilon)+\sum_{j=i+2}^{k}S_j(\alpha_j-\varepsilon)+(\mu_f-\varepsilon)+\sum_{j=1}^{r}(\mu_j-\varepsilon)\right\}P_{2,n}(t)\\ &+\left\{(\mu_f-\varepsilon)+\sum_{j=1}^{r}(\mu_j-\varepsilon)\right\}P_{2,n+1}(t)\\ &+\left\{M(\lambda-\varepsilon)+\left(S^{(i+1)}-n+1\right)(\alpha_{i+1}-\varepsilon)+\sum_{j=i+2}^{k}S_j(\alpha_j-\varepsilon)\right\}\\ &\times P_{2,n-1}(t);\quad S^{(i)}+1\le n\le S^{(i+1)}-2, r\le i\le k-1\end{aligned} \tag{3.20}$$

$$\begin{aligned}\frac{\mathrm{d}}{\mathrm{d}t}P_{2,S^{(k)}}(t) = &-\left\{M(\lambda_d-\varepsilon)+(\mu_f-\varepsilon)+\sum_{j=1}^{r}(\mu_j-\varepsilon)\right\}P_{2,S^{(k)}}+\left\{(\mu_f-\varepsilon)+\sum_{j=1}^{r}(\mu_j-\varepsilon)\right\}P_{2,S^{(k)}+1}(t)\\ &+\{M(\lambda-\varepsilon)+(\alpha_k-\varepsilon)\}P_{2,S^{(k)}-1}(t)\end{aligned} \tag{3.21}$$

$$\begin{aligned}\frac{\mathrm{d}}{\mathrm{d}t}P_{2,n}(t) = &-\left\{\left(M+S^{(k)}-n\right)(\lambda_d-\varepsilon)+(\mu_f-\varepsilon)+\sum_{j=1}^{r}(\mu_j-\varepsilon)\right\}P_{2,n}(t)\\ &+\left\{(\mu_f-\varepsilon)+\sum_{j=1}^{r}(\mu_j-\varepsilon)\right\}P_{2,n+1}(t)+\left\{\left(M+S^{(k)}-n+1\right)(\lambda_d-\varepsilon)\right\}\\ &\times P_{2,n-1}(t);\quad S^{(k)}+1\le n\le L-1\end{aligned} \tag{3.22}$$

$$\frac{\mathrm{d}}{\mathrm{d}t}P_{2,L}(t) = -\left\{(\mu_f - \varepsilon) + \sum_{j=1}^{r}(\mu_j - \varepsilon)\right\} P_{2,L}(t) + \{m\,(\lambda_d - \varepsilon)\}\, P_{2,L-1}(t). \tag{3.23}$$

To solve the above set of equations (1)−(23), we employ Runge−Kutta method of fourth order and get the transient state probabilities. The set of equations (1)-(23) can be written as:

$$\frac{\mathrm{d}P(t)}{\mathrm{d}t} = f\,(t, P(t)) \tag{3.24}$$

where unknown probability vector $\mathbf{P}(t) = [P_{0,0}(t), \ldots P_{0,N-1}(t), P_{1,1}(t), \ldots P_{S_1-1,1}(t), P_{2,N_0+1}(t), \ldots, P_{2,L}(t)]$ is computed by using routine ode45 of MATLAB 7 with the initial condition $P_{0,0}(0) = 1$. This method is based on iterative computation as follows:

$$\mathbf{P}(t_{w+1}) = \mathbf{P}(t_w) + \frac{h}{6}\,(f_{w_1} + 2f_{w_2} + 2f_{w_3} + 2f_{w_4}) \tag{3.25}$$

where

$$f_{W_1} = f\,(t_W, \mathbf{P}(t_W)) \tag{3.26}$$

$$f_{W_2} = f\left(t_W + \frac{h}{2}, \mathbf{P}(t_W) + \frac{f_{W_1}h}{2}\right) \tag{3.27}$$

$$f_{W_3} = f\left(t_W + \frac{h}{2}, \mathbf{P}(t_W) + \frac{f_{W_2}h}{2}\right) \tag{3.28}$$

$$f_{W_4} = f\,(t_W + h, \mathbf{P}(t_W)) + f_{W_3}h. \tag{3.29}$$

4. Performance measures

In order to validate the utility of the present model in real time system and to analyze the performance of the concerned system, it is important to establish some measures of performance. Now, we formulate some performance measures in terms of state probabilities of machining system supported by standbys and repair crew as follows:

