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NEWSBOY PROBLEM: VIABILITY OF OPTIMAL INITIAL SELLING PRICE AND ORDERING POLICIES IN THE PRESENCE OF EXOGENOUS PRICE DECLINE AND RANDOM LEAD TIME

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Abstract. Analysis of empirical sales data lead us to consider newsboy model for four practical market conditions arising from the presence/absence of stochastic lead time and exogenous linear temporal decline in selling price when distribution of the stochastic demand depends upon initial selling price. Viability of the solutions is discussed for three strategies of obtaining optimal initial selling price and/or ordering quantity. Numerical studies are conducted to assess the effects of lead time and price decline.

Keywords. Stochastic lead-time, exogenous price decline, pricing, initial selling price, viable policies.

Mathematics Subject Classification. 91B24.

1. Introduction

The single period problem (SPP) or the newsboy problem – as addressed by [1] has become one of the most versatile inventory models encompassing a wide variety of products – such as fashion apparels, hi-tech goods *etc.* – that are characterized by short demand season, single ordering opportunity and time independent demand. Although this model has been extended in various directions [2–6], rapidly evolving market conditions entail further extensions of the existing models.

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The inevitability of the presence of lead time in the markets for such goods has been pointed out by [7]. Ordering policy for SPP with continuous price decline in the presence of stochastic lead time has been presented by [8].

In this paper, we first present market reports and sales data from a developing country, which motivated us to consider different pricing and/or ordering policies in the presence of exogenous decline in selling price and random lead time. We also consider the situation where (a) the season begins with the product being sold at an initial selling price (ISP), which at times is the same as MRP (maximum retail price) of the product, and affects the demand distribution (see Sect. 1.3); (b) subsequently, market situations compel the retailer to offer progressive discounts that are effective for periods with random lengths of time; (c) it is not possible to control the salvage value. This limits the pricing policy to setting of only the ISP.

Considering the short product life cycles (market reports and data presented in Sect. 1.1), newsboy model can be extended to one that is reflective of the current market scenarios. Further, a theoretical optimal price/order quantity may not be practical if it exceeds the reference price/target sales volume (Market reports presented in Sect. 1.2). Thus, the SPP where both price and quantity are set simultaneously [9–11] may not cater to the challenges posed by contemporary markets. Hence, practical considerations make it imperative to check the viability of the theoretical optimal values. [12] considers optimization of initial price for announced discounts For exogenous price decline at random time epochs by random amounts, we, in this paper, extend the model of [8] to consider the important problem of optimization of ISP in addition to ordering policy. The contribution of this paper to the existing literature is three folds (i) four different market conditions are considered, namely, combinations of (a) replenishment with/without random lead time and (b) selling price with/without exogenous price decline. (ii) Three types of market dependent strategies are considered, namely, optimization of ISP only, optimization of order quantity only and joint optimization of ISP and order quantity. (iii) A major practical consideration is checking the viability of the optimal policies. To the best of our knowledge, pricing and ordering policies for such models with stochastic ISP dependent demand, random lead time and viability considerations for the policies have not yet been addressed in published literature.

The remainder of this paper is organized as follows: Sections 1.1 and 1.2 provide a brief survey of market reports that enabled us to identify the factors such as exogenous lead time, shortening of demand season and the influence of reference price. Section 1.3 provides the empirical data collected from retail outlets of fashion apparel and statistical analyses to investigate the discount pathways as well as the price-demand curve. Section 2 presents the model, notations and assumptions. Brief explanation of the four different market situations and three profit maximization strategies considered in this paper are presented. Section 3 deals with the characteristics of profit function. Section 4 presents numerical examples and managerial insights that illustrate various analyses pertaining to profit maximization and viability of the theoretical optimal solutions under realistic circumstances. Finally, Section 5 provides conclusions.

1.1. Competition and innovations lead to exogenous price decline and shortening of product life cycle

From early times, increasing the sales volume in order to outclass competitors is the main reason for indiscriminate price cuts being indulged in by various companies [13].This trend is still evident in the current market. For example, the price of laptops of various brands in India dropped by 20% during the span of six months as shown below:

Brand Name	RAM	HDD	Processor	Price 6 months ago	Price on 31st July 2011
HP(MINI)	1 GB	160 GB	Atom	20 500	17 000
HCL	1 GB	320 GB	Celeron	25 500	21 000
COMPAQ	1 GB	320 GB	Dual Core	27 500	22 500
SONY	2 GB	320 GB	Dual Core	31 000	25 000

The prices are in INR and all these models have Intel processors and Windows Operating Systems.

