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THE NONLINEAR COMPLEMENTARITY MODEL OF INDUSTRIAL SYMBIOSIS NETWORK EQUILIBRIUM PROBLEM

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Abstract. In this paper, we propose an industrial symbiosis network equilibrium model by using nonlinear complementarity theory. The industrial symbiosis network consists of industrial producers, industrial consumers, industrial decomposers and demand markets, which imitates natural ecosystem by means of exchanging by-products and recycling useful materials exacted from wastes. The industrial producers and industrial consumers are assumed to be concerned with maximization of economic profits as well as minimization of emissions. We describe the optimizing behavior, derive optimality conditions of the various decision-makers along with respective economic interpretations and establish the nonlinear complementarity model in accordance with the industrial symbiosis network equilibrium conditions. Based on the existence proof of the corresponding nonlinear complementarity model under reasonable assumptions, two groups of numerical examples are given to illustrate the rationality as well as the effectiveness of the model.

Keywords. Industrial symbiotic networks, nonlinear complementarity, theory equilibrium conditions, multicriteria decision-making.

Mathematics Subject Classification. 49J40, 90B06, 91A40.

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Introduction

With sustainable development and corporate social responsibility prevailing, the industrial ecology is becoming as a brand-new research field. In particular, the eco-industrial park emerges as a particular type of an industrial ecosystem and has drawn attention as a promising approach for retrofitting existing industrial parks to improve energy efficiency and reduce total emissions in the whole system. The eco-industrial park is defined as an industrial park in which enterprises cooperate with each other by using other members' by-products and wastes; through this kind of cooperation, they form an industrial symbiotic network (ISN) (*cf.* Lowe and Evans [24], Korhonen [22], Harper and Graedel [19], Zheng *et al.* [45], and the references therein). In other words, different from linear sum of individual enterprises in traditional industrial system, an eco-industrial system usually exchanges and reuses materials, energy, water and by-products, thus constituting the complex network.

The ISN is based on industrial symbiosis, a form of industrial organization in eco-industrial parks. In the previous research literature, industrial symbiosis systems were often expressed as eco-industrial parks [8], eco-industrial development [7] and circular economy [16]. The focus of industrial symbiosis which bears a resemblance to natural ecosystems is that companies and other economic entities form suppliers and customers in networks. In order to survive and maintain their productivity, these members rely on resources available in the natural environment (*e.g.* [39]). The *Kalundborg Complex* in Denmark is the most representative example of the industrial symbiosis, which is not pre-planned or facilitated by an authority but spontaneously evolved into an eco-industrial park over many decades. As a matter of fact, Chertow [5] analyzed initiatives of the individual participating companies and found that, in many cases, these companies were motivated by opportunities to save costs or to meet local resource availability limits. Spontaneous networks are driven by the economic advantages offered by the present market conditions and demands, with companies acting on their own behalf [4]. By cooperating with each other in an industrial ecosystem, enterprises would improve their combined environmental performance by measures that could increase profit margins and thereby potentially boost economic development [17].

Financial gains from symbiotic operations are generally the main drive behind the emergence of ISN [13, 37, 41, 44]. Enterprises cultivate symbiotic relationships by developing waste and by-product networks in a mutual and systematic manner. It can reduce costs and achieve an overall reduction in eco-park wastes through using by-products of other economic entities in the eco-industrial park, avoiding transport costs and purchasing goods below market prices [6]. Significant cost savings are experienced by each partner in the cooperation, making it feasible to carry out without losing economic profits. The reduction of emissions and the subsequent consumption of virgin raw materials and energy inputs are beneficial for enterprises as well as for the society as a whole [21]. What's more, an ISN also includes the exchanging of information or services among different organizational units, such as logistics, training and system planning [28].

According to Mirata [27], particularly, it is worth noting that ISNs are attractive due to the following potentials for environmental, economic and social benefits among others:

- Reducing resource use, dependence on the non-renewable, pollutant emissions, and waste discharges;
- Reducing inputs, production, and waste management costs, and by generating additional income due to value added to by-product and waste streams;
- Improving relationships with external parties, and by facilitating development of new products and their markets;
- Generating new employment, and helping to create a safer and cleaner working environment.

Even though, recently, the initiatives in eco-industrial parks are promoted worldwide, there are still only limited studies furnished with respect to ISNs. We have chosen a portion displayed in chronological order in Table 1. Despite much attention that ISNs have attracted and economic and environmental benefits that ISNs have created, most of the contributions focus on the engineering and technical feasibility of the exchanges, but few attempts have been made so far to investigate the equilibrium of the ISN. The perspective of complex network's equilibrium, in general, has widely applied in the field of supply chain networks by using variational inequality theory (*cf.* [12, 18, 29–35]). We have developed a supply chain network equilibrium model through variational inequalities, which embraces multiperiod decision-making, multicriteria decision-making and electronic commerce in the presence of B2B (business-to-business) as well as B2C (business-to-consumer) transactions [25]. The essential difference between our former work and this one lies in both distinct objects and theories. On one hand, in particular terms, this means that the object that the former paper emphasizes is supply chain networks within an equilibrium context, whereas the latter one analyzes the equilibrium conditions of industrial symbiotic networks. On the other hand, the former paper uses the variational inequality theory while the theory that the latter one applies is the nonlinear complementarity theory. These two theories are not isolated from each other because a variational inequality model is equivalent to a nonlinear complementarity model under the condition that the feasible region is defined on a nonnegative orthant.

Therefore, the remarkable feature of the paper is mainly attributed to an analogue that we attempt to make a conversion from the field of the supply chain network to the field of the industrial symbiosis network. In spite of common characteristics between the above two networks, there also exist remarkable differences such as independence *vs.* dependence, low diversity *vs.* high diversity, low awareness of environmental protection *vs.* high awareness of environmental protection, extensive networks *vs.* regional networks. We explore network equilibrium of the industrial symbiosis system in eco-industrial parks. However, it is necessary to

Table 1. Representative references about ISNs displayed in chronological order.

Author(s)	Contribution(s)
Fleig (2000) [15]	studied the risk of industrial symbiosis network in eco-industrial park and proposed that the higher material exchange and technological innovation among enterprises require, the stronger dependence and risk would turn to.
Majumdar (2001) [26]	suggested that the legal, institutional, economic, organizational, technical factors were essential to maintain stability of industrial symbiosis network in eco-industrial park, determining whether the industrial symbiosis network succeeded or not.
Wang (2002) [42]	described an ISN as a nonlinear complex system and constructed model of the ISN by imitating the producer, consumer and decomposer in natural ecological system. Then, it put forward four kinds of operation model of the ISN and analyzed them in detail by using the case study.
Mirata (2004) [27]	reviewed the factors influencing the development and sustained operation of regional industrial symbiosis networks and discussed the roles a coordination body can play to alter these factors so as to catalyze the development and function of such networks.
Mirata and Emtairah (2005) [28]	discussed industrial symbiosis networks from the perspective of innovation studies, which held that industrial symbiosis networks could contribute to fostering environmental innovation at the local or regional level by providing inter-sectoral interfaces and promoting a culture of inter-organisational collaboration oriented towards environmental challenges.
Song (2006) [40]	analyzed network complexity and the features of ideal material networks as well as enterprise symbiotic networks from aspects of scale, aggregation, connectedness, complexity, small world characteristics and node influences.
Yuan and Jun (2009) [44]	noted that industrial symbiosis networks could achieve the optimization of the economic, social and environmental benefits and illustrated that the purpose of the enterprises was the improvement of productivity as well as eco-efficiency.
Posch (2010) [36]	investigated whether industrial recycling networks or industrial symbiosis projects could be used as a starting point for broader inter-company cooperation for sustainable development.
Domenecha and Davies (2011) [11]	adopted Social Network Analysis as the main methodological framework and applied it to the IS network in *Kalundborg* (Denmark), proving useful in the analysis of the structural characteristics of the network and the understanding of the roles that different actors play.
Behera *et al.* (2012) [3]	presented the detailed mechanism of Korean EIP initiative for transforming the conventional industrial complexes into EIP and stimulated the systematic development of 'designed' symbiotic networks.

state that the equilibrium conditions of the ISN are established by the nonlinear complementarity theory rather than the variational inequality theory.

In the light of the aim of ISNs to reduce the intake of virgin materials and lower wastes, this paper contributes to understanding of the operation of the ISN by focusing on equilibrium conditions and how the industrial symbiosis influences the collaboration, both in economic and environmental terms. It has a remarkable significance in the aspect of enhancing awareness of sustainable development among enterprisers in eco-industrial parks, which benefits the society as a whole. Consequently, this paper is to present a nonlinear complementarity model of the industrial symbiosis network equilibrium, which consists of four-tier decision-makers including industrial producers, industrial consumers, industrial decomposers and demand markets. With more enterprises being more environmentally-friendly, specifically, the proposed optimization is performed with consideration of multicriteria decision-making, that is, the integration of environmental decision-making into pursuing for economic profits. Indeed, environmental decision-making affects the behavior of most industrial members to a great extent so that they attempt to minimize their emissions, produce more environmentally-friendly products and establish sound network systems, which can make a positive impact on the reduction of pollution and then protect the environment.

This paper is organized as follows. In Section 2, we describe the optimizing behavior of the various decision-makers in the industrial symbiosis network, derive the governing optimality conditions with the corresponding economic interpretations and establish the nonlinear complementarity conditions formulation. In Section 3, we propose the definition concerning the equilibrium state of industrial symbiosis network and establish the existence of solutions to the nonlinear complementarity model under reasonable assumptions. In Section 4, several numerical examples are presented to further illustrate the model. We conclude with Section 5 in which we summarize and suggest possibilities for future research.

1. The nonlinear complentarity conditions of the various decision-makers in the ISN

In this section, we develop the nonlinear complementarity conditions in accordance with the industrial symbiosis network equilibrium conditions through integrated environmental decision-making. This complex network consists of industrial producers, industrial consumers, industrial decomposers and demand markets, which focuses on the equilibrium state in statics surroundings. It is assumed to be a non-cooperative and competitive network structure, and then, the governing equilibrium conditions can be formulated as a series of nonlinear complementarity conditions.

As it was previously mentioned, the industrial symbiosis network, an organized form of the industrial ecosystem, are increasingly mimicking the natural ecosystem in order to follow the rules of sustainable development in the everyday industrial practice. In this complex network, each entity incorporated into an eco-industrial park is regarded as a living organism. This may be due to the fact that they need organic or inorganic input materials and energy from renewable or non-renewable sources. As a result, two kinds of products are formed. First of all, there are main products defined as the desired products with market value, which in this paper are in the form of semi-products and finished products. The second group is composed of by-products and wastes. Some by-products and exacted wastes can be potentially used as input materials in the recycling.

What's more, it is significant to define the term 'symbiosis' and clarify it in detail. Two close definitions formulated by Ashworth [1] and Côté *et al.* [10] fit our model concerning the industrial symbiosis network. Symbiosis is well-known positive interactions between species in the natural ecosystems. Symbiosis is defined as a relationship between two organisms from which both derive some benefits [1]. Côté *et al.* [10] defined the symbiosis as the situation in which two business coexist in a physiological mutually beneficial relationship. As for the categories of the symbiosis, the work of Liwarska–Bizukojc *et al.* [23] is deserved to be reviewed and cited.

'Symbiosis is well-known positive interactions between species in the natural ecosystems... However, we would like to distinguish two kinds of symbiosis: obligatory and facultative. Symbiosis can be obligatory, when one organism is unable to exist in the absence of another one or facultative when both organisms can exist independently. In the industrial ecosystems, the facultative symbiosis dominates, which occurs, for example, when a by-product of one enterprise becomes an input material for another. Both sides benefit: one enterprise achieves a cheaper input material, while the other resolves its waste disposal problem. This way the industrial metabolism of the enterprises is coupled together. An obligatory symbiosis is a rare case in the industrial ecosystem as the industrial metabolism of two different types of enterprises is seldom coupled inseparably. An example of an obligatory symbiosis in the industrial ecosystems can be the cooperation between the coal power station and the gypsum sheetrock producer. The latter utilizes gypsum produced within flue gas desulphurisation in the coal power station. If there are no natural gypsum mines in the neighbourhood, sheetrock producer could not exist. Furthermore, a coal power plant that cannot find a gypsum purchaser, would find itself in organizational, technical and financial troubles connected with the management of its waste gypsum...'

Liwarska–Bizukojc *et al.* [23] also discussed the classification of industrial ecosystems or eco-industrial parks.

'Functionally two components of the ecosystem can be recognized (autotrophs and heterotrophs) and usually four constituents, *i.e.* abiotic, producers, consumers and decomposers. Following this functional classification, the enterprises, which are a biotic part of the industrial ecosystem, can be divided into industrial producers, consumers and decomposers...'

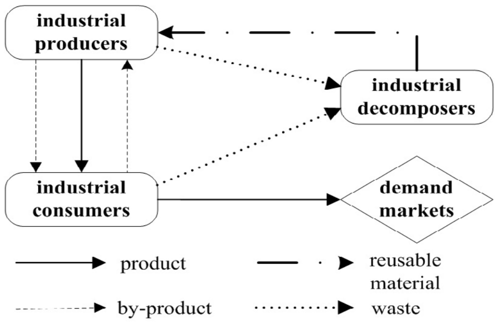

FIGURE 1. The structure of the industrial symbiosis network.