- The probability that at time t, the system is in accumulation state when the repair is not yet started and the failed units are joining the queue,

$$P_A(t) = \sum_{n=0}^{N-1} P_{0,n}(t). \tag{4.1}$$

- The probability that the permanent repairman is operating in normal mode at time t

$$P_{PN}(t) = \sum_{n=1}^{S_1-1} P_{1,n}(t). \tag{4.2}$$

- The probability that the permanent repairman is rendering service with faster rate and the additional repairmen are also activated at time t

$$P_{PF}(t) = \sum_{n=N_0+1}^{L} P_{2,n}(t). \tag{4.3}$$

- The probability that the system is working in short mode at time t

$$P_S(t) = \sum_{n=S^{(k)}+1}^{L-1} P_{2,n}(t). \tag{4.4}$$

- The expected number of failed units in the system at time t

$$E_N(t) = \sum_{n=1}^{N-1} nP_{0,n}(t) + \sum_{n=1}^{S_1-1} nP_{1,n}(t) + \sum_{n=N_0+1}^{L} nP_{2,n}(t). \tag{4.5}$$

- The expected number of available standby units in the system at time t

$$E_S(t) = \sum_{n=0}^{N-1} \left(S^{(k)} - n\right) P_{0,n}(t) + \sum_{n=1}^{S_1-1} \left(S^{(k)} - n\right) P_{1,n}(t) + \sum_{n=N_0+1}^{S^{(k)}-1} \left(S^{(k)} - n\right) P_{2,n}(t). \tag{4.6}$$

- The expected number of additional repairmen in the system at time t

$$E_A(t) = \sum_{n=N_0+1}^{S_2-1} P_{2,n}(t) + \sum_{i=2}^{r-1} \sum_{n=S^{(i)}}^{S^{(i+1)}-1} iP_{2,n}(t) + \sum_{n=S^{(r)}}^{L} rP_{2,n}(t). \tag{4.7}$$

- The throughput of the system at time t

$$T(t) = \sum_{n=1}^{N_0} \mu P_{1,n}(t) + \sum_{n=N_0+1}^{S_1-1} \mu P_{1,n}(t) + \sum_{n=N_0+1}^{S_2-1} (\mu_f + \mu_1) P_{2,n}(t) + \sum_{i=2}^{r-1} \sum_{n=S^{(i)}}^{S^{(i+1)}-1} \left(\mu_f + \sum_{j=1}^{i} \mu_j\right) P_{2,n}(t)$$

$$+ \sum_{n=S^{(r)}}^{L} \left(\mu_f + \sum_{j=1}^{r} \mu_j\right) P_{2,n}(t). \tag{4.8}$$

- The reliability of the system at time t is

$$R(t) = 1 - P_{2,L}(t). \tag{4.9}$$

- The failure frequency of the system at time t

$$F(t) = \lambda_{L-1} P_{2,L-1}(t). \tag{4.10}$$

- ***Cost function***

In order to obtain the optimal number of additional removable repairmen and standbys required to maintain the system availability and reliability, the cost function has been developed. To construct the cost function, the different cost factors taken into consideration are as follows:

C_0 = Cost per unit time of the operating units when the system is working in normal mode.
C_S = Cost per unit time of the available standby units in the system.
C_A = Cost per unit time of the additional repairmen while repairing the failed units.
C_μ = Cost per unit time of the repair done by the permanent repairman at normal rate.
C_f = Cost per unit time of the repair done by the permanent repairman at faster rate.

TABLE 1. Fixed parameters for Figures 2–6.