The rapid decline in laptop prices have been attributed to innovation in technology and cut throat competitions [14]. With the arrival of new trend/technology, companies start offering discounts in order to sell out remaining stock of the old items. Such trend of competition and innovation induced price decline is also observed in fashion apparel markets in India. For example, companies like Koutons, Pantaloons, Cotton County, *etc.* offer successive discount rates as the selling season advances. As a result of different responses by companies in the wake of competition and obsolescence of products, the offered discounts are exogenous in nature.

1.2. Reference price

Consumers are sensitive to price deviations from prices of competing products, at times obligating managers to fix the price of a product on the basis of reference price. [15] pointed out that competitor pricing was the strongest determinant of retail pricing strategies. Take a cue from the market trend for sales of iPhones in India. The then latest 8 Gbyte 3 G iPhone from Apple was launched in India with a price tag of Rs. 31 000 *i.e.*, approximately $715 as per currency value of INR on 22nd August 2008 as compared to $199 in the US markets [16]. Market reports revealed that only 1500 units of iPhones were sold in the first week of its launch in India [17] and only 20 000 units were sold in one year despite the huge smartphone market in India, including iPhones, estimated at 6 million units in 2009 [18]. This poor sales performance, despite the initial hype along with the then novelty factor of iPhone was attributed to the disproportionately high ISP in India as compared to its US retail price which acted as reference price [19].

1.3. Data collection and analysis

In order to investigate the sales pattern of fashion apparels in the presence of exogenously declining prices, data was collected for sales of trendy wears of three different brands from retail outlets located at Indore (India).

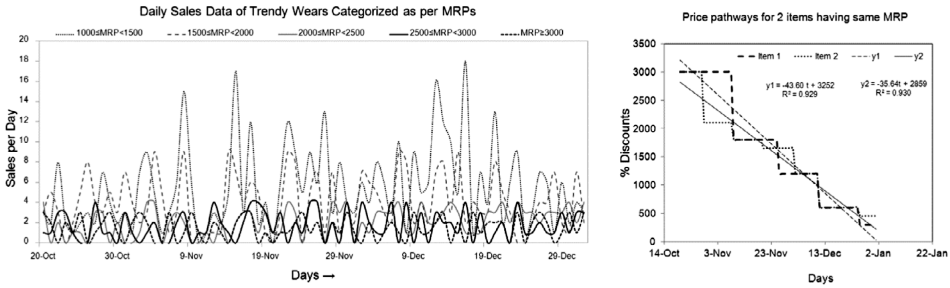

FIGURE 1. (a) Daily sales of trendy wears grouped into five categories as per MRPs. (b) Price pathways for two items and linear trends fitted to the stepwise decline of selling price over time.

Figure 1a shows the daily sales for a season which approximately lasts for 74 days wherein items are classified into five categories on the basis of their maximum retail prices (MRPs). During the season, item specific price discounts have been offered on random occasions. Figure 1b shows the price pathways for 2 such items. Number of discounts offered in a season for each of the items ranged from 2 to 4. Each new discount rate was valid for at least a week except for the last discount rate – generally the maximum discount rate of a season – which at times was offered for about a couple of days. Contrary to general perception, no surge in the sales is observed from Figure 1a in any category for any discount offer. On performing run test with mean as the cut-off point, the sample sales data in each of the five categories is found to be random with p-values 0.45, 0.86, 0.11, 0.62 and 0.1. χ^2-test of goodness of fit shows that the sales data for each category of items follow uniform distributions (p-values: 0.1, 0.33, 0.62, 0.26 and 0.98).

Further, it is observed that the maximum realized demand rate (demand potential, say D) varies for different categories and hence the probability distributions of the sales for different categories vary. This gives rise to the necessity of investigating the demand potential as a function of ISP (MRP in this case) which we denote by S_0. Figure 2 shows the price-demand curve *i.e.*, S_0 *vs.* D. Since, as per the statistical analysis presented above, the demand for each category follows $U(0, D)$, where D depends upon the ISP S_0, we call D as $D(S_0)$ and estimate it from the mean of daily sales for each category. We observed that $D(S_0)$ decreases as S_0 increases, implying that sales of items decrease stochastically as S_0 increases. Three different functions – power, exponential and linear – are fitted on the price-demand curve. On fitting the power function $D(S_0) = K\ S_0^{-a}$, we find that for $K = 97\,674$ and $a = 1.3$ with $R^2 = 0.97$. Similarly, on fitting the exponential function $D(S_0) = K\ \mathrm{Exp}[\,-a\,S_0]$, we get $K = 21.43$, $a = 0.0007$ with $R^2 = 0.95$ and finally, for the linear function $D(S_0) = K - a\,S_0$, we get $K = 14.405$, $a = 0.0042$ with $R^2 = 0.89$. All the three functions reasonably explain the price demand curve. A linear or exponential functional form of $D(S_0)$ will provide theoretically easy functions to deal with. However, for detailed study, we have chosen the iso-elastic