Furthermore, in terms of Schwarz and Steininger [38], industrial ecosystems are open systems. Indeed, there exist material or information exchanges and cash flows between the industrial symbiosis system and the outside of eco-industrial parks, such as government, demand markets. It is indispensable to consider these interactions especially when discussing the network equilibrium of the whole industrial symbiosis system. For the sake of simplicity, nevertheless, we just take demand markets into account for product transactions.

On the other hand, Côté and Smolenaars [9] discussed that lacking of diversity would create the potential instability of the eco-industrial parks. Indeed, diversity is the key for not only creating symbiotic relationships but also functioning eco-industrial parks efficiently. The importance of the diversity was confirmed in the related literature (*cf.* [2, 6]). In the paper, the diversity is mainly embodied by the various semi-products, finished products, by-products and wastes, which are treated as numbers of decision variables of the optimization model.

The model considers m industrial producers involved in the production of X semi-products, who can sell merely to n industrial consumers for deep processing (links represented by solid arcs in Fig. 1) rather than to demand markets. With regard to industrial consumers, their deep processing associated with producing U finished products is mainly to be supplied for L demand markets (links represented by solid arcs in Fig. 1). What's more, there are also exchange activities (links represented by dashed arcs in Fig. 1) between industrial producers and industrial consumers to obtain by-products for their respective necessities. Specially, there are Y by-products derived from industrial producers and V by-products originated from industrial consumers.

It is assumed that Z wastes generated in the process of industrial producers' producing or remanufacturing and W wastes generated in the process of industrial consumers' deep processing. All of these wastes (links represented by dotted arcs in Fig. 1) are sold to o industrial decomposers who conduct some separation and detection to exact G reusable materials (links represented by bold arc of dashes in Fig. 1) to return to the industrial producers for remanufacturing. We assume, for

TABLE 2. Decision variables of the various decision-makers in the ISN.

Decision variables	Interpretation
$\tilde{q}_{ix}$	nonnegative amount of raw materials purchased by industrial producer i for producing semi-product x. Group these shipments of all raw materials into the column vector $\tilde{q} \in R_+^{mX}$
q_{ijx}	nonnegative shipment of semi-product x from industrial producer i to industrial consumer j. Group these volumes of all semi-product shipments between all industrial producers and all industrial consumers into the column vector $Q_1 \in R_+^{mnX}$
q_{ijy}	nonnegative shipment of by-product y from industrial producer i to industrial consumer j. Group these volumes of all by-product shipments between all industrial producers and all industrial consumers into the column vector $Q_2 \in R_+^{mnY}$
q_{jiv}	nonnegative shipment of by-product v from industrial consumer j to industrial producer i. Group these volumes of all by-product shipments between all industrial consumers and all industrial producers into the column vector $Q_3 \in R_+^{nmV}$
q_{ikz}	nonnegative shipment of waste z from industrial producer i to industrial decomposer k. Group these volumes of all waste shipments between all industrial producers and all industrial decomposers into the column vector $Q_4 \in R_+^{moZ}$
q_{jkw}	nonnegative shipment of waste w from industrial consumer j to industrial decomposer k. Group these volumes of all waste shipments between all industrial consumers and all industrial decomposers into the column vector $Q_5 \in R_+^{noW}$
q_{kig}	nonnegative shipment of reusable material g from industrial decomposer k to industrial producer i. Group these volumes of all reusable material shipments between all industrial decomposers and all industrial producers into the column vector $Q_6 \in R_+^{omG}$
q_{jlu}	nonnegative shipment of finished product u from industrial consumer j to demand market l. Group these volumes of all finished product shipments between all industrial consumers and all demand markets into the column vector $Q_7 \in R_+^{nLU}$

the sake of generality, there is no perceived quality depreciation of semi-products made from both raw and reusable materials as well.

We denote a typical industrial producer by i, a typical industrial consumer by j, a typical industrial decomposer by k and a typical demand market by l, respectively. Furthermore, a typical semi-product is denoted by x, a typical finished product is denoted by u, a typical by-product derived from industrial producers is denoted by y, a typical by-product originated from industrial consumers is denoted by v, a typical reusable material is denoted by g, a typical waste relevant to industrial producers is denoted by z and a typical waste related to industrial consumers is denoted by w. The general industrial symbiosis network investigated is illustrated in Figure 1. The decision variables and parameters associated with the various decision-makers are defined in Tables 2 and 3, respectively. Moreover,

TABLE 3. Parameters of the various decision-makers in the ISN.

Parameters	Interpretation
α_{ix}	fraction of one unit of raw material wholly transformed into semi-product x for industrial producer i, $\alpha_{ix} \in [0,\ 1]$
δ_{ix}	a binary parameter which is introduced to indicate whether industrial producer i does ($\delta_{ix} = 1$) or does not ($\delta_{ix} = 0$) incur a production associated with semi-product x
δ_{ijx}	a binary parameter which is introduced to indicate whether does ($\delta_{ijx} = 1$) exist or does not ($\delta_{ijx} = 0$) exist transaction link of semi-product x between industrial producer i and industrial consumer j
δ_{ijy}	a binary parameter which is introduced to indicate whether does ($\delta_{ijy} = 1$) exist or does not ($\delta_{ijy} = 0$) exist a transaction link of by-product y between industrial producer i and industrial consumer j
δ_{ikz}	a binary parameter which is introduced to indicate whether does ($\delta_{ikz} = 1$) exist or does not ($\delta_{ikz} = 0$) exist a transaction link of waste w between industrial producer i and industrial decomposer k
δ_{jiv}	a binary parameter which is introduced to indicate whether does ($\delta_{jiv} = 1$) exist or does not ($\delta_{jiv} = 0$) exist a transaction link of by-product v between industrial producer i and industrial consumer j
α_v	fraction of per unit of by-product v wholly transformed into semi-product x, $\alpha_v \in [0,\ 1]$
δ_{kig}	a binary parameter which is introduced to indicate whether does ($\delta_{kig} = 1$) exist or does not ($\delta_{kig} = 0$) exist a transaction link of reusable material g between industrial decomposer k and industrial producer i
α_g	fraction of per unit of reusable material g wholly transformed into semi-product x, $\alpha_g \in [0,\ 1]$
η_{jkw}	a binary parameter which is introduced to indicate whether does ($\eta_{jkw} = 1$) exist or does not ($\eta_{jkw} = 0$) exist a transaction link of waste w between industrial consumer j and industrial decomposer k
$\bar{\rho}$	disposing fee of per unit of useless material to the landfill
β_x	fraction of per unit of semi-product x wholly transformed into finished product, $\beta_x \in [0,\ 1]$
β_y	fraction of per unit of by-product y wholly transformed into finished product, $\beta_y \in [0,\ 1]$
χ_z	fraction of per unit of waste z wholly transformed into reusable material, $\chi_z \in [0,\ 1]$
χ_w	fraction of per unit of waste w wholly transformed into reusable material, $\chi_w \in [0,\ 1]$

the majority of the cost functions related to the model are given in Table 4. The equilibrium solution is marked by ‘ * ’.

In this section, we develop the nonlinear complementarity conditions equivalent to the optimality conditions of the various decision-makers in the industrial symbiosis network. As mentioned in the Introduction, the first two-tier decision-makers are multicriteria ones with integrated environmental decision-making. We first

TABLE 4. Cost functions of the various decision-makers in the ISN.

Cost functions	Interpretation
f_{ix1}	cost of producing semi-product x from raw materials by industrial producer i, $f_{ix1} = f_{ix1}(\tilde{q}_{ix}, \alpha_{ix})$
f_{ix2}	cost of remanufacturing semi-product x from reusable materials by industrial producer i, $f_{ix2} = f_{ix2}(q_{kig}, \alpha_g)$
f_{ix3}	cost of producing semi-product x from by-products by industrial producer i, $f_{ix3} = f_{ix3}(q_{jiv}, \alpha_v)$
c_{ijx}	cost of transacting associated with semi-product x for industrial producer i conducting with industrial consumers j, $c_{ijx} = c_{ijx}(q_{ijx})$
c_{ijy}	cost of transacting associated with by-product y for industrial producer i conducting with industrial consumer j, $c_{ijy} = c_{ijy}(q_{ijy})$
c_{ikz}	cost of transacting associated with waste z for industrial producer i conducting with industrial decomposer k, $c_{ikz} = c_{ikz}(q_{ikz})$
$\hat{c}_{jiv}$	cost of transacting associated with by-product v for industrial producer i conducting with industrial consumer j, $\hat{c}_{jiv} = \hat{c}_{jiv}(q_{jiv})$
$\hat{c}_{kig}$	cost of transacting associated with reusable material g for industrial producer i conducting with industrial decomposer k, $\hat{c}_{kig} = \hat{c}_{kig}(q_{kig})$
f_{ju}	cost of producing finished product u from semi-products by industrial consumer j, $f_{ju} = f_{ju}(q_{ijx}, q_{ijy}, \beta_x, \beta_y)$
$\hat{c}_{ijx}$	cost of transacting associated with semi-product x for industrial consumer j conducting with industrial producer i, $\hat{c}_{ijx} = \hat{c}_{ijx}(q_{ijx})$
$\hat{c}_{ijy}$	cost of transacting associated with by-product y for industrial consumer j conducting with industrial producer i, $\hat{c}_{ijy} = \hat{c}_{ijy}(q_{ijy})$
$\hat{c}_{jiv}$	cost of transacting associated with by-product v for industrial producer i conducting with industrial consumer j, $\hat{c}_{jiv} = \hat{c}_{jiv}(q_{jiv})$
c_{jiv}	cost of transacting associated with by-product v for industrial consumer j conducting with industrial producer i, $c_{jiv} = c_{jiv}(q_{jiv})$
c_{jlu}	cost of transacting associated with finished product u for industrial consumer j conducting with demand market l, $c_{jlu} = c_{jlu}(q_{jlu})$
c_{jkw}	cost of transacting associated with waste w for industrial consumer j conducting with industrial decomposer k, $c_{jkw} = c_{jkw}(q_{jkw})$
c_{kig}	cost of transacting associated with reusable material g for industrial decomposer k conducting with industrial producer i, $c_{kig} = c_{kig}(q_{kig})$
$\hat{c}_{jkw}$	cost of transacting associated with waste w for industrial decomposer k conducting with industrial consumer j, $\hat{c}_{jkw} = \hat{c}_{jkw}(q_{jkw})$
ϕ_k	cost of separating and detecting wastes associated with industrial decomposer k, $\phi_k = \phi_k(q_{ikz}, q_{jkw})$

focus on the industrial producers, then turn to the industrial consumers, subsequently to the industrial decomposers, and finally to the demand markets.

1.1. The behavior of the industrial producers and their nonlinear complementarity conditions

We assume that each industrial producer is faced with two criteria: the maximization of profit and the minimization of total emissions generated in producing and remanufacturing. Let ρ_{ijx} denote the selling price of per unit of semi-product x from industrial producer i to industrial consumer j; let ρ_{ijy} denote the selling price of per unit of by-product y from industrial producer i to industrial consumer j and let ρ_{jiv} denote the selling price of per unit of by-product v from industrial consumer j to industrial producer i; let ρ_{ikz} denote the selling price of per unit of waste z from industrial producer i to industrial decomposer k and let ρ_{kig} denote the selling price of per unit of reusable material g from industrial decomposer k to industrial producer i. The economic profit maximization problem, hence, faced by industrial producer i can be expressed as

$$
\begin{aligned}
\text{Maximize } & \sum_{j=1}^{n}\sum_{x=1}^{X}\delta_{ijx}\rho_{ijx}q_{ijx} + \sum_{j=1}^{n}\sum_{y=1}^{Y}\delta_{ijy}\rho_{ijy}q_{ijy} + \sum_{k=1}^{o}\sum_{z=1}^{Z}\delta_{ikz}\rho_{ikz}q_{ikz} \\
& - \sum_{k=1}^{o}\sum_{g=1}^{G}\delta_{kig}\rho_{kig}q_{kig} - \sum_{j=1}^{n}\sum_{v=1}^{V}\delta_{jiv}\rho_{jiv}q_{jiv} \\
& - \sum_{x=1}^{X}\delta_{ix}\left[f_{ix1}\left(\tilde{q}_{ix},\alpha_{ix}\right) + f_{ix2}\left(q_{kig},\alpha_{g}\right) + f_{ix3}\left(q_{jiv},\alpha_{v}\right) + \tilde{f}_{ix}\left(\tilde{q}_{ix}\right)\right] \\
& - \sum_{j=1}^{n}\sum_{x=1}^{X}\delta_{ijx}c_{ijx}\left(q_{ijx}\right) - \sum_{j=1}^{n}\sum_{y=1}^{Y}\delta_{ijy}c_{ijy}\left(q_{ijy}\right) - \sum_{k=1}^{o}\sum_{z=1}^{Z}\delta_{ikz}c_{ikz}\left(q_{ikz}\right) \\
& - \sum_{k=1}^{o}\sum_{g=1}^{G}\delta_{kig}\hat{c}_{kig}\left(q_{kig}\right) - \sum_{j=1}^{n}\sum_{v=1}^{V}\delta_{jiv}\hat{c}_{jiv}\left(q_{jiv}\right)
\end{aligned}
\tag{1.1}
$$

subject to:

$$
\sum_{x=1}^{X}\alpha_{ix}\delta_{ix}\tilde{q}_{ix} + \sum_{j=1}^{n}\sum_{v=1}^{V}\alpha_{v}\delta_{jiv}q_{jiv} + \sum_{k=1}^{o}\sum_{g=1}^{G}\alpha_{g}\delta_{kig}q_{kig} = \sum_{j=1}^{n}\sum_{x=1}^{X}\delta_{ijx}q_{ijx} \tag{1.2}
$$

and the nonnegativity constraints that:

$$
\tilde{q}_{ix} \geq 0, \quad q_{ijx} \geq 0, \quad q_{ijy} \geq 0, \quad q_{jiv} \geq 0, \quad q_{ikz} \geq 0, \quad q_{kig} \geq 0. \tag{1.3}
$$

Note that the objective function (1.1) states that an industrial producer's economic profit is equal to sale revenues less costs associated with producing and remanufacturing, the payout to raw material suppliers, the payout to the industrial consumers, the payout to the industrial decomposers and the various costs of transacting. As to the illustration of 'cost of transacting', what's more, Liu and Xu [25] have defined it at the beginning of Section 1 in their paper. Constraint (1.2) reflects that each industrial producer i must satisfy the conservation of flow equation which states that the semi-product volumes shipped to the industrial consumers must be equal to the sum of the semi-product volumes devived from raw materials, by-products and reusable materials.