	Figure 2		Figure 3	Figure 4	Figure 5	Figure 6	
	(ii-iv)	(i)				(i-iv,vi)	(v)
$\boldsymbol{\mu_f}$	1	4	2	2	2	2	2
$\boldsymbol{\lambda}$	0.4		0.4	1	0.4	0.4	1
$\boldsymbol{\alpha}$	[0.3,0.2,0.1]		[0.3,0.2,0.1]	[1,0.8,0.5]	[0.3,0.2,0.1]	[0.3,0.2,0.1]	[1,0.8, 0.5]
$\boldsymbol{\lambda_d}$	0.8		0.8	2	0.8	0.8	2

To find out the optimal number of additional repairmen and standbys subject to the reliability constraint, the expected total cost per unit time is given by:

$$TC(t) = C_0M - E_N(t) + C_SE_S(t) + C_AE_A(t) + C_\mu\mu + C_f\mu_f \tag{4.11}$$

subject to

$$R(t) \geq R_0(t) \tag{4.12}$$

where $R_0(t)$ is a minimum value of the reliability of the system to be achieved at time t.

5. Sensitivity analysis

After establishing the performance indices given in the previous section, it is worthwhile to compute these measures by taking suitable illustration. By keeping some default parameters fixed as $M = 10, k = 3, r = 3, N = 3, N_0 = 5, m = 2, \mu = 1$ and $\varepsilon = 0.05$ and other parameters as given in Table 1, we compute performance indices which are interpreted from the sensitivity analysis view points, are as follows:

(i) Effect of the time (t):
The expected number of failed units $E_N(t)$ in the system and the throughput of the system $T(t)$ increases (Figs. 2 and 5) as time t grows. On the contrary, the expected number of available standby units $E_S(t)$ and the reliability of the system $R(t)$ decrease (Figs. 3 and 4) with the increase in time t which is what we expect in the real time system also.

(ii) Effect of the failure rate (λ), repair rates (μ & μ_f) and interdependence rate (ε):
At any specific time epoch t, with the increase in the failure rate λ of the operating units, the expected number of failed units $E_N(t)$, the expected number of additional repairmen available in the system $E_A(t)$, the throughput of the system $T(t)$ and the failure frequency of the system $F(t)$ increase which is quite obvious as depicted in Figures 6(i), 6(iii), 6(iv) and 6(vi), respectively. The expected number of available standby units $E_S(t)$ and the reliability of the system $R(t)$ decrease with the increase in the failure rate λ which can be seen in Figures 6(ii) and 6(v), respectively.

The repair rate of the permanent repairman μ affects the expected number of failed units in the system $E_N(t)$ abruptly. In Figure 2(i), for the lower values of t, it is almost constant but for the higher value of t, it decreases on increasing the values of μ. The expected number of available standby units $E_S(t)$ remains unaffected by the increase in the repair rate μ_f as depicted in Figure 3(iv). The reliability of the system $R(t)$ and throughput of the system $T(t)$ increase with the increase in the value of μ_f as shown in Figures 4(iv) and 5(iv), respectively, which is quite understandable and obvious too.

The mean dependence rate of the failure and repair rates ε affects the performance measures significantly. The expected number of failed units $E_N(t)$ and the throughput $T(t)$ show a decreasing trend in Figures 2(iii)