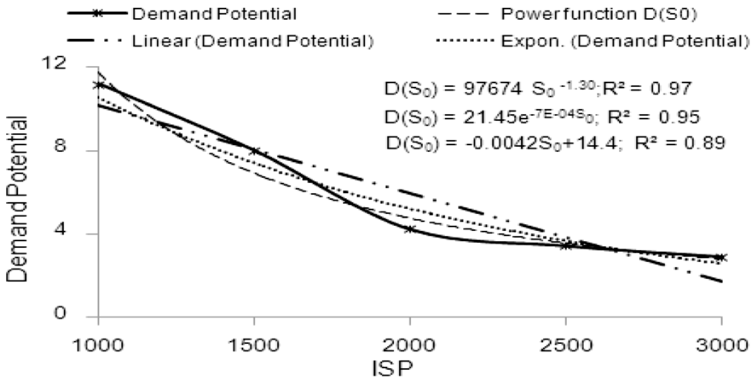

FIGURE 2. Price-demand curve: three functions *viz.* linear, exponential and power fitted to the curve with iso-elastic power function as the best fit.

price-demand curve which is popular in empirical studies as reported by [20]. Incidentally the power function has the highest coefficient of determination for the sample under consideration.

2. Model and notations

We consider a situation where retailer faces stochastic price dependent demand from the end-customer market. The retailer places an order of q units (decision variable) at a unit cost c before the beginning of the demand season, say at time epoch 0 and the demand begins at time epoch $t \geqslant 0$. Replenishment of q units arrives at a random time $L \in (0, t_1)$. If there is no lead time, demand begins at time epoch $t_0 = 0$. The season ends at time epoch t_1. Unsold goods fetch a salvage value of R per unit. Some of the consumers are price sensitive while others are time sensitive. The advancing season off puts the time sensitive consumers from buying the product, nullifying the possible surge in sales due to discounts. The stochastic demand rate X is thus assumed to be of the commonly used multiplicative form $X = D(S_0)\xi$ independent of t, where ξ >0 is the random component, $\xi \in (0, 1]$ and the demand potential $D(S_0)$ is a declining function of S_0. Both mean and variance of the demand function depend upon S_0. Although the total demand during the season, say $Y(S_0)$, is a random variable, we assume it to be evenly distributed over the season such that at any epoch t, for a given value of $Y(S_0)$, $X(t) = Y(S_0)/(t_1 - t_0)$ is constant.

The model assumptions are as follows:

(i) Random lead time $L \sim U(0, t_1)$.
(ii) Selling price $S(t)$ at epoch t

$$S(t) = \begin{cases} S_0 - b(t - t_0)t_0 \leqslant t \leqslant t_1 \\ R \text{ when } t = \ t_1;\ R \leqslant S(t_1) \end{cases}$$

where the rate of decline b is a known constant and S_0 (the decision variable) is the selling price at t_0. Unsold inventory at the end of the season *i.e.*, at $t = t_1$, is sold off at a salvage price R per unit. t_0, t_1, b, R fixed and known to the retailer

(iii) Shortage cost $p > 0$ per unit (this includes pre arrival shortages, as well as stock out shortages) and holding cost $h > 0$ per unit per unit time.
(iv) X demand rate at any epoch during the season, a random variable with probability distribution $U(0, D(S_0))$, where $D(S_0)$ decreases stochastically with the increase in S_0.

$D(S_0) = KS_0^{-a}$, such that the price elasticity $a > 1$ and $q \leqslant D(S_0)\ \Delta$ (the maximum total demand possible for a season of length Δ). For convenience, we use the notation D and $D(S_0)$ interchangeably. As in [8], we consider four market conditions.

M1: selling price declines during the season and lead time is present;
M2: selling price is static over time and lead time is present;
M3: selling price declines during the season and there is no lead time;
M4: selling price is static over the season and there is no lead time.

For these market conditions, in view of the influence of the reference price and pressure to achieve target sales volume discussed in Section 1.2, we introduce three strategies ST1, ST2 and ST3 as described below and discuss viability of the solutions for each of them.

ST1. Find the joint optimal ISP and sales volume.
This is the conventional profit maximization strategy generally applicable in the absence of both target sales and the influence of reference price.
ST2. Find the optimal sales volume for a fixed ISP.
This enables the practitioner to optimize his profit returns in competitive markets wherein an external factor like reference price obligates the practitioner to fix ISP of the product. Minimum values of profitable ISP are also identified.
ST3. Find the optimal ISP for a given sales volume.