In addition to the criterion of economic profit maximization, each industrial producer is supposed to seek to minimize emissions composed by its by-products and wastes, which are generated in producing as well as remanfacturing. The second criterion of industrial producer i, thus, can be expressed mathematically as

$$\text{Minimize} \sum_{j=1}^{n}\sum_{y=1}^{Y}\delta_{ijy}q_{ijy}+\sum_{k=1}^{o}\sum_{z=1}^{Z}\delta_{ikz}q_{ikz} \tag{1.4}$$

$$\text{subject to:} \qquad q_{ijy}\geq 0,\ q_{ikz}\geq 0. \tag{1.5}$$

We can now construct the multicriteria decision-making problem facing an industrial producer which allows to weight the criteria of profit maximization and emission minimization in an individual manner. Assume that industrial producer i assigns a nonnegative weight γ_i to the emission minimization, and the weight associated with the profit maximization serves as the numeraire and is set equal to 1. The nonnegative weights measure the importance of emission and transform these values into monetary units. Thus, the multicriteria decision-making problem of industrial producers i can be expressed as

$$\begin{aligned}\text{Maximize} \ &\sum_{j=1}^{n}\sum_{x=1}^{X}\delta_{ijx}\rho_{ijx}q_{ijx}+\sum_{j=1}^{n}\sum_{y=1}^{Y}\delta_{ijy}\rho_{ijy}q_{ijy}+\sum_{k=1}^{o}\sum_{z=1}^{Z}\delta_{ikz}\rho_{ikz}q_{ikz}\\ &-\sum_{k=1}^{o}\sum_{g=1}^{G}\delta_{kig}\rho_{kig}q_{kig}-\sum_{j=1}^{n}\sum_{v=1}^{V}\delta_{jiv}\rho_{jiv}q_{jiv}\\ &-\sum_{x=1}^{X}\delta_{ix}\left[f_{ix1}\left(\tilde{q}_{ix},\alpha_{ix}\right)+f_{ix2}\left(q_{kig},\alpha_{g}\right)+f_{ix3}\left(q_{jiv},\alpha_{v}\right)+\tilde{f}_{ix}\left(\tilde{q}_{ix}\right)\right]\\ &-\sum_{j=1}^{n}\sum_{x=1}^{X}\delta_{ijx}c_{ijx}\left(q_{ijx}\right)-\sum_{j=1}^{n}\sum_{y=1}^{Y}\delta_{ijy}c_{ijy}\left(q_{ijy}\right)\\ &-\sum_{k=1}^{o}\sum_{z=1}^{Z}\delta_{ikz}c_{ikz}\left(q_{ikz}\right)-\sum_{k=1}^{o}\sum_{g=1}^{G}\delta_{kig}\hat{c}_{kig}\left(q_{kig}\right)\\ &-\sum_{j=1}^{n}\sum_{v=1}^{V}\delta_{jiv}\hat{c}_{jiv}\left(q_{jiv}\right)-\gamma_i\left[\sum_{j=1}^{n}\sum_{y=1}^{Y}\delta_{ijy}q_{ijy}+\sum_{k=1}^{o}\sum_{z=1}^{Z}\delta_{ikz}q_{ikz}\right]\end{aligned} \tag{1.6}$$

subject to: the conservation of flow equation (1.2) and non-negativity constraints (1.3).

It is necessary to assume that the functions f_{ix1}, f_{ix2}, f_{ix3}, c_{ijx}, c_{ijy}, c_{ikz} and $\hat{c}_{jiv}$, are convex and continuously differentiable with regard to their respective decision variables. Then, it can be easily seen that the feasible region of the above nonlinear optimization problem is defined on a nonnegative orthant and can be equivalently transformed into a nonlinear complementarity problem through *Karush–Kuhn–Tucker* conditions. Hence, the formulation of the nonlinear complementarity conditions (represented by $IPNCC$) for all the industrial producers is expressed by: determine $(\tilde{q}^*, Q_1^*, Q_2^*, Q_3^*, Q_4^*, Q_6^*) \geq 0$ satisfying

$$\begin{cases} \delta_{ix} \cdot \left[\dfrac{\partial \tilde{f}_{ix}\left(\tilde{q}_{ix}^*\right)}{\partial \tilde{q}_{ix}} + \dfrac{\partial f_{ix1}\left(\tilde{q}_{ix}^*, \alpha_{ix}\right)}{\partial \tilde{q}_{ix}} - \alpha_{ix}\lambda_{ix}^* \right] \times \tilde{q}_{ix}^* = 0 \\ \delta_{ix} \cdot \left[\dfrac{\partial \tilde{f}_{ix}\left(\tilde{q}_{ix}^*\right)}{\partial \tilde{q}_{ix}} + \dfrac{\partial f_{ix1}\left(\tilde{q}_{ix}^*, \alpha_{ix}\right)}{\partial \tilde{q}_{ix}} - \alpha_{ix}\lambda_{ix}^* \right] \geq 0 \\ \tilde{q}_{ix}^* \geq 0 \qquad \forall i = 1, 2\ldots, m; x = 1, 2\ldots, X. \end{cases} \tag{1.7}$$

$$\begin{cases} \delta_{ijx} \cdot \left[\dfrac{\partial c_{ijx}\left(q_{ijx}^*\right)}{\partial q_{ijx}} + \lambda_{ix}^* - \rho_{ijx}^* \right] \times q_{ijx}^* = 0 \\ \delta_{ijx} \cdot \left[\dfrac{\partial c_{ijx}\left(q_{ijx}^*\right)}{\partial q_{ijx}} + \lambda_{ix}^* - \rho_{ijx}^* \right] \geq 0 \\ q_{ijx}^* \geq 0 \qquad \forall i = 1, 2\ldots, m; j = 1, 2\ldots, n; x = 1, 2\ldots, X. \end{cases} \tag{1.8}$$

$$\begin{cases} \delta_{ijy} \cdot \left[\dfrac{\partial c_{ijy}\left(q_{ijy}^*\right)}{\partial q_{ijy}} - \rho_{ijy}^* + \gamma_i^* \right] \times q_{ijy}^* = 0 \\ \delta_{ijy} \cdot \left[\frac{\partial c_{ijy}\left(q_{ijy}^*\right)}{\partial q_{ijy}} - \rho_{ijy}^* + \gamma_i^* \right] \geq 0 \\ q_{ijy}^* \geq 0 \qquad \forall i = 1, 2\ldots, m; j = 1, 2\ldots, n; y = 1, 2\ldots, Y. \end{cases} \tag{1.9}$$

$$\begin{cases} \delta_{jiv} \cdot \left[\rho_{jiv}^* + \dfrac{\partial f_{ix3}\left(q_{jiv}^*, \alpha_v\right)}{\partial q_{jiv}} + \dfrac{\hat{\partial} c_{jiv}\left(q_{jiv}^*\right)}{\partial q_{jiv}} - \alpha_v\lambda_{ix}^* \right] \times q_{jiv}^* = 0 \\ \delta_{jiv} \cdot \left[\rho_{jiv}^* + \dfrac{\partial f_{ix3}\left(q_{jiv}^*, \alpha_v\right)}{\partial q_{jiv}} + \dfrac{\partial \hat{c}_{jiv}\left(q_{jiv}^*\right)}{\partial q_{jiv}} - \alpha_v\lambda_{ix}^* \right] \geq 0 \\ q_{jiv}^* \geq 0 \qquad \forall j = 1, 2\ldots, n; i = 1, 2\ldots, m; v = 1, 2\ldots, V. \end{cases} \tag{1.10}$$

$$\begin{cases} \delta_{ikz} \cdot \left[\dfrac{\partial c_{ikz}\left(q_{ikz}^*\right)}{\partial q_{ikz}} - \rho_{ikz}^* + \gamma_i^* \right] \times q_{ikz}^* = 0 \\ \delta_{ikz} \cdot \left[\dfrac{\partial c_{ikz}\left(q_{ikz}^*\right)}{\partial q_{ikz}} - \rho_{ikz}^* + \gamma_i^* \right] \geq 0 \\ q_{ikz}^* \geq 0 \qquad \forall i = 1, 2\ldots, m; k = 1, 2\ldots, o; z = 1, 2\ldots, Z. \end{cases} \tag{1.11}$$

$$\begin{cases} \left[\delta_{kig}\rho^*_{kig} + \delta_{ix} \cdot \dfrac{\partial f_{ix2}\left(q^*_{kig}, \alpha_g\right)}{\partial q_{kig}} + \delta_{kig} \cdot \dfrac{\partial \hat{c}_{kig}\left(q^*_{kig}\right)}{\partial q_{kig}} \right. \\ \qquad \left. -\delta_{kig}\alpha_g\lambda^*_{ix} \right] \times q^*_{kig} = 0 \\ \delta_{kig}\rho^*_{kig} + \delta_{ix} \cdot \dfrac{\partial f_{ix2}\left(q^*_{kig}, \alpha_g\right)}{\partial q_{kig}} + \delta_{kig} \cdot \dfrac{\partial \hat{c}_{kig}\left(q^*_{kig}\right)}{\partial q_{kig}} - \delta_{kig}\alpha_g\lambda^*_{ix} \geq 0 \\ q^*_{kig} \geq 0 \qquad \forall k = 1, 2\ldots, o; i = 1, 2\ldots, m; g = 1, 2\ldots, G. \end{cases} \tag{1.12}$$

Note that λ^*_{ix} is the Lagrange multiplier associated with constraint (1.2) for industrial producer i. Such a Lagrange multiplier has an interpretation as the minimum supply cost that industrial producer i is willing to pay for producing or remanufacturing a unit of semi-product x at most. The economic interpretations of the above *IPNCC* are now highlighted. It is necessary to illustrate that all the binary parameters involved with productions or transactions in the following economic interpretations are equal to 1.

From condition (1.7), we can see that, if industrial producer i purchases a positive number of raw materials, then its marginal purchasing cost plus marginal production cost (relevant to the raw material) should be equal to the minimum supply cost that it is willing to pay for a unit of semi-product times the transformation ratio associated with the raw material. Otherwise, industrial producer i will purchase zero volume of the raw material.

Condition (1.8) states that, if there is a positive shipment of the semi-product transacted from industrial producer i to industrial consumer j, then the sum of its minimum supply cost and marginal cost of transacting (relevant to semi-product transactions) must be equal to the price that industrial consumer j is willing to pay for a unit of semi-product. Otherwise, there will be zero volume of the semi-product flow between the particular industrial producer and industrial consumer pair. Condition (1.10) has a similar interpretation.

Condition (1.9) notes that if there is a positive shipment of the by-product generated by industrial producer i to industrial consumer j, then the sum of the marginal cost of transacting (relevant to these by-product transactions) and the environmental weight of industrial producer i is equal to the the price that industrial consumer j is willing to pay for a unit of by-product. Condition (1.11) has a similar interpretation.

Moreover, it can be shown from conditions (1.11) that if industrial producer i purchases a positive amount of the reusable material, then the sum of its marginal remanufacturing cost and marginal cost of transacting (relevant to these reusable material transactions) should be equal to the minimum supply cost times the transformation ratio associated with the reusable material.