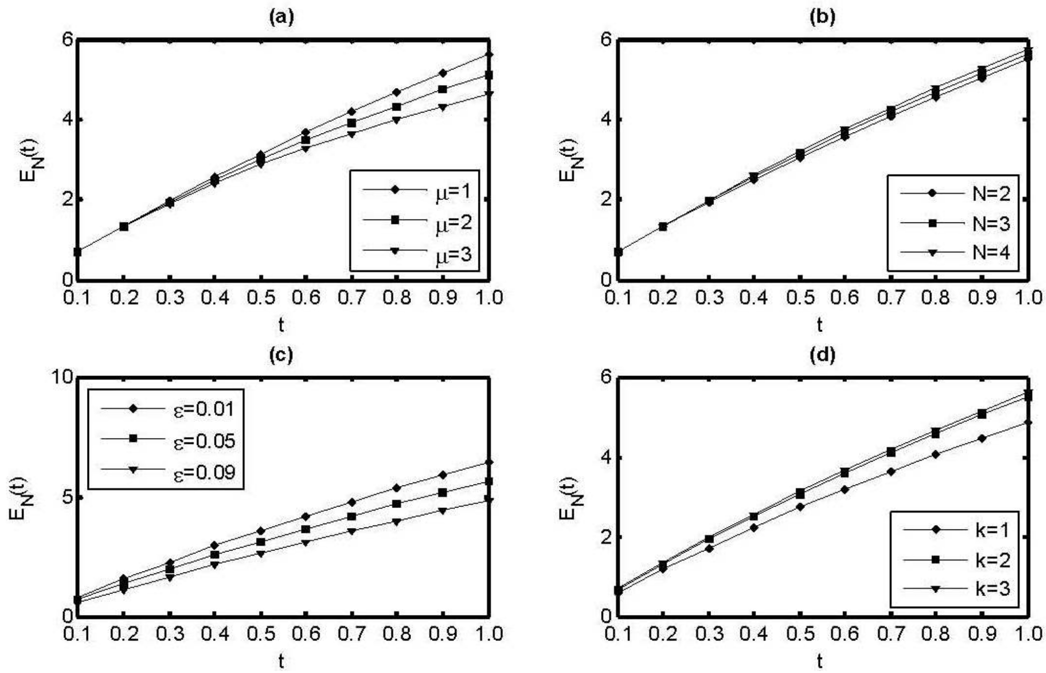

FIGURE 2. Effect of (a) μ (b) N (c) ε (d) k on the expected number of failed units in the system.

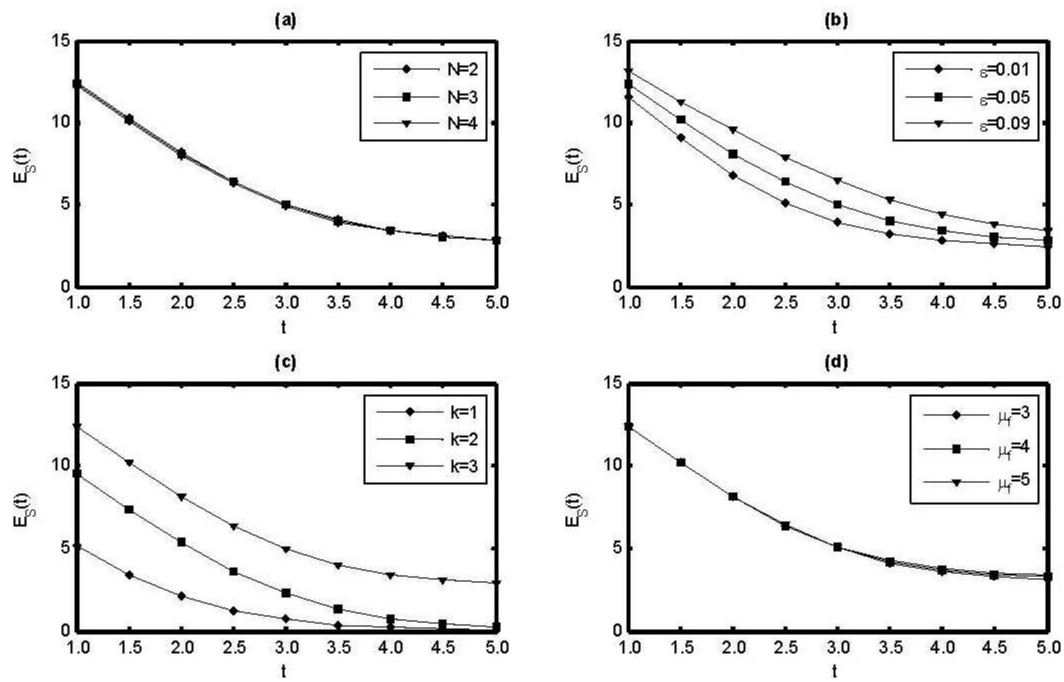

FIGURE 3. Effect of (a) N (b) ε (c) k (d) μ_f on the expected number of standby units in the system.