This allows practitioners to identify the viability of a target sales volume by checking if it is profitable under the prevailing business constraints.

Notations.

$g(L) = 1/t_1$; $L \in (0, t_1)$; pdf of L; Note1: we use the same notation for a particular value of L.

$\Delta = t_1 - t_0$:	length of the season;
$\Delta_1 = t_1 - L$:	time over which demand is served in the absence of stock out shortage;
x:	random variate corresponding to demand rate X;

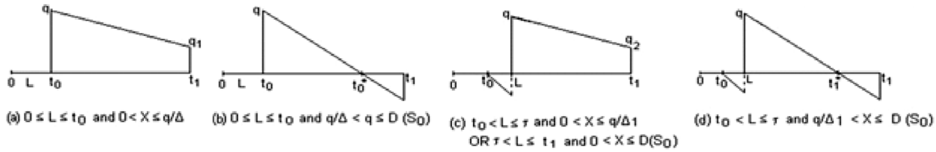

FIGURE 3. Time-inventory graph for different conditions of stochastic lead time and demand rate.

$q_1 = q - x\Delta$: unsold items at the season end, when $L < t_0$, $q \geqslant x\Delta$;
$q_2 = q - x\Delta_1$: unsold items at the season end, when $t_0 \leqslant L \leqslant t_1$, $q \geqslant x\,\Delta_1$;
$t_0^* = t_0 + q/x$: random time epoch at which inventory is exhausted when $L < t_0$, $q < x\,\Delta$;
$t_1^* = L + q/x$: random time epoch at which inventory is exhausted when $t_0 < L, q < x\,\Delta_1$;
q/x: length of time for which q units of inventory last if demand rate is x;
$\tau = t_1 - q/D$: time epoch such that inventory arriving at this epoch or later will not result in stock out shortage.

It must be noted that Δ_1, t_0^* and t_1^* are random variables while τ is a constant depending upon the value of the decision variable q.

$TEP_{L=0}$: Case without lead time. This is the case for market conditions $M3$ and $M4$. We simply use the notation TEP for the case with lead time.
j^*: With a decision variable denotes its optimal value for the market condition Mj; $j = 1, 2, 3$ and 4.
$TEP|_{\mathrm{STi}}$: TEP calculated using strategy STi; $i = 1, 2.$ & 3.
$q|_{\mathrm{STi}}$: q calculated using strategy STi; $i = 1, 2$ & 3.
$TEP|_{v=y}$: TEP calculated for a given $v = y$; $y = 0$, $v=$ any parameter.

Note 2. Since the entire amount purchased will be sold including the amount sold at salvage price, we use the term q for both "sales volume" as well as "order quantity".

3. TOTAL EXPECTED PROFIT

Time inventory graphs for different situations based on the inventory replenishment time are presented in Figure 3.

This enables us to derive the holding cost, shortage cost, selling price and hence the net profit functions. For D constant, [8] obtained the TEP functions for the cases with and without lead time (see Appendix E in supplementary material) and showed that both these functions are concave w.r.t. q.

In this paper, we calculate the TEP functions when $D = KS_0^{-a}$ for the market conditions

(a) M1 and $M2$ *i.e.*, when lead time is present

$$TEP = -\frac{q^3 S_0^{2a}(b+h)}{6K^2 t_1} - \frac{S_0^{-a} K p \Delta}{2} + \frac{S_0^a q^2 \left((S_0 - R + p)(t_0 - 3t_1) - \Delta h(2t_0 + t_1) + \Delta b(2t_1 - 3t_0)\right)}{4Kt_1\Delta} + \frac{q(-ht_0^2 - 2ct_1 + 2pt_1 + b\Delta^2)}{2t_1} - \frac{S_0^a q^2 (S_0 - R + p + bt_0 + ht_1)\mathrm{Log}\left[\frac{S_0^{-a}K\Delta}{q}\right]}{2Kt_1} \tag{3.1}$$

(b) $M3$ and $M4$ *i.e.*, when lead time is not present

$$TEP_{L=0} = -\frac{KpS_0^{-a}t_1}{2} - q\left(S_0 + p - c\right) - \frac{q^2 S_0^a}{4Kt_1}\left(2p - 2R + S_0 + bt_1 + 3ht_1 + 2t_1\left(b+h\right)\mathrm{Log}\left[\frac{KS_0^{-a}t_1}{q}\right] + 2S_0\right) \tag{3.2}$$

We now present further properties of the two TEP functions.