1.2. The behavior of the industrial consumers and their nonlinear complementarity conditions

Let ρ_{jlu} denote the selling price of per unit of finished product u from industrial consumer j to demand market l and let ρ_{jkw} denote the selling price of per unit of waste w from industrial consumer j to industrial decomposer k. The economic profit maximization problem, hence, faced by industrial consumer j can be expressed as

$$\begin{aligned}\text{Maximize } & \sum_{u=1}^{U}\sum_{l=1}^{L}\rho_{jlu}q_{jlu} + \sum_{i=1}^{m}\sum_{v=1}^{V}\delta_{jiv}\rho_{jiv}q_{jiv} + \sum_{k=1}^{o}\sum_{w=1}^{W}\eta_{jkw}\rho_{jkw}q_{jkw} \\ & - \sum_{i=1}^{m}\sum_{x=1}^{X}\delta_{ijx}\rho_{ijx}q_{ijx} - \sum_{i=1}^{m}\sum_{y=1}^{Y}\delta_{ijy}\rho_{ijy}q_{ijy} - \sum_{u=1}^{U}\eta_{ju}f_{ju}\left(q_{ijx}, q_{ijy}, \beta_x, \beta_y\right) \\ & - \sum_{i=1}^{m}\sum_{x=1}^{X}\delta_{ijx}\hat{c}_{ijx}\left(q_{ijx}\right) - \sum_{i=1}^{m}\sum_{y=1}^{Y}\delta_{ijy}\hat{c}_{ijy}\left(q_{ijy}\right) - \sum_{u=1}^{U}\sum_{l=1}^{L}c_{jlu}\left(q_{jlu}\right) \\ & - \sum_{i=1}^{m}\sum_{v=1}^{V}\delta_{jiv}c_{jiv}\left(q_{jiv}\right) - \sum_{k=1}^{o}\sum_{w=1}^{W}\eta_{jkw}c_{jkw}\left(q_{jkw}\right) \end{aligned} \tag{1.13}$$

$$\text{subject to: } \sum_{i=1}^{m}\sum_{y=1}^{Y}\beta_y\delta_{ijy}q_{ijy} + \sum_{i=1}^{m}\sum_{x=1}^{X}\beta_x\delta_{ijx}q_{ijx} = \sum_{u=1}^{U}\sum_{l=1}^{L}q_{jlu} \tag{1.14}$$

and the nonnegativity constraints that:

$$q_{ijx} \geq 0, \quad q_{ijy} \geq 0, \quad q_{jiv} \geq 0, \quad q_{jkw} \geq 0, \quad q_{jlu} \geq 0. \tag{1.15}$$

Note that in (1.13) the first three terms represent the revenue, whereas the subsequent eight terms state the various costs. Constraints (1.14) reflects that each industrial consumer must satisfy the conservation of flow equation which states that the product volumes shipped to demand markets must be equal to the sum of the product volumes produced from semi-products and by-products.

In addition to the criterion of profit maximization, each industrial consumer j is supposed to seek to minimize emissions including its by-products and wastes, which are generated in the part of deep processing. The second criterion of industrial consumer j can be expressed mathematically as

$$\text{Minimize } \sum_{i=1}^{m}\sum_{v=1}^{V}\delta_{jiv}q_{jiv} + \sum_{k=1}^{o}\sum_{w=1}^{W}\eta_{jkw}q_{jkw} \tag{1.16}$$

$$\text{subject to: } q_{jiv} \geq 0, \quad q_{jkw} \geq 0. \tag{1.17}$$

It is assumed that industrial consumer j assigns a nonnegative weight ω_j to the emissions according to its individual preference. Similar with industrial producers, the weight associated with profit maximization of industrial consumer j serves as the numeraire and it is set equal to 1. Thus, the multicriteria decision-making problem of industrial consumer j can be expressed as

$$\begin{aligned}\text{Maximize } & \sum_{u=1}^{U}\sum_{l=1}^{L}\rho_{jlu}q_{jlu}+\sum_{i=1}^{m}\sum_{v=1}^{V}\delta_{jiv}\rho_{jiv}q_{jiv}+\sum_{k=1}^{o}\sum_{w=1}^{W}\eta_{jkw}\rho_{jkw}q_{jkw}\\ & -\sum_{i=1}^{m}\sum_{y=1}^{Y}\delta_{ijy}\rho_{ijy}q_{ijy}-\sum_{i=1}^{m}\sum_{x=1}^{X}\delta_{ijx}\rho_{ijx}q_{ijx}-\sum_{u=1}^{U}\eta_{ju}f_{ju}\left(q_{ijx},q_{ijy},\beta_x,\beta_y\right)\\ & -\sum_{i=1}^{m}\sum_{x=1}^{X}\delta_{ijx}\hat{c}_{ijx}\left(q_{ijx}\right)-\sum_{i=1}^{m}\sum_{y=1}^{Y}\delta_{ijy}\hat{c}_{ijy}\left(q_{ijy}\right)-\sum_{u=1}^{U}\sum_{l=1}^{L}c_{jlu}\left(q_{jlu}\right)\\ & -\sum_{i=1}^{m}\sum_{v=1}^{V}\delta_{jiv}c_{jiv}\left(q_{jiv}\right)-\sum_{k=1}^{o}\sum_{w=1}^{W}\eta_{jkw}c_{jkw}\left(q_{jkw}\right)\\ & -\omega_j\left(\sum_{i=1}^{m}\sum_{v=1}^{V}\delta_{jiv}q_{jiv}+\sum_{k=1}^{o}\sum_{w=1}^{W}\eta_{jkw}q_{jkw}\right)\end{aligned} \tag{1.18}$$

subject to: the conservation of flow equation (1.14) and non-negativity constraints (1.15).

It is necessary to assume that the functions f_{ju}, $\hat{c}_{ijx}$, $\hat{c}_{ijy}$, c_{jlu}, c_{jiv} and c_{jkw}, are convex and continuously differentiable with regard to their respective decision variables. Then, it can be easily seen that above nonlinear optimization problem can be equivalently transformed into a nonlinear complementarity problem through *Karush–Kuhn–Tucker* conditions. Hence, the nonlinear complementarity conditions (represented by *ICNCC*) for all the industrial consumers are expressed by:

determine $(Q_1^*, Q_2^*, Q_3^*, Q_5^*, Q_7^*) \geq 0$ satisfying

$$\begin{cases}\left[\eta_{ju}\cdot\dfrac{\partial f_{ju}\left(q_{ijx}^*,q_{ijy}^*,\beta_x,\beta_y\right)}{\partial q_{ijx}}+\delta_{ijx}\cdot\dfrac{\partial\hat{c}_{ijx}\left(q_{ijx}^*\right)}{\partial q_{ijx}}+\delta_{ijx}\rho_{ijx}^*\right.\\ \qquad\left.-\beta_x\delta_{ijx}\mu_{ju}^*\right]\times q_{ijx}^*=0\\ \eta_{ju}\cdot\dfrac{\partial f_{ju}\left(q_{ijx}^*,q_{ijy}^*,\beta_x,\beta_y\right)}{\partial q_{ijx}}+\delta_{ijx}\cdot\dfrac{\partial\hat{c}_{ijx}\left(q_{ijx}^*\right)}{\partial q_{ijx}}+\delta_{ijx}\rho_{ijx}^*\\ \qquad-\beta_x\delta_{ijx}\mu_{ju}^*\geq 0\\ q_{ijx}^*\geq 0\qquad\forall i=1,2\ldots,m;j=1,2\ldots,n;x=1,2\ldots,X.\end{cases} \tag{1.19}$$

$$\begin{cases} \left[\delta_{ijy}\rho^*_{ijy} + \eta_{ju} \cdot \dfrac{\partial f_{ju}\left(q^*_{ijx}, q^*_{ijy}, \beta_x, \beta_y\right)}{\partial q_{ijy}} + \delta_{ijy} \cdot \dfrac{\partial \hat{c}_{ijy}\left(q^*_{ijy}\right)}{\partial q_{ijy}} \right. \\ \qquad \left. -\delta_{ijy}\beta_y\mu^*_{ju}\right] \times q^*_{ijy} = 0 \\ \delta_{ijy}\rho^*_{ijy} + \eta_{ju} \cdot \dfrac{\partial f_{ju}\left(q^*_{ijx}, q^*_{ijy}, \beta_x, \beta_y\right)}{\partial q_{ijy}} + \delta_{ijy} \cdot \dfrac{\partial \hat{c}_{ijy}\left(q^*_{ijy}\right)}{\partial q_{ijy}} \\ \qquad -\delta_{ijy}\beta_y\mu^*_{ju} \geq 0 \\ q^*_{ijy} \geq 0 \qquad \forall i = 1, 2 \ldots, m; j = 1, 2 \ldots, n; y = 1, 2 \ldots, Y. \end{cases} \tag{1.20}$$

$$\begin{cases} \delta_{jiv} \cdot \left[\dfrac{\partial c_{jiv}\left(q^*_{jiv}\right)}{\partial q_{jiv}} - \rho^*_{jiv} + \omega^*_j\right] \times q^*_{jiv} = 0 \\ \delta_{jiv} \cdot \left[\dfrac{\partial c_{jiv}\left(q^*_{jiv}\right)}{\partial q_{jiv}} - \rho^*_{jiv} + \omega^*_j\right] \geq 0 \\ q^*_{jiv} \geq 0 \qquad \forall j = 1, 2 \ldots, n; i = 1, 2 \ldots, m; v = 1, 2 \ldots, V. \end{cases} \tag{1.21}$$

$$\begin{cases} \eta_{jkw} \cdot \left[\dfrac{\partial c_{jkw}\left(q^*_{jkw}\right)}{\partial q_{jkw}} - \rho^*_{jkw} + \omega^*_j\right] \times q^*_{jkw} = 0 \\ \eta_{jkw} \cdot \left[\dfrac{\partial c_{jkw}\left(q^*_{jkw}\right)}{\partial q_{jkw}} - \rho^*_{jkw} + \omega^*_j\right] \geq 0 \\ q^*_{jkw} \geq 0 \qquad \forall j = 1, 2 \ldots, n; k = 1, 2 \ldots, o; w = 1, 2 \ldots, W. \end{cases} \tag{1.22}$$

$$\begin{cases} \left[\dfrac{\partial c_{jlu}\left(q^*_{jlu}\right)}{\partial q_{jlu}} + \mu^*_{ju} - \rho^*_{jlu}\right] \times q^*_{jlu} = 0 \\ \dfrac{\partial c_{jlu}\left(q^*_{jlu}\right)}{\partial q_{jlu}} + \mu^*_{ju} - \rho^*_{jlu} \geq 0 \\ q^*_{jlu} \geq 0 \qquad \forall j = 1, 2 \ldots, n; l = 1, 2 \ldots, L; u = 1, 2 \ldots, U. \end{cases} \tag{1.23}$$

Note that μ^*_{ju} is the Lagrange multiplier associated with constraint (1.14) for industrial consumer j. Such a Lagrange multiplier has an interpretation as the minimum supply cost that industrial consumer j is willing to pay for deep processing to obtain a unit of finished product u at most. The economic interpretations of the above *ICNCN* are now highlighted. Similar with the industrial producers, it is also necessary to illustrate that all the binary parameters involved with the following economic interpretations are equal to 1.

Condition (1.19) states that if there is a positive shipment of the semi-product transacted from industrial producer i to industrial consumers j, then the sum of the marginal production cost (related to semi-product transactions), the marginal cost of transacting (relevant to semi-product transactions) from industrial consumer j's perspective and the price that it is willing to pay for a unit of semi-product must be equal to its minimum supply cost times the transformation ratio of the semi-product. Condition (1.20) has a similar economic interpretation.

It can be shown from conditions (1.21) that if there is a positive shipment of the by-product transacted from industrial consumer j to industrial producer i, then the sum of its marginal cost of transacting (relevant to these by-product transactions) and the environmental weight of industrial consumer j is equal to the the price that industrial producer i is willing to pay for a unit of by-product. Condition (1.22) has a similar economic interpretation.

From condition (1.23), we can see that if there is a positive shipment of the finished product transacted from industrial consumer j to demand market l, then the sum of its minimum supply cost and the marginal cost of transacting (relevant to these finished transactions) is equal to the the price that customers at demand market l are willing to pay for a unit of finished product.