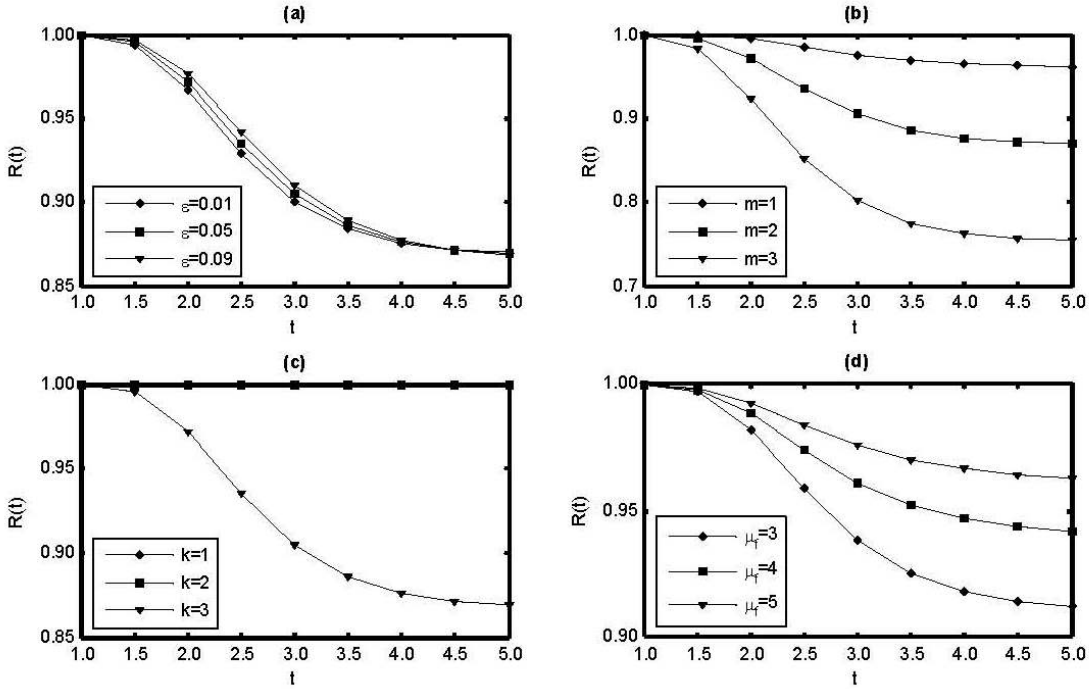

Figure 4. Effect of different set of (a) ε (b) m (c) k (d) μ_f on the reliability of the system up.