Theorem 3.1. *TEP is a concave function of S_0 iff*

$$f(S_0) \leqslant 2t_0(a+1) - \Delta(2a^2 + a - 3) + \frac{4(a-1)Kt_1}{q},$$

where

$$f(S_0) = -\frac{4a^2(b+h)qS_0^a}{3KS_0} - \frac{4a^2K^2pt_1\Delta}{q^2S_0^{2a+1}} + \frac{2a^2(p - R + bt_0 + ht_1)}{S_0} - 2(a+1)\mathrm{Log}\left[\frac{K\Delta}{qS_0^a}\right]$$

Proof. Please see Appendix A1.

A practical range of the values of S_0 for which Theorem 3.1 holds is given below in Corollary 3.2. □

Corollary 3.2. *TEP is a concave function of S_0 in the range*

$$\left(\frac{3K(bt_0 + ht_1)}{2q(b+h)}\right)^{1/a} \leqslant S_0 \leqslant \left(\frac{4Kt_1(a-1)}{q(2a^2 + a - 3)}\right)^{1/a}.$$

Proof. Please see Appendix A2.

We now derive a range of values of S_0 for which $TEP_{L=0}$ is concave. □

Theorem 3.3. *$TEP_{L=0}$ is a concave function of S_0 iff $g(S_0) \geqslant (b+h)\, t_1$, where*

$$g(S_0) = 2pK_1^2 + \frac{S_0}{a^2}\left(2(a-1)K_1 + (a+1)\right),$$

such that

$$K_1 = \frac{Kt_1}{qS_0^a}.$$

Proof. Please see Appendix B1. □

Note 3. $K_1 = \frac{Kt_1}{qS_0^a} \geqslant 1$ since $qS_0^{\mathrm{a}} \leqslant Kt_1$.

The condition in Theorem 3.3 depends upon both the decision variables S_0 and q and hence is not of much practical use. For a specified value of S_0, Theorem 3.4 enables one to obtain the range of values of q for which $TEP|_{L=0}$ is a concave function of q.

Theorem 3.4. *A necessary and sufficient condition for $TEP_{L=0}$ to be a concave function of q is $q \leqslant \frac{K(S_0-c+p)}{(b+h)S_0^a}$.*

Proof. Please see Appendix C1. □

The practical range of S_0 for which Theorem 3.4 will hold is given by Corollary 3.5 below.

Corollary 3.5. *$TEP_{L=0}$ is concave in q if $S_0 \geqslant c - p + (b+h)t_1$*

Proof. Please see Appendix C2. □

Since TEP and $TEP|_{L=0}$ are transcendental functions in the decision variables, it is not possible to get closed form solutions for S_0 and q. Hence numerical techniques have to be employed for finding q^* and S_0^*. Corollaries 3.2, 3.5 and Theorem 3.4 can be used for finding initial guess value(s) while obtaining the roots for S_0.

Theorem 3.6 presents a necessary and sufficient condition for joint optimality of TEP with respect to both the decision variables, which can be verified for a given set of parameter values.

Theorem 3.6. *$TEP_{L=0}$ is a concave function in $(q,\, S_0)$ iff $g(S_0, K_1) \geqslant 2$ where*

$$\begin{aligned} g\left(S_0, K_1\right) = {} & 2a^2(S_0 - c + p)\frac{K_1^3}{S_0^2} + \frac{\left((3a-1)(b+h)t_1S_0 - \left(a(b+h)t_1\right)^2\right)}{S_0^2} \\ & + \frac{K_1}{S_0^2}\begin{pmatrix} S_0\left((3a-1)(c-p) + (3a^2-6a+2)(b+h)t_1\right) \\ -(3a-5)S_0^2 - 3a^2(b+h)(c-p)t_1 \end{pmatrix} \\ & + \frac{2K_1^2}{S_0^2}\left(-a^2\left((c-p)^2 + p(b+h)t_1\right)\right. \\ & \left. + S_0(2a^2-3a+1)(c-p) - S_0^2(a^2+3a-2)\right). \end{aligned}$$

Proof. Please see Appendix D1. □

4. Numerical illustrations and managerial insights

In this section, based on the empirical data analyzed in Section 1.3, we present profit analysis using the strategies ST1, ST2 and ST3 where we identify optimal and viable policies for market conditions $M1$, $M2$, $M3$ and $M4$.