1.3. The behavior of the industrial decomposers and their nonlinear complementarity conditions

Given the notations previously mentioned, each industrial decomposer k wishes to fulfill its economic profit maximization that can be expressed as:

$$\begin{aligned}\text{Maximize } &\sum_{k=1}^{o}\sum_{g=1}^{G}\delta_{kig}\rho_{kig}q_{kig}-\sum_{i=1}^{m}\sum_{z=1}^{Z}\delta_{ikz}\rho_{ikz}q_{ikz}\\ &-\sum_{j=1}^{n}\sum_{w=1}^{W}\eta_{jkw}\rho_{jkw}q_{jkw}-\sum_{i=1}^{m}\sum_{g=1}^{G}\delta_{kig}c_{kig}\left(q_{kig}\right)\\ &-\sum_{i=1}^{m}\sum_{z=1}^{Z}\delta_{ikz}\hat{c}_{ikz}\left(q_{ikz}\right)-\sum_{j=1}^{n}\sum_{w=1}^{W}\eta_{jkw}\hat{c}_{jkw}\left(q_{jkw}\right)-\phi_k\left(q_{ikz},q_{jkw}\right)\\ &-\bar{\rho}\cdot\left[\sum_{i=1}^{m}\sum_{z=1}^{Z}\left(1-\chi_z\right)\delta_{ikz}q_{ikz}+\sum_{j=1}^{n}\sum_{w=1}^{W}\left(1-\chi_w\right)\eta_{jkw}q_{jkw}\right]\end{aligned}\tag{1.24}$$

$$\text{subject to: } \sum_{i=1}^{m}\sum_{z=1}^{Z}\chi_z\delta_{ikz}q_{ikz}+\sum_{j=1}^{n}\sum_{w=1}^{W}\chi_w\eta_{jkw}q_{jkw}=\sum_{i=1}^{m}\sum_{g=1}^{G}\delta_{kig}q_{kig}\tag{1.25}$$

and the nonnegativity constraints that:

$$q_{ikz}\geq 0,\quad q_{jkw}\geq 0,\quad q_{kig}\geq 0.\tag{1.26}$$

Note that the objective function (1.24) states that an industrial decomposer's profit is equal to sales revenues less costs associated with the payout to the industrial producers, the payout to the industrial consumers, the various costs of transacting and the disposal fee for sending useless materials to the landfill after extracting. Constraints (1.25) reflects that each industrial decomposer k must satisfy the conservation of flow equation which notes that the reusable material volumes shipped to the industrial producers must be equal to the sum of the waste volumes from the industrial producers as well as the industrial consumers.

It is necessary to assume that the functions $\hat{c}_{ikz}$, c_{kig}, $\hat{c}_{jkw}$ and ϕ_k, are convex and continuously differentiable concerning their respective decision variables. Then, it is easy to obtain that above nonlinear optimization problem can be equivalently transformed into a nonlinear complementarity problem through *Karush–Kuhn–Tucker* conditions. Hence, the formulation of the nonlinear complementarity conditions (represented by *IDNCC*) all the industrial decomposers is expressed by:

determine $(Q_4^*, Q_5^*, Q_6^*) \geq 0$ satisfying

$$\begin{cases} \left[\delta_{ikz}\rho_{ikz}^* + \delta_{ikz} \cdot \dfrac{\partial \hat{c}_{ikz}\left(q_{ikz}^*\right)}{\partial q_{ikz}} + \dfrac{\partial \phi_k\left(q_{ikz}^*, q_{jkw}^*\right)}{\partial q_{ikz}} \right. \\ \qquad \left. + \delta_{ikz}\bar{\rho}\left(1-\chi_z\right) - \delta_{ikz}\chi_z\upsilon_{kg}^* \right] \times q_{ikz}^* = 0 \\ \delta_{ikz}\rho_{ikz}^* + \delta_{ikz} \cdot \dfrac{\partial \hat{c}_{ikz}\left(q_{ikz}^*\right)}{\partial q_{ikz}} + \dfrac{\partial \phi_k\left(q_{ikz}^*, q_{jkw}^*\right)}{\partial q_{ikz}} \\ \qquad + \delta_{ikz}\bar{\rho}\left(1-\chi_z\right) - \delta_{ikz}\chi_z\upsilon_{kg}^* \geq 0 \\ q_{ikz}^* \geq 0 \qquad \forall i = 1, 2 \ldots, m; k = 1, 2 \ldots, o; z = 1, 2 \ldots, Z. \end{cases} \tag{1.27}$$

$$\begin{cases} \left[\eta_{jkw}\rho_{jkw}^* + \eta_{jkw} \cdot \dfrac{\partial \hat{c}_{jkw}\left(q_{jkw}^*\right)}{\partial q_{jkw}} + \dfrac{\partial \phi_k\left(q_{ikz}^*, q_{jkw}^*\right)}{\partial q_{jkw}} + \eta_{jkw}\bar{\rho}\left(1-\chi_w\right) \right. \\ \qquad \left. - \eta_{jkw}\chi_w\upsilon_{kg}^* \right] \times q_{jkw}^* = 0 \\ \eta_{jkw}\rho_{jkw}^* + \eta_{jkw} \cdot \dfrac{\partial \hat{c}_{jkw}\left(q_{jkw}^*\right)}{\partial q_{jkw}} + \dfrac{\partial \phi_k\left(q_{ikz}^*, q_{jkw}^*\right)}{\partial q_{jkw}} \\ \qquad + \eta_{jkw}\bar{\rho}\left(1-\chi_w\right) - \eta_{jkw}\chi_w\upsilon_{kg}^* \geq 0 \\ q_{jkw}^* \geq 0 \qquad \forall j = 1, 2 \ldots, n; k = 1, 2 \ldots, o; w = 1, 2 \ldots, W. \end{cases} \tag{1.28}$$

$$\begin{cases} \delta_{kig} \cdot \left[\dfrac{\partial c_{kig}\left(q^*_{kig}\right)}{\partial q_{kig}} + \upsilon^*_{kg} - \rho^*_{kig} \right] \times q^*_{kig} = 0 \\ \delta_{kig} \cdot \left[\dfrac{\partial c_{kig}\left(q^*_{kig}\right)}{\partial q_{kig}} + \upsilon^*_{kg} - \rho^*_{kig} \right] \geq 0 \\ q^*_{kig} \geq 0 \qquad \forall k = 1, 2 \ldots, o; i = 1, 2 \ldots, m; g = 1, 2 \ldots, G. \end{cases} \tag{1.29}$$

It is worth noting that υ^*_{kg} is the Lagrange multiplier associated with constraint (1.25) for industrial decomposer k. Such a Lagrange multiplier also has an interpretation as the minimum supply cost that industrial decomposer k is willing to pay for gaining a unit of reusable material g at most. The economic interpretations of the above *IDNCC* are now highlighted. Similar with the former two-tier decision-makers in the industrial symbiosis network, the binary parameters involved with the following transactions are also equal to 1.

Condition (1.27) states that if there is a positive shipment of the waste transacted from industrial producer i to industrial decomposer k, then the sum of its marginal cost of transacting (relevant to waste transactions), the marginal cost of separating and detecting, the disposing fee of useless materials to the landfill and the price that it is willing to pay for a unit of waste must be equal to its minimum supply cost times the transformation ratio associated with wastes. Condition (1.28) has a similar economic interpretation.

It can be shown from condition (1.29) that if there is a positive shipment of the reusable material transacted from industrial decomposer k to industrial producer i, then the sum of its marginal cost of transacting (relevant to these reusable material transactions) and minimum supply cost is equal to the the price that industrial producer i is willing to pay for a unit of reusable material.

1.4. The behavior of customers at the demand markets and the nonlinear complementarity conditions

Customers at different demand markets take into account in making their consumption decisions not only the charged price by the sellers (*i.e.* the industrial consumers) but also the cost of transacting associated with obtaining finished products from their perspective.

Let $\hat{c}_{jlu}$ denote the cost of transacting associated with demand market l obtaining finished product u from industrial consumer j, which is assumed to be continuous and of the general form: $\hat{c}_{jlu} = \hat{c}_{jlu}\left(q_{jlu}\right)$. In addition, ρ_u is the L-dimensional column vector with the component of ρ_{lu} which denotes the price of demand market l for obtaining finished product u from the industrial consumers. Denote the demand of finished product u at demand market l by d_{lu} and assume, as given, the continuous function: $d_{lu} = d_{lu}\left(\rho_u\right)$. It is assumed that the function $\hat{c}_{jlu}$ is convex and continuous concerning the finished product shipment q_{jlu}, then the equilibrium conditions for customers at demand market l take the following form: for $j = 1, 2\ldots, n$ and $u = 1, 2\ldots, U$:

$$\rho_{jlu}^{*} + \hat{c}_{jlu}\left(q_{jlu}^{*}\right) \begin{cases} = \rho_{lu}^{*}, \ if \ q_{jlu}^{*} > 0 \\ \geq \rho_{lu}^{*}, \ if \ q_{jlu}^{*} = 0 \end{cases} \tag{1.30}$$

and

$$d_{lu}\left(\rho_{u}^{*}\right) \begin{cases} = \sum_{j=1}^{n} q_{jlu}^{*}, \ if \ \rho_{lu}^{*} > 0 \\ \leq \sum_{j=1}^{n} q_{jlu}^{*}, \quad if \quad \rho_{lu}^{*} = 0. \end{cases} \tag{1.31}$$

Conditions (1.30) state that, in equilibrium, customers at demand market l will purchase finished product u from industrial consumer j, if the price charged by industrial consumer j plus the cost of transacting (from the perspective of customers) does not exceed the price that customers are willing to pay. Conditions (1.31) represent that if the equilibrium price customers are willing to pay for finished products is positive, then the quantity of finished product u pursued by demand market l is precisely equal to the demand at that market.

In equilibrium, all the demand markets must satisfy conditions (1.30) and (1.31), that is, the equilibrium conditions of the demand markets are equivalent to the following nonlinear complementarity conditions (represented by *DMNCC*):

determine $(Q_7^{*}, \rho_u^{*}) \geq 0$ satisfying

$$\begin{cases} \left[\rho_{jlu}^{*} + \hat{c}_{jlu}\left(q_{jlu}^{*}\right) - \rho_{lu}^{*}\right] \times q_{jlu}^{*} = 0 \\ \rho_{jlu}^{*} + \hat{c}_{jlu}\left(q_{jlu}^{*}\right) - \rho_{lu}^{*} \geq 0 \\ q_{jlu}^{*} \geq 0 \qquad \forall j = 1, 2\ldots, n; l = 1, 2\ldots, L; u = 1, 2\ldots, U. \end{cases} \tag{1.32}$$

$$\begin{cases} \left[\sum_{j=1}^{n} q_{jlu}^{*} - d_{lu}\left(\rho_{u}^{*}\right)\right] \times \rho_{lu}^{*} = 0 \\ \sum_{j=1}^{n} q_{jlu}^{*} - d_{lu}\left(\rho_{u}^{*}\right) \geq 0 \\ \rho_{lu}^{*} \geq 0 \qquad \forall l = 1, 2\ldots, L; u = 1, 2\ldots, U. \end{cases} \tag{1.33}$$

2. The nonlinear complementarity model of the industrial symbiosis network equilibrium conditions

Definition 2.1 (The equilibrium state of the industrial symbiosis network)**.** The equilibrium state of the industrial symbiosis network is one where the product flows between four tiers of the decision-makers coincide and the product outputs, shipments associated with semi-products, finished products, by-products, wastes as well as reusable materials, and prices satisfy the sum of the nonlinear complementarity conditions (IPNCC), (ICNCC), (IDNCC) and (DMNCC).

The equilibrium state is equivalent to the following:

Theorem 2.2 (The nonlinear complementarity model of the industrial symbiosis network equilibrium conditions)**.** *The equilibrium conditions governing the industrial symbiosis network according to Definition 2.1 are equivalent to the solution to the nonlinear complementarity model (NCM) given by: determine* $X^* \in R_+^{mX+mnX+mnY+nmV+moZ+omG+noW+nLU+LU}$ *satisfying*

$$X^* \geq 0,\ F(X^*) \geq 0,\ \ X^{*\mathrm{T}} F(X^*) = 0. \tag{2.1}$$

where the row vector X*:* $X = (\tilde{q}, Q_1, Q_2, Q_3, Q_4, Q_5, Q_6, Q_7, \rho_u) \in R_+^{mX+mnX+mnY+nmV+moZ+omG+noW+nLU+LU}$ *and the row vector function*

$$F(X) : F(X) = \big(F^1(X), F^2(X), F^3(X), F^4(X), F^5(X), F^6(X), F^7(X), F^8(X), F^9(X)\big)^{\mathrm{T}} : R_+^{mX+mnX+mnY+nmV+moZ+omG+noW+nLU+LU} \mapsto R_+^{mX+mnX+mnY+nmV+moZ+omG+noW+nLU+LU}$$

where vector functions are given by:

$$\begin{aligned}
F^1(X) &= \big(\ldots, F^1_{ix}(X), \ldots\big) \in R_+^{mX};\\
F^2(X) &= \big(\ldots, F^2_{ijx}(X), \ldots\big) \in R_+^{mnX};\\
F^3(X) &= \big(\ldots, F^3_{ijy}(X), \ldots\big) \in R_+^{mnY};\\
F^4(X) &= \big(\ldots, F^4_{jiv}(X), \ldots\big) \in R_+^{nmV};\\
F^5(X) &= \big(\ldots, F^5_{ikz}(X), \ldots\big) \in R_+^{moZ};\\
F^6(X) &= \big(\ldots, F^6_{kig}(X), \ldots\big) \in R_+^{omG};\\
F^7(X) &= \big(\ldots, F^7_{jkw}(X), \ldots\big) \in R_+^{now};\\
F^8(X) &= \big(\ldots, F^8_{jlu}(X), \ldots\big) \in R_+^{nLU};\\
F^9(X) &= \big(\ldots, F^9_{lu}(X), \ldots\big) \in R_+^{LU}.
\end{aligned}$$