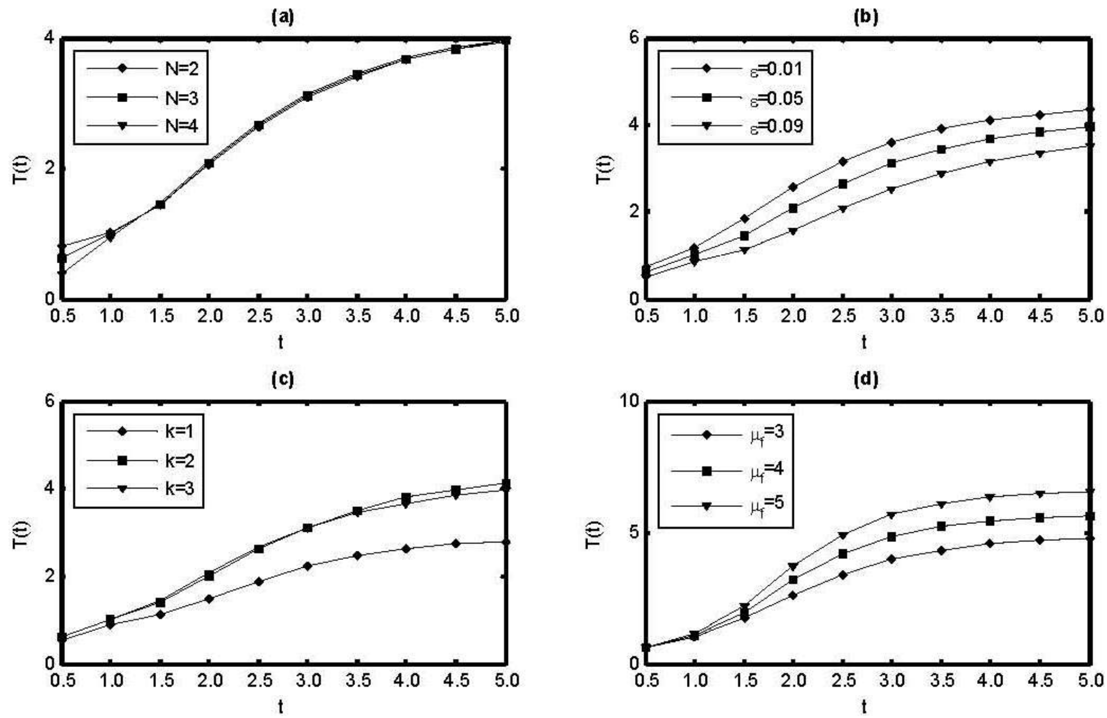

Figure 5. Effect of (a) N (b) ε (c) k (d) μ_f on the throughput of the system.