Example 4.1. The empirical data discussed in Section 1.3 can be summarized as the case where $D(S_0) = KS_0^{-a}$ with $K = 97\,674$, $a = 1.3$ (Fig. 2) and rate of price decline $b = 35.64$ ($y2$ of Fig. 1b), $t_0 = 10$, $t_1 = 84$, $c = 300$, $R = 200$ and $p = 30$. This means that orders are placed 10 time units before the actual beginning of the season and the season lasts for approximately 74 days, goods are purchased at Rs. 300/unit and unsold units at the end of the season are disposed off at Rs. 200/unit. Penalty cost is assumed to be 10% of the purchase price per unit.

For the stated set of parameter values, Table 1 shows the values of S_0^*, q^* and TEP^* calculated using ST1 for $M1$, $M2$, $M3$ and $M4$ for different values of holding cost.

4.1. Conducive levels of holding cost

In Table 1, we observed that, as h increases, $TEPj^*$ and q_j^* decrease while S_0j^* increases for each $j = 1, 2, 3 \,\&\, 4$. The values of $TEPj^*$, S_0j^* and qj^* may be used to decide whether a business venture should be taken up or called off for a given level of holding cost. For example, if the target profit is above Rs. 90 000, it is unachievable if $h \geqslant 9$ for $M1$. Alternatively, if the ISP of an item needs to be set below Rs. 5000 due to competition, and $h \geqslant 9$ for the business set up, optimal ISPs obtained for none of the market conditions are viable indicating that the business will not be conducive in these situations.

4.2. ST1: Not viable in $M1$ and $M3$

If the fixed value of S_0 for ST2 and the fixed value of q for ST3 deviate from the respective optimal values calculated using ST1 then $TEP^*|_{\mathrm{ST1}} > \mathrm{Max}(TEP^*|_{\mathrm{ST2}}, TEP^*|_{\mathrm{ST3}})$. However, in practice, ST1 may not be viable in all the market conditions. Table 1 shows that $S_{0\mathrm{j}}{}^*|_{\mathrm{ST1}}$ for $M1$ and $M3$ are much higher than viable MRPs. For example, even at $h = 1$, $S_01^* = 10009.09$ and $S_03^* = 8708.0$ that are much higher than the actual MRPs of the items under consideration which are generally between Rs. 1000 to Rs. 5000 as per our observation. Although not appropriate in $M1$ and $M3$, ST1 is an effective strategy for $M2$ and $M4$ *e.g.*, $S_{0j}{}^*$ <5000 for any for $h < 9$. These ISPs are acceptable as per existing market MRPs.

4.3. Sensitivity of market conditions with unit change in holding cost

In this section, we investigate the sensitivity of $TEPj^*$, qj^* and $S_{0j}{}^*$ with unit increase in the value of h *i.e.*, from $h = i$ to $i + 1$ across a range of penalty cost p.

TABLE 1. $TEPj^*$, qj^* and $S_{0j}{}^*$; $j = 1, 2, 3$ and 4 calculated using ST1 for different values of h.

	M1			M2			M3			M4		
H	TEP1*	q1*	$S_0$1*	TEP2*	q2*	$S_0$2*	TEP3*	q3*	$S_0$3*	TEP4*	q4*	$S_0$4*
1	97771.4	40.4	10009.1	144769.1	203.1	2542.1	188235.1	60.1	7872.5	278280.7	317.6	2089.0
2	96474.7	37.6	10415.1	138902.9	168.6	2870.9	185075.2	55.5	8306.5	263271.9	248.8	2486.2
3	95271.0	35.2	10808.3	134004.8	143.7	3191.7	182161.5	51.5	8731.7	251332.4	203.1	2876.5
4	94146.8	33.1	11191.4	129813.3	124.9	3506.8	179458.2	48.0	9150.0	241482.4	170.7	3262.4
5	93091.3	31.2	11566.1	126160.3	110.3	3817.6	176937.2	45.0	9562.8	233142.0	146.6	3645.3
6	92096.1	29.6	11933.7	122930.8	98.5	4125.2	174576.0	42.4	9971.0	225939.7	128.0	4026.0
7	91154.4	28.2	12295.3	120042.9	88.8	4430.1	172356.0	40.0	10375.5	219623.8	113.3	4405.0
8	90260.6	26.8	12651.7	117435.9	80.8	4733.0	170262.0	37.8	10776.8	214016.1	101.3	4782.7
9	89409.8	25.7	13003.5	115063.9	73.9	5034.2	168280.9	35.9	11175.4	208986.1	91.4	5159.5
10	88598.2	24.6	13351.3	112891.2	68.1	5334.0	166401.8	34.2	11571.6	204435.5	83.2	5535.4
11	87822.2	23.6	13695.4	110889.3	63.0	5632.6	164615.3	32.6	11965.8	200288.3	76.1	5910.7
12	87078.8	22.6	14036.4	109035.6	58.6	5930.1	162913.1	31.1	12358.1	196485.0	70.1	6285.5
13	86365.5	21.8	14374.4	107311.3	54.7	6226.8	161288.1	29.8	12748.9	192977.9	64.9	6659.8
14	85679.9	21.0	14709.8	105701.1	51.3	6522.7	159734.2	28.5	13138.3	189728.3	60.3	7033.8
15	85020.0	20.3	15042.8	104192.1	48.2	6818.0	158245.7	27.4	13526.4	186704.3	56.3	7407.4
16	84383.9	19.6	15373.6	102773.5	45.5	7112.6	156817.7	26.3	13913.4	183879.5	52.7	7780.7
17	83770.1	18.9	15702.3	101436.0	43.0	7406.8	155445.9	25.3	14299.3	181231.8	49.5	8153.8
18	83177.1	18.3	16029.2	100171.6	40.7	7700.5	154126.4	24.4	14684.3	178742.4	46.6	8526.7
19	82603.6	17.8	16354.4	98973.5	38.7	7993.8	152855.5	23.5	15068.5	176395.1	44.0	8899.3
20	82048.4	17.2	16678.0	97835.8	36.8	8286.7	151630.2	22.7	15451.9	174176.3	41.7	9271.9