Each entity in these nine vector functions is defined by:

$$F^1_{ix}(X) = \delta_{ix} \cdot \left[\frac{\partial \tilde{f}_{ix}(\tilde{q}_{ix})}{\partial \tilde{q}_{ix}} + \frac{\partial f_{ix1}(\tilde{q}_{ix}, \alpha_{ix})}{\partial \tilde{q}_{ix}} - \alpha_{ix}\lambda_{ix}\right]$$
$$for\ \ i = 1, 2 \ldots, m; x = 1, 2 \ldots, X.$$

$$\begin{aligned}
F^2_{ijx}(X) = {}& \delta_{ijx} \cdot \frac{\partial c_{ijx}(q_{ijx})}{\partial q_{ijx}} + \delta_{ijx}\lambda_{ix} + \eta_{ju} \cdot \frac{\partial f_{ju}(q_{ijx}, q_{ijy}, \beta_x, \beta_y)}{\partial q_{ijx}}\\
&+ \delta_{ijx} \cdot \frac{\partial \hat{c}_{ijx}(q_{ijx})}{\partial q_{ijx}} - \beta_x \delta_{ijx}\mu_{ju}
\end{aligned}$$
$$for\ \ i = 1, 2 \ldots, m; j = 1, 2 \ldots, n; x = 1, 2 \ldots, X.$$

$$F_{ijy}^{3}(X) = \delta_{ijy} \cdot \frac{\partial \hat{c}_{ijy}(q_{ijy})}{\partial q_{ijy}} + \eta_{ju} \cdot \frac{\partial f_{ju}(q_{ijx}, q_{ijy}, \beta_x, \beta_y)}{\partial q_{ijy}}$$
$$+ \delta_{ijy} \cdot \frac{\partial c_{ijy}(q_{ijy})}{\partial q_{ijy}} - \delta_{ijy}\beta_y\mu_{ju} + \delta_{ijy}\gamma_i$$
$$for \;\; i = 1, 2\ldots, m; j = 1, 2\ldots, n; y = 1, 2\ldots, Y.$$

$$F_{jiv}^{4}(X) = \delta_{jiv} \cdot \left[\frac{\partial f_{ix3}(q_{jiv}, \alpha_v)}{\partial q_{jiv}} + \frac{\partial c_{jiv}(q_{jiv})}{\partial q_{jiv}} + \frac{\partial \hat{c}_{jiv}(q_{jiv})}{\partial q_{jiv}} - \alpha_v\lambda_{ix} + \omega_j\right]$$
$$for \; j = 1, 2\ldots, n; i = 1, 2\ldots, m; v = 1, 2\ldots, V.$$

$$F_{ikz}^{5}(X) = \delta_{ikz} \cdot \frac{\partial c_{ikz}(q_{ikz})}{\partial q_{ikz}} + \delta_{ikz} \cdot \frac{\partial \hat{c}_{ikz}(q_{ikz})}{\partial q_{ikz}} + \frac{\partial \phi_k(q_{ikz}, q_{jkw})}{\partial q_{ikz}}$$
$$+ \delta_{ikz}\bar{\rho}(1 - \chi_z) - \delta_{ikz}\chi_z\upsilon_{vg} + \delta_{ikz}\gamma_i,$$
$$for \;\; i = 1, 2\ldots, m; \;\; k = 1, 2\ldots, o; z = 1, 2\ldots, Z.$$

$$F_{kig}^{6}(X) = \delta_{ix} \cdot \frac{\partial f_{ix2}(q_{kig}, \alpha_g)}{\partial q_{kig}} + \delta_{kig} \cdot \frac{\partial \hat{c}_{kig}(q_{kig})}{\partial q_{kig}} + \delta_{kig} \cdot \frac{\partial c_{kig}(q_{kig})}{\partial q_{kig}}$$
$$- \delta_{kig}\alpha_g\lambda_{ix} + \delta_{kig}\upsilon_{kg}$$
$$for \;\; k = 1, 2\ldots, o; i = 1, 2\ldots, m; g = 1, 2\ldots, G.$$

$$F_{jkw}^{7}(X) = \eta_{jkw} \cdot \frac{\partial c_{jkw}(q_{jkw})}{\partial q_{jkw}} + \eta_{jkw} \cdot \frac{\partial \hat{c}_{jkw}(q_{jkw})}{\partial q_{jkw}} + \frac{\partial \phi_k(q_{ikz}, q_{jkw})}{\partial q_{jkw}}$$
$$+ \eta_{jkw}\bar{\rho}(1 - \chi_w) - \eta_{jkw}\chi_w\upsilon_{kg} + \omega_j$$
$$for \; j = 1, 2\ldots, n; k = 1, 2\ldots, o; w = 1, 2\ldots, W.$$

$$F_{jlu}^{8}(X) = \frac{\partial c_{jlu}(q_{jlu})}{\partial q_{jlu}} + \mu_{ju} + \hat{c}_{jlu}(q_{jlu}) - \rho_{ju}$$
$$for \; j = 1, 2\ldots, n; \;\; l = 1, 2\ldots, L; \; u = 1, 2\ldots, U.$$

$$F_{lu}^{9}(X) = \sum_{j=1}^{n} q_{jlu} - d_{lu}(\rho_u), \qquad for \;\; l = 1, 2\ldots, L; \;\; u = 1, 2\ldots, U.$$

Concerning solving the nonlinear complementarity model, one of the efficient methods is the merit function approach. It is a function that can be used to constitute an equivalent minimization problem for a nonlinear complementarity problem, which aims to transform a nonlinear complementarity problem into an unconstrained minimization problem [20]. It is extraordinarily notable that the following simple smoothing function introduced by Fischer [14], which plays a vital role in constructing a merit function for a nonlinear complementarity problem.

$$\tau\left(a,\ b\right)=\left[\sqrt{a^2+b^2}-(a+b)\right]^2:\ R^2\mapsto R_+$$

It is easy to verify that function $\tau\left(a,\ b\right)$ is continuously differentiable with regard to its variables, that is, a and b. It also has a favorable property:

$$\tau\left(a,\ b\right)=0\quad\text{if and only if}\quad a\geq 0,\ b\geq 0,\ a\times b=0.$$

Consequently, the corresponding nonnegative real function in terms of the *Fischer function* can be constructed and defined as given:

$$\begin{aligned}\mathit{\Gamma}\left(X\right)=&\sum_{i=1}^{m}\sum_{x=1}^{X}\tau\left(q_{ix},F^1_{ix}(X)\right)+\sum_{i=1}^{m}\sum_{j=1}^{n}\sum_{x=1}^{X}\tau\left(q_{ijx},F^2_{ijx}(X)\right)\\&+\sum_{i=1}^{m}\sum_{j=1}^{n}\sum_{y=1}^{Y}\tau\left(q_{ijy},F^3_{ijy}(X)\right)+\sum_{j=1}^{n}\sum_{i=1}^{m}\sum_{v=1}^{V}\tau\left(q_{jiv},F^4_{jiv}(X)\right)\\&+\sum_{i=1}^{n}\sum_{k=1}^{o}\sum_{\pounds=1}^{Z}\tau\left(q_{ikz},F^5_{ikz}(X)\right)+\sum_{k=1}^{o}\sum_{i=1}^{m}\sum_{g=1}^{G}\tau\left(q_{kig},F^6_{kig}(X)\right)\\&+\sum_{j=1}^{n}\sum_{k=1}^{o}\sum_{w=1}^{W}\tau\left(q_{jkw},F^7_{jkw}(X)\right)+\sum_{j=1}^{n}\sum_{l=1}^{L}\tau\left(q_{jlu},F^8_{jlu}(X)\right)\\&+\sum_{l=1}^{L}\sum_{u=1}^{U}\tau\left(q_{lu},F^9_{lu}(X)\right).\end{aligned}\tag{2.2}$$

Proposition 2.3. $\mathit{\Gamma}\left(X^*\right)=0$ *if and only if* X^* *is the solution to the nonlinear complementarity model (2.1).*

In the light of $\mathit{\Gamma}\left(X\right)\geq 0$, Proposition 2.3 implies that seeking for a solution to the nonlinear complementarity model (2.1) is equivalent to finding a global minimum of the unconstrained minimization problem (2.3) as follows:

$$\text{Minimize }\mathit{\Gamma}\left(X\right),\quad X\in R_+^{mX+mnX+mnY+nmV+moZ+omG+noW+nLU+LU}\tag{2.3}$$

Proposition 2.4. *Conditions in Theorem 2 of Nagurney et al. (2002) [32] can also make sure that the unconstrained minimization problem (2.3) at least admits one solution.*

Proof. Conditions in Theorem 2 of Nagurney *et al.* (2002) [32] demonstrate that there is a nonnegative vector $B\in R_+^{mX+mnX+mnY+nmV+moZ+omG+noW+nLU+LU}$ such that the following variational inequality (2.4) has a solution represented by X^B which fulfills the condition $0\leq X^B<B$.

Searching for $0 \le X^* \le B$ such that

$$F\left(X^*\right)\left(X - X^*\right)^{\mathrm{T}} \ge 0, \ \ \forall\, 0 \le X \le B \tag{2.4}$$

Due to the condition that $0 \le X^B < B$, there exists a sufficient small but positive number ξ_1 and a row vector $E = (\xi_2, \ldots, \xi_2) \in R_+^{mX+mnX+mnY+nmV+moZ+omG+noW+nLU+LU}$ with element $\xi_2>0$ such that the following conditions simultaneously:

$$0 \le (1+\xi_1)\, X^B \le B \quad \text{and} \quad 0 \le X^B + E \le B$$

When taking three row vectors: $X_1 = (1+\xi_1)\, X^B$, $X_2 = 0.5X^B$ and $X_3 = X^B + E$, we can easily find that $0 \le X_1, X_2, X_3 \le B$. Then, we can obtain through substituting such vectors into the variational inequality (2.4):

$$F\left(X^B\right)\left(X_1 - X^B\right)^{\mathrm{T}} \ge 0;\ F\left(X^B\right)\left(X_2 - X^B\right)^{\mathrm{T}} \ge 0;\ F\left(X^B\right)\left(X_3 - X^B\right)^{\mathrm{T}} \ge 0;$$

that is,

$$\xi_1 F\left(X^B\right)\left(X^B\right)^{\mathrm{T}} \ge 0; -0.5 \times F\left(X^B\right)\left(X^B\right)^{\mathrm{T}} \ge 0; F\left(X^B\right) E^{\mathrm{T}} \ge 0.$$

It is easy to acquire that $F\left(X^B\right)\left(X^B\right)^{\mathrm{T}} = 0$, $F\left(X^B\right) \ge 0$.

Therefore, X^B satisfies $\varGamma\left(X^B\right) = 0$ on the basis of the property of the merit function mentioned. Then, we can conlude that X^B is a global minimum for the unconstrained minimization problem (2.3). The proof is complete. □

It can be shown that Theorem 2.2, Proposition 2.3 and Proposition 2.4 will guarantee the existence of a solution to the network equilibrium of the industrial symbiosis system, that is, the following Theorem 2.5.

Theorem 2.5. *The nonlinear complementarity model (2.1) that is accordance with the network equilibrium of the industrial symbiosis system has at least one solution.*

3. Numerical examples

The numerical examples consist of three industrial producers, three industrial consumers, one industrial decomposer and two demand markets, that is $m = 3$, $n = 3$, $o = 1$, $L = 2$. In addition, it is assumed that $X = 2$, $Y = 1$, $Z = 2$, $U = 2$, $V = 2$, $W = 2$, $G = 2$. Specially, the first and the second industrial producers are involved in producing homogenous semi-product $x = 1$, whereas the third industrial producer is concerned with producing the semi-product $x = 2$. The situation about the three industrial consumers is similar with the three industrial producers, that is, the first two industrial consumers are concerned with deep processing to produce homogenous finished product $u = 1$, whereas the third industrial consumer is involved in producing the finished product $u = 2$. As to

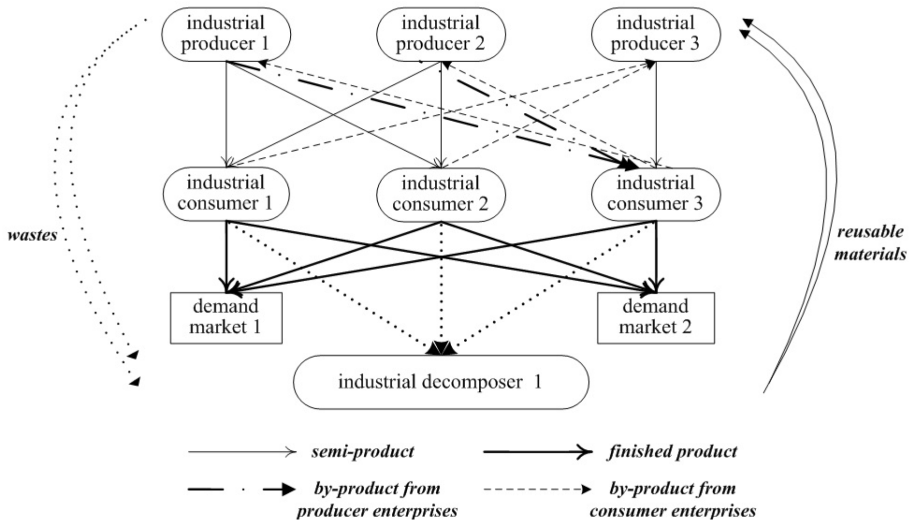

FIGURE 2. The structure of the industrial symbiosis network for numerical examples.

by-products, we only consider the by-product $y = 1$ generated in producing or remanufacturing of the first two industrial producers, but consider two kinds of by-products ($z = 1$ and $z = 2$) generated in the deep processing of the first and the second industrial consumers. The structure of the industrial symbiosis network for the following examples is depicted in Figure 2. The detailed description is given below.