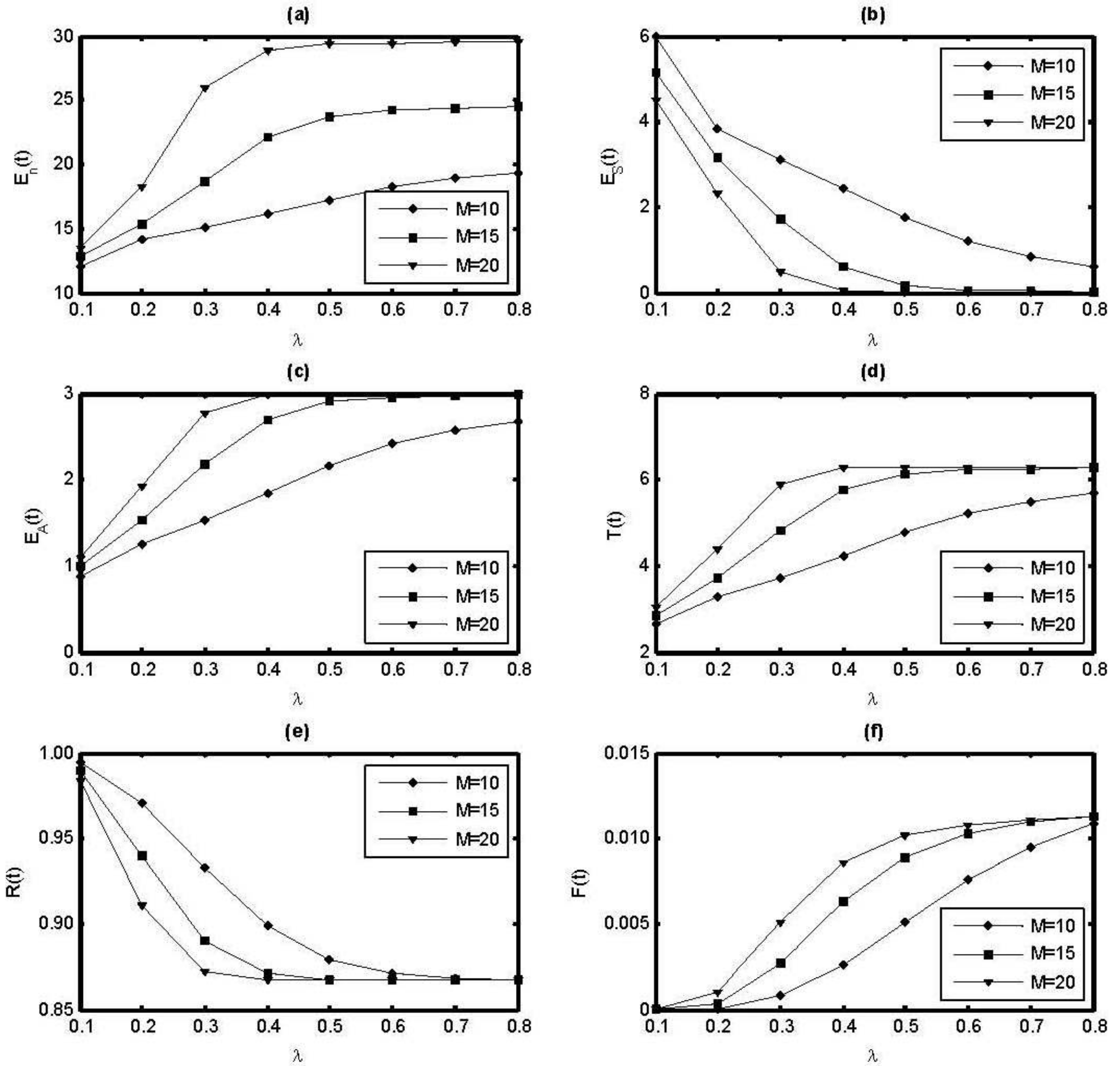

FIGURE 6. On varying the failure rate λ of the operating units, the effect of values of M on (a) $E_N(t)$ (b) $E_S(t)$ (c) $E_A(t)$ (d) $T(t)$ (e) $R(t)$ (f) $F(t)$.

and 5(ii), respectively. On the contrary, the expected number of available standby units $E_S(t)$ and the reliability $R(t)$ show an increasing trend with the increase in ε as displayed in Figures 3(ii) and 4(i), respectively.

(iii) Effect of N:
The expected number of failed units $E_N(t)$ and $T(t)$ increase with the increasing values of N as shown in Figures 2(ii) and 5(ii), respectively. On the other hand, the expected number of available standby units $E_S(t)$ remains almost constant on increasing the values of N as demonstrated in Figure 3(i).

(iv) Effect of the number of types of standbys (k):
With the increase in the number of the standbys k, the expected number of failed units $E_N(t)$ increases (see Fig. 2(iv)) whereas the expected number of available standby units $E_S(t)$ and the reliability of the system $R(t)$

decrease as depicted in Figures 3(iii) and 4(iii), respectively. In Figure 5(iii), we notice that the throughput $T(t)$ first increases and then decreases with increasing values of k.

(v) Effect of (m, M):

The system will be less reliable if the minimum number (m) of operating units required to keep the system working, will be more. The reliability of the system $R(t)$ varies inversely with the values of m as depicted in Figure 4(ii). In Figures 6(i), 6(ii) and 6(iv), it is observed that the expected number of failed units $E_N(t)$, the expected number of available standby units $E_S(t)$ and the throughput of the system $T(t)$ respectively, increase with the increase in the number of operating units M. It is further noticed in Figure 6(ii) that the expected number of available standby units $E_S(t)$ decreases with the increase in M.

Based on the trends of numerical results displayed we conclude that the system will be more reliable if more number of standbys are deployed in the system and the repair is done at a faster pace. It is also visible from the graphs that if the units fail with a higher rate, then the failure frequency of the system will be more and there will be more chances of the system failure. To cope up with this situation, it is recommended that the optimal combination of maintainability and redundancy should be kept based on queueing and reliability performance indices.

6. Conclusion

The study of the performance measures enables us to control the threshold parameters as per requirement of the machining system which operates under certain techno-economic constraints. This investigation will be helpful in developing an efficient and economical machining system which is equipped with multi-type of standbys and maintenance crew to achieve a higher grade of service. Such type of machining system can be commonly found in shopping malls, transportation systems, manufacturing systems, production systems, computer and communication systems, *etc.* The present study can be further extended in the direction of providing a cost optimization model for a machine repair problem with controlled rates and additional repairmen.

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