TABLE 2. Sensitivity of $TEPj^*$, qj^* and S_0j^* to the change in holding cost.

	TEP1*	q1*	$S_0$1*	TEP2*	q2*	$S_0$2*	TEP3*	q3*	$S_0$3*	TEP4*	q4*	$S_0$4*
APC_{30}	−0.92	−4.37	2.73	−2.04	−8.53	6.45	−1.13	−4.98	3.62	−2.43	−10.01	8.23
ACP_{10}	−0.92	−4.39	2.74	−2.07	−8.63	6.56	−1.13	−4.98	3.62	−2.43	−10.01	8.23
APC_{50}	−0.91	−4.36	2.72	−2.01	−8.44	6.34	−1.13	−4.98	3.62	−2.43	−10.02	8.23
APC	−0.92	−4.37	2.73	−2.04	−8.53	6.45	−1.13	−4.98	3.62	−2.43	−10.01	8.23

In Table 2, for each column, APC u is the average percentage change calculated as

$$\frac{1}{19}\sum_{i=1}^{19}\frac{100\times(Uj^*|_{h=i+1}-Uj^*|_{h=i})}{U^*|_{h=i}}$$

where $U = TEP$, S_0 and q. APC is the average of APC u for all u. Negative APC indicates decrease in the value with unit increase in holding cost. For interpretation, we consider only the magnitude of the average. We observe that with change in h, $M4$ is most sensitive with APC in $TEP4^*$, q_4^*, S_04^* being (2.43, 10.1, 8.23) followed by $M2$ (2.04, 8.53, 6.45), $M3$ (1.13, 4.98, 3.62) and $M1$ (0.92, 4.37, 2.73). On comparing APC's in $M3$ and $M2$, it may be noted that presence of lead time makes the business much more sensitive to change in holding cost than the presence of price decline.

TABLE 3. Sensitivity of Mj to the presence/absence of random lead time and/or price decline.

	$M1$-$M2$	$M1$-$M3$	$M1$-$M4$	$M2$-$M3$	$M2$-$M4$	$M3$-$M4$
	$TEP1^*$-$TEP2^*$	$TEP1^*$-$TEP3^*$	$TEP1^*$-$TEP4^*$	$TEP2^*$-$TEP3^*$	$TEP2^*$-$TEP4^*$	$TEP3^*$-$TEP4^*$
AC_{30}	−26 205.6	−78 259.6	−121 245	−52 054.1	−95 039.2	−42 985.2
AC_{10}	−26 625.6	−78 118.2	−121 124	−51 492.6	−94 498.3	−43005.7
AC_{50}	−25 802.6	−78 399.9	−121 365	−52 597.3	−95 562.2	−42 964.9
AC	26211.2	78259.2	121244	52048	95033.2	42985.3
	$S_0 1^*$-$S_0 2^*$	$S_0 1^*$-$S_0 3^*$	$S_0 1^*$-$S_0 4^*$	$S_0 2^*$-$S_0 3^*$	$S_0 2^*$-$S_0 4^*$	$S_0 3^*$-$S_0 4^*$
AC_{30}	8000.28	1728.65	7751.205	−6271.63	−249.074	6022.553
AC_{10}	8026.76	1650.22	7676.479	−6376.53	−350.278	6026.256
AC_{50}	7975.53	1806.88	7825.799	−6168.66	−149.733	6018.922
AC	8000.86	1728.584	7751.161	6272.27	249.695	6022.577
	$q1^*$-$q2^*$	$q1^*$-$q3^*$	$q1^*$-$q4^*$	$q2^*$-$q3^*$	$q2^*$-$q4^*$	$q3^*$-$q4^*$
AC_{30}	−56.3936	−10.4331	−82.6338	45.9605	−26.2402	−72.2
AC_{10}	−58.2049	−10.2646	−82.519	47.94033	−24.314	−72.3
AC_{50}	−54.6973	−10.5994	−82.7478	44.09784	−28.0505	−72.1
AC	−56.4319	−10.4324	−82.6335	45.99956	−26.2016	−72.2