The purchasing cost functions faced by the three industrial producers were:

$$\tilde{f}_{11} = \tilde{q}_{11}^2 + \tilde{q}_{11}\tilde{q}_{21} + 2\tilde{q}_{11}; \quad \tilde{f}_{21} = \tilde{q}_{21}^2 + \tilde{q}_{11}\tilde{q}_{21} + 2\tilde{q}_{21}; \quad \tilde{f}_{32} = \tilde{q}_{32}^2 + 3\tilde{q}_{32}.$$

The cost functions of producing semi-products from raw materials by the industrial producers were:

$$f_{111} = 2.5\left(0.6\tilde{q}_{11}\right)^2 + 0.6\tilde{q}_{11} \cdot 0.6\tilde{q}_{21} + 2\left(0.6\tilde{q}_{11}\right);$$
$$f_{211} = 2.5\left(0.6\tilde{q}_{21}\right)^2 + 0.6\tilde{q}_{11} \cdot 0.6\tilde{q}_{21} + 2\left(0.6\tilde{q}_{21}\right);$$
$$f_{321} = 1.5\left(0.6\tilde{q}_{32}\right)^2 + 5\left(0.6\tilde{q}_{32}\right).$$

The cost functions of remanufacturing to obtain semi-products from reusable materials by the industrial producers were given by:

$$f_{112} = 5\left(0.4q_{111}\right)^2 + 4\left(0.4q_{111}\right);$$
$$f_{212} = 5\left(0.4q_{121}\right)^2 + 4\left(0.4q_{121}\right).$$

The costs of producing semi-product from by-products by the industrial producers:

$$f_{113} = 2.5\left(0.8q_{312}\right)^2 + 1.5\left(0.8q_{312}\right);$$
$$f_{213} = 2.5\left(0.8q_{322}\right)^2 + 1.5\left(0.8q_{322}\right);$$
$$f_{323} = 2\left(0.8q_{131}\right)^2 + 0.8q_{131} \cdot 0.8q_{231} + 2\left(0.8q_{231}\right)^2.$$

The cost of transacting associated with semi-product x for industrial producer i conducting with industrial consumer j, c_{ijx}, and its corresponding binary parameter δ_{ijx}:

$$c_{111} = 0.5\left(q_{111}\right)^2 + 3.5q_{111}; \quad c_{121} = 0.5\left(q_{121}\right)^2 + 3.5q_{121};$$

$$c_{211} = q_{211}^2 + 2q_{211}; \quad c_{221} = q_{221}^2 + 2q_{221};$$
$$c_{332} = 0.5q_{332}^2 + 3q_{332}.$$
$$\delta_{111} = \delta_{121} = \delta_{211} = \delta_{221} = \delta_{332} = 1.$$

The cost of transacting associated with by-product y for industrial producer i conducting with industrial consumer j, c_{ijy}, and its corresponding binary parameter δ_{ijy}:

$$c_{131} = q_{131}^2 + 2q_{131}; \quad c_{231} = 0.5q_{231}^2 + 2.5q_{231}; \quad \delta_{131} = \delta_{231} = 1.$$

The cost of transacting associated with waste z for industrial producer i conducting with industrial decomposer k, c_{ikz}, and its corresponding binary parameter δ_{ikz}:

$$c_{111} = q_{111}^2 + 3q_{111}; \quad c_{211} = q_{211}^2 + q_{211}; \quad c_{312} = q_{312}^2 + 5q_{312}. \quad \delta_{111} = \delta_{211} = \delta_{312} = 1.$$

The cost of producing finished product u from semi-products by industrial consumer j and its corresponding binary parameter η_{ju}:

$$f_{11} = 1.5\left(0.8q_{111} + 0.8q_{211}\right)^2 + 2 \cdot \left(0.8q_{111} + 0.8q_{211}\right)\left(0.8q_{121} + 0.8q_{221}\right);$$

$$f_{21} = 1.5\left(0.8q_{121} + 0.8q_{221}\right)^2 + 2 \cdot \left(0.8q_{111} + 0.8q_{211}\right)\left(0.8q_{121} + 0.8q_{221}\right);$$
$$f_{32} = 2.5\left(0.8q_{332}\right)^2 + 5\left(0.8q_{332}\right) + 2.5\left(0.4q_{131} + 0.4q_{231}\right)^2.$$

$$\eta_{11} = \eta_{21} = \eta_{32} = 1.$$

The cost of transacting associated with by-product v for industrial consumer j conducting with industrial producer i, c_{jiv}, and its corresponding binary parameter δ_{jiv}:

$$c_{131} = 2q_{131}^2 + 2.5q_{131}; \quad c_{231} = 1.5q_{231}^2 + 2.5q_{231};$$

$$c_{312} = 1.5q_{312}^2 + 3q_{312}; \quad c_{322} = 2q_{322}^2 + 2q_{322}.$$
$$\delta_{131} = \delta_{231} = \delta_{312} = \delta_{322} = 1.$$

The cost of transacting associated with waste w for industrial consumer j conducting with industrial decomposer k, c_{jkw}, and its corresponding binary parameter η_{jkw}:

$$c_{111} = q_{111}^2 + 3q_{111}; \quad c_{211} = 0.5q_{211}^2 + 2.5q_{211}; \quad c_{312} = 1.5q_{312}^2 + 2q_{312}.$$

$$\eta_{111} = \eta_{211} = \eta_{312} = 1.$$

The cost of transacting associated with finished product u for industrial consumer j conducting with demand market l, c_{jlu}, and its corresponding binary parameter δ_{jlu}:

$$c_{111} = 0.5q_{111}^2 + 3q_{111}; \quad c_{121} = 0.5q_{121}^2 + 3q_{121}; \quad c_{211} = 1.5q_{211}^2 + 2.5q_{211};$$

$$c_{221} = 1.5q_{221}^2 + 2.5q_{221}; \quad c_{312} = 2.5q_{312}^2 + q_{312}; \quad c_{322} = 2.5q_{322}^2 + q_{322}.$$

$$\delta_{111} = \delta_{121} = \delta_{211} = \delta_{221} = \delta_{312} = \delta_{322} = 1.$$

The cost of transacting associated with reusable material g for industrial decomposer k conducting with industrial producer i, c_{kig}, and its corresponding binary parameter δ_{kig}:

$$c_{111} = 1.5q_{111}^2 + 2.5q_{111}; \quad c_{121} = 2.5q_{121}^2 + 2q_{121}; \quad c_{132} = 3q_{132}^2 + q_{132}.$$

$$\delta_{111} = \delta_{121} = \delta_{132} = 1.$$

The cost of separating and detecting wastes associated with the industrial decomposer 1, ϕ_1:

$$\phi_1 = \left(\sum_{i=1}^{3}\sum_{z=1}^{2} q_{i1z}\right)^2 + \left(\sum_{j=1}^{3}\sum_{w=1}^{2} q_{j1w}\right)^2.$$

The disposing fee of per unit of useless material to the landfill is given by: $\bar{\rho} = 2$.

The cost of transacting associated with demand market l obtaining the finished product u from industrial consumer j, $\hat{c}_{jlu}$:

$$\hat{c}_{jlu} = q_{jlu} + 5, \quad \forall j = 1,2,3; \quad l = 1,2; \quad u = 1,2.$$

The demand functions about two finished products at the two demand markets were:

$$d_{11} = -2\rho_{11} - 1.5\rho_{21} + 1000; \quad d_{21} = -2\rho_{21} - 1.5\rho_{11} + 1000;$$
$$d_{12} = -2.5\rho_{12} - 2\rho_{22} + 1000; \quad d_{22} = -2.5\rho_{22} - 2\rho_{12} + 1000.$$

All other functions were set equal to zero (*e.g.* Yang *et al.* [43]). In addition, the parameters were given by:

$$\alpha_{ix} = 0.8,\ \alpha_v = 0.6,\ \alpha_g = 0.4; \quad \beta_x = 0.8, \beta_y = 0.4; \quad \chi_z = 0.3, \chi_w = 0.3.$$

Example 3.1. The data for the first example were constructed for easy interpretation purposes serving as a base line. For the first example, we assumed that all the weights associated with the environment decision-making were set equal to zero, that is, $\gamma_i = \omega_j = 0$, for $i = 1, 2, 3$; $j = 1, 2, 3$. This meant that the industrial producers and the industrial consumers were concerned with profit maximization exclusively.

Let set $\alpha_{ix} = 0.8$, $\alpha_v = 0.6$, $\beta_x = 0.8$, $\beta_y = 0.4$, $\chi_z = 0.3$, $\chi_w = 0.3$ and vary the parameter α_g from 0.2 to 0.6 using 0.2 as the interval. It meant that Example 1a corresponded to the situation where $\alpha_g = 0.2$, Example 1b corresponded to the situation where $\alpha_g = 0.4$ and Example 1c corresponded to the situation where $\alpha_g = 0.6$. It was necessary to note that the parameter α_g (fraction of a unit of the reusable material wholly transformed into the semi-product) was reasonable to be less than α_{ix} (fraction of a unit of the raw material wholly transformed into the semi-product).

Then, LINGO 9.0 version is run on a personal computer with the CPU of Intel Core 1.80 GHZ and RAM 2.00 GB. Global solver inherent in LINGO 9.0 is used to solve a couple of nonlinear complementarity conditions that hold simultaneously. The LINGO 9.0 converged in 76 iterations (corresponding to Exp. 1a), 40 iterations (corresponding to Exp. 1b) and 33 iterations (corresponding to Exp. 1c), respectively, and yielded the following equilibrium patterns which were given in Table 5.

From the results in Table 5, an interesting fact to note here is that the change of an invidual parameter affected the entire industrial symbiosis network and the prices throughout the whole system. It could be easily to obtain that the majority of decision variables such as $\tilde{q}_{ix}$, q_{ijx}, q_{ikz}, q_{jlu}, q_{jkw} and q_{kig} changed obviously along with the increase in the transformation ratio of reusable materials. It might be due to the following fact. As for industrial producers i, the demand for raw materials was relatively high when the fraction α_g was relatively low; but when the value of this parameter became higher and higher, industrial producers were inclined to purchase reusable materials rather than raw materials to remanufacture homogenous semi-products because the former ones were far cheaper than the latter. In other words, the increase in the transformation ratio of reusable materials resulted in the decrease of the amount of raw materials and the increase of the flow of reusable materials. It was also worth noting that the effects on the corresponding selling prices associated with the above two materials were the opposite.

Due to the the conservation of flow equation that each industrial producer i must satisfy (*cf.* (1.2)), we could observe that the first term in the left of this equation decreased and the second one maintained unchanged, but the third one increased. The amount that the third term increased was greater than the amount that the first term decreased, which led to the value of the term in the right of the equation increasing. For the industrial producers, as semi-product flows increased, the total emissions composed of by-products and wastes also climbed. For the industrial consumers, on the other hand, it would lead to the amount of finished product further increased which added their total emissions. Note that reusable materials

TABLE 5. Equilibrium solutions of Example 3.1.

Variable	Example 1a	Example 1b	Example 1c
Raw material transactions between material suppler and industrial producers $\tilde{q}_{ix}$			
$\tilde{q}^*_{11}$	19.19	19.05	18.82
$\tilde{q}^*_{21}$	17.01	16.90	16.71
$\tilde{q}^*_{32}$	10.51	10.57	10.57
Product transactions and corresponding prices between industrial producers and industrial consumers $q_{ijx}\|\rho_{ijx}$			
$q^*_{111}\|\rho^*_{111}$	12.62|140.12	12.78|139.48	13.04|138.31
$q^*_{121}\|\rho^*_{121}$	9.53|137.11	9.66|136.35	9.86|135.13
$q^*_{211}\|\rho^*_{211}$	10.38|140.20	10.42|139.48	10.49|138.31
$q^*_{221}\|\rho^*_{221}$	8.84|137.11	8.86|136.35	8.90|135.13
$q^*_{332}\|\rho^*_{332}$	13.25|64.23	13.22|64.39	13.22|64.40
By-product transactions and corresponding prices from industrial producers to industrial consumers $q_{ijy}\|\rho_{ijy}$			
$q^*_{131}\|\rho^*_{131}$	12.21|26.42	12.21|26.43	12.21|26.43
$q^*_{231}\|\rho^*_{231}$	23.92|26.42	23.93|26.43	23.93|26.43
By-Product transactions and corresponding prices from industrial consumers to industrial producers $q_{jiv}\|\rho_{jiv}$			
$q^*_{131}\|\rho^*_{131}$	3.59|16.84	3.60|16.91	3.60|16.91
$q^*_{231}\|\rho^*_{231}$	4.31|15.44	4.33|15.50	4.33|15.50
$q^*_{312}\|\rho^*_{312}$	11.33|36.99	11.24|36.73	11.11|36.32
$q^*_{322}\|\rho^*_{322}$	9.34|39.37	9.27|39.10	9.17|38.66
Product transaction shipments and corresponding prices between industrial consumers and demand markets $q_{jlu}\|\rho_{jlu}$			
$q^*_{111}\|\rho^*_{111}$	7.12|269.96	7.20|269.85	7.31|269.68
$q^*_{211}\|\rho^*_{211}$	5.58|271.51	5.64|271.41	5.73|271.26
$q^*_{312}\|\rho^*_{312}$	12.53|201.91	12.51|201.93	12.51|201.93
$q^*_{121}\|\rho^*_{121}$	7.12|269.96	7.20|269.85	7.31|269.68
$q^*_{221}\|\rho^*_{221}$	5.58|271.51	5.64|271.41	5.73|271.26
$q^*_{322}\|\rho^*_{322}$	12.53|201.91	12.51|201.93	12.51|201.93
Waste transaction shipments and corresponding prices between industrial producers and industrial decomposers $q_{ikz}\|\rho_{ikz}$			
$q^*_{111}\|\rho^*_{111}$	0.19|3.39	1.31|5.63	2.51|8.02
$q^*_{211}\|\rho^*_{211}$	0.60|3.39	1.15|5.63	1.76|8.02
$q^*_{312}\|\rho^*_{312}$	0.00|3.57	0.00|2.83	0.00|0.05
Waste transaction shipments and corresponding prices between industrial consumers and industrial decomposers $q_{jkw}\|\rho_{jkw}$			
$q^*_{111}\|\rho^*_{111}$	0.12|3.24	0.82|4.64	1.57|6.14
$q^*_{211}\|\rho^*_{211}$	0.74|3.24	2.14|4.64	3.64|6.14
$q^*_{312}\|\rho^*_{312}$	0.00|1.30	0.00|0.00	0.00|0.00
Reusable material transactions and corresponding prices from industrial decomposers to industrial producers $q_{kig}\|\rho_{kig}$			
$q^*_{111}\|\rho^*_{111}$	0.00|23.25	1.15|45.84	1.98|68.30
$q^*_{121}\|\rho^*_{121}$	0.00|21.88	0.49|44.29	0.87|66.21
$q^*_{132}\|\rho^*_{132}$	0.50|7.76	0.00|19.271	0.00|28.91
Price at demand markets ρ_{lu}			
ρ^*_{11}	282.08	282.04	281.99
ρ^*_{12}	219.44	219.44	219.44
ρ^*_{21}	282.08	282.04	281.99
ρ^*_{22}	219.44	219.44	219.44

were derived from wastes generated by both industrial producers and industrial consumers. From Table 5, then, it was obvious to see that the wastes involved in activities of the industrial producers and the industrial consumers increased accordingly.

In the light of the symbiotic relationships in the industrial symbiosis network, it was mainly embodied by the activities of exchanging by-products between industrial producers and industrial consumers. These activities brought about not only economic revenues to these enterprises but also environmental benefits to the society. For the part of the industrial producers, when we set $\alpha_g = 0.4$, the total economic revenue owing to exchanging their by-products with the industrial consumers and selling wastes to the industrial decomposers was:

$$12.21 \times 26.43 + 1.31 \times 5.63 + 23.93 \times 26.431.15 \times 5.63 = 969.03;$$

whereas, as far as industrial consumers were concerned, the total economic revenue on account of exchanging their by-products with the industrial producers and selling wastes to the industrial decomposers was:

$$3.60 \times 16.91 + 0.82 \times 4.64 + 4.33 \times 15.50 + 2.14 \times 4.64 + 11.24 \times 36.73 + 9.27 \times 39.10 = 917.04.$$

As expected, in addition, we could observe that the demand prices exceeded the prices for finished products at the industrial consumers, which might be due to the fact that the prices increased as these products propagated down through the industrial symbiosis network when costs accumulated.

Example 3.2. Example 3.2 had the identical data as in Example 1b except that a part of the former two-tier decision-makers was more concerned about environment. In Example 2a, the weights regarding the environmental decision-making of all the industrial producers were equal to one. On the contrary, all the weights concerning environmental decision-making of the industrial consumers were equal to zero. Then, on the basis of Example 2a, Example 2b made the weights associated with the industrial consumers take value from 0 to 1. Example 2c had the same data as in Example 2b but we increased the weights associated with the industrial producers so that they took value from 1 to 5. Based on Example 2c, finally, Example 2d transformed the weights relevant to the industrial consumers into 5.

The LINGO 9.0 converged in 77 iterations (corresponding to Exp. 2a), 85 iterations (corresponding to Exp. 2b), 63 iterations (corresponding to Exp. 2c) and 109 iterations (corresponding to Exp. 2d), respectively, and yielded the following equilibrium patterns which were given in Table 6.

The total emissions generated by the industrial producers and the industrial decomposers are given by the expression:

$$\sum_{i=1}^{3}\sum_{j=1}^{3}\sum_{y=1}^{1}\delta_{ijy}q_{ijy}+\sum_{i=1}^{3}\sum_{k=1}^{1}\sum_{z=1}^{2}\delta_{ikz}q_{ikz}+\sum_{j=1}^{3}\sum_{i=1}^{3}\sum_{v=1}^{2}\delta_{jiv}q_{jiv}+\sum_{j=1}^{3}\sum_{k=1}^{1}\sum_{w=1}^{2}\eta_{jkw}q_{jkw}.$$

Table 6. Equilibrium solutions of Example 3.2.

Weight	Example 2a	Example 2b	Example 2c	Example 2d
	1	1	5	5
γ_i	1	1	5	5
	1	1	5	5
	0	1	1	5
ω_j	0	1	1	5
	0	1	1	5
Product transaction shipments between industrial producers and industrial consumers q_{ijx}				
q^*_{111}	12.83	12.80	12.76	12.55
q^*_{121}	9.70	9.68	9.64	9.49
q^*_{211}	10.44	10.42	10.40	10.27
q^*_{221}	8.88	8.86	8.84	8.74
q^*_{332}	13.27	13.21	13.44	13.18
By-product transaction shipments from industrial producers to industrial consumers q_{ijy}				
q^*_{131}	12.03	12.05	11.33	11.37
q^*_{231}	23.57	23.59	22.16	22.24
By-Product transaction shipments from industrial consumers to industrial producers q_{jiv}				
q^*_{131}	3.62	3.51	3.57	3.14
q^*_{231}	4.35	4.22	4.30	3.78
q^*_{312}	11.21	11.07	11.10	10.59
q^*_{322}	9.25	9.13	9.15	8.72
Product transaction shipments between industrial consumers and demand markets q_{jlu}				
q^*_{111}	7.22	7.21	7.18	7.08
q^*_{211}	5.66	5.64	5.63	5.54
q^*_{312}	12.43	12.41	12.07	11.99
q^*_{121}	7.22	7.21	7.18	7.08
q^*_{221}	5.66	5.64	5.63	5.54
q^*_{322}	12.43	12.41	12.07	11.99
Waste transaction shipments between industrial producers and industrial decomposers q_{ikz}				
q^*_{111}	0.00	0.00	0.00	0.49
q^*_{211}	0.00	0.00	0.00	0.74
q^*_{312}	4.00	4.42	2.58	0.00
Waste transaction shipments between industrial consumers and industrial decomposers q_{jkw}				
q^*_{111}	0.01	0.00	0.00	0.30
q^*_{211}	0.50	0.00	0.50	1.11
q^*_{312}	3.00	3.25	2.75	0.00
Reusable material transactions from industrial decomposers to industrial producers q_{kig}				
q^*_{111}	1.52	1.55	1.22	0.65
q^*_{121}	0.74	0.76	0.53	0.14
q^*_{132}	0.00	0.00	0.00	0.00
Price at demand markets ρ_{lu}				
ρ^*_{11}	282.03	282.04	282.05	282.11
ρ^*_{12}	219.46	219.47	219.54	219.56
ρ^*_{21}	282.03	282.04	282.05	282.11
ρ^*_{22}	219.46	219.47	219.54	219.56

According to the above expression, the total emissions generated in Example 2a were equal to:

$$12.03 + 23.57 + 3.62 + 4.35 + 11.21 + 9.25 + 4.00 + 0.01 + 0.50 + 3.00 = 71.54.$$

In Example 2b, the result was that the total emissions decreased further to:

$$12.05 + 23.59 + 3.51 + 4.22 + 11.07 + 9.13 + 4.42 + 3.25 = 71.24.$$

In Example 2c, the total emissions decreased and were equal to:

$$11.33 + 22.16 + 3.57 + 4.30 + 11.10 + 9.15 + 2.58 + 0.50 + 2.75 = 67.44.$$

In Example 2d, the total emissions generated were further reduced and were equal to:

$$11.37 + 22.24 + 3.14 + 3.78 + 10.59 + 8.72 + 0.49 + 0.74 + 0.30 + 1.11 = 62.49.$$

Due to the higher weight on total emissions, the amount of emissions decreased step by step compared with the amount generated in Example 2a. Consequently, as expected, along with the weights increased, environmentally conscious industrial producers and industrial consumers both could reduce their respective emissions generated through the underlying decision-making behavior in the whole industrial symbiosis network. For the three industrial consumers, the main effects of the varying weights were demonstrated by the two changes which were involved with the transformation of Example 2a into Example 2b and the transformation of Example 2c into Example 2d. These two conversions were both accompanied by the increase of the demand price at demand markets. Maybe it could be explained that these enterprises would become more environmentally-friendly with the increase in awareness of lower emissions. Then the lower emissions that resulted from increasingly environmental consciousness was helpful to improve their reputation, which would appeal to customers and then lift the demanding price at markets. As an independent economic entity, focusing on social interests would make a positive effect on its long-term brand image, influencing its economic interests in the long run. Indeed, poor environmental performance in the ISN may damage an enterprise's most important asset, that is reputation.

By far, it is feasible to obtain some suggestion or managerial insights into operating enterprises in industrial symbiosis networks. From the analysis of Example 3.1, the total emission increased along with the rising transformation ratio of reusable materials. From the result of Example 3.2, in contrast, the rising environmental weights led to the declining total emission generated by more environmentally-friendly enterprises in the industrial symbiosis network. Then, this contradictory phenomenon will raise a number of thorny issues to managers in these enterprises, that is, what if the rising transformation ratio of reusable materials and the rising of the environmental weights occurred at the same time? Which one would be emphasized on? Whether economic interests should give way to social interests? How to tradeoff? Maybe answers to the above complicated problems depend on these enterprises' development stage and market orientation. If an enterprise in industrial symbiosis network is small-scale and it is at the primary stage, even though it is more concerned about environmental protection, they are likely to place more emphasis on economic profits rather than social interests. But if an enterprise in industrial symbiosis network is large-scale and it is at the mature stage, it will tend to pay more attention to its own brand in order to occupy a higher position in market orientation. Therefore, when the environmental weights climb, it may value social interests even if it is at the cost of economic interests. In a word, the managerial staffs relevant to making significantly strategical decisions should balance short-term economic interests and long-term social interests, which may have much to do with identifying different strategies and tactics when enterprises are at different development stages.

Obviously, the above examples are stylized but they demonstrate the efficacy of the model. Indeed, different input data and dimensions of the problems solved will affect the equilibrium outputs, transactions and price patterns. One could also explore the effects of data as well as the effects of changes in the number of various decision-makers in the industrial symbiosis network.

4. Conclusion and directions for future research

In the paper, a framework for the formulation of the industrial symbiosis network equilibrium was proposed. The industrial symbiosis network consisted of industrial producers, industrial consumers, industrial decomposers and demand markets in which the interactive competition and their independent decision-making were considered. As for the industrial producers and the industrial consumers in the complex network, in particular, we described their multicriteria decision-making behavior including the maximization of profit as well as the minimization of emission. Specifically, these two tiers of decision-makers were permitted to weight objective functions according to their individual preferences. Then, we established the optimality conditions of four-tier decision-makers along with economic interpretations, which were equivalent to a series of nonlinear complementarity conditions. Then, it would be easy to provide the nonlinear complementarity model in accordance with the industrial symbiosis network equilibrium conditions. Existence under suitable assumptions on the underlying functions was presented to guarantee the validility of the modeling. Finally, seven illustrative examples categorized by two groups were considered to verify the rationality of the model and obtain some managerial insight to decision-makers.

For further research, the model may take into account the coupling among enterprises located at the identical tier in the industrial symbiosis network. Moreover, the paper may include the consideration about how to calculate those parameters of weights associated with environmental decision-making. An application of the algorithm to concrete numerical examples should also be integrated. The dynamic industrial symbiosis network equilibrium problem is the authors' intention to explore in the future.

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