4.4. Sensitivity of TEP^*, $S0^*$ and q^* to lead time and decline in selling price

We now segregate the effect of lead time and price decline on TEP^*, q^* and S_0^*. Table 3 – provides AC_p, the average change calculated using the values in Table 1 which is calculated as $\sum_{k=1}^{20}(Ui^*|_{h=k} - Uj^*|_{h=k})$ for $U = TEP$, S_0, or q; $i = 1, 2, 3$, $j = 2, 3, 4$ and for penalty cost p. "AC" is the overall average across the values of p. Negative AC indicates the decrease in the optimal values of profit, order quantity or ISP due to the presence of either lead time or price decline. Table 3 provides the following information. Mi-Mj reveals the effect of change from market Mi to market Mj; i, $j = 1, 2, 3, 4$. For example, in the column $M1$-$M2$, we observe that the presence of exogenous decline in selling price leads to lowering of the average TEP^* by Rs. 26211.2, increase in average S_0^* by Rs. 8000.9 and lowering of the average optimal order quantity by 56.4 units.

4.5. ST2: Sustaining business in competitive market

This strategy enables practitioners to identify optimal order quantity when ISP needs to be set competitively. The graph of qj^* *vs.* ISP is presented in Figure 4a where qj^*; $j = 1, 2, 3$ & 4 for different values of ISP are calculated using parameters in Example 4.1 with $h = 10$. "#" marked on each curve indicates the ISP at which unique maximum qj^* is obtained and decreasing ISP below this value will lead to decreased qj^*. This shows that decreasing the ISP does not always lead to increase in optimal sales volume. For example, for the market condition $M1$, the maximum optimal sales volume $q1^* = 72.93$ is achieved at $S_0 = 3170$ and any value of ISP below this will yield lower optimal sales volume.

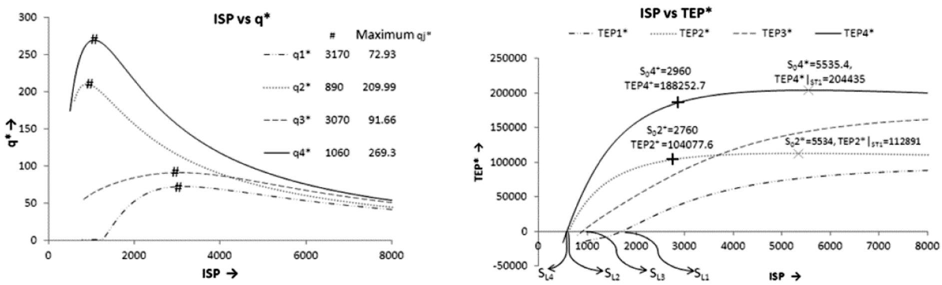

FIGURE 4. (a) Reducing ISP does not necessarily increase optimal sales volume (q^*). # denotes the ISP below which q^* will decline. (b) Minimum values of ISPj (*i.e.*, SLj) below which business will incur loss. $\times$ denotes theoretical optimal ISP while + denotes the practical ISP.

Our finding is consistent with that of the article in [22] which had also pointed out that the general perception that lowering price will bring more consumers is not always true.

4.6. ST2 to identify minimum ISP below which business will eat red ink

The graph of ISP *vs.* TEP_j^* in Figure 4b shows the minimum value of ISP for each market condition below which a business venture will eat red ink. We label these minimum values as S_{Lj} ($j = 1, 2, 3, 4$ representing the market condition Mj) in the graph. For $M1$, lowering ISP below $S_{\mathrm{L1}} = R$s. 1720/unit will make the business a losing venture and so will $S_{\mathrm{L2}} = R$s. 620, $S_{\mathrm{L4}} = R$s. 1060 while $S_{\mathrm{L3}} = R$s. 900.

4.7. Theoretical optimal *vs.* viable pricing policy

The graph of $TEPj^*$ against ISP in Figure 4b shows that, for $M2$ and $M4$ *i.e.*, when there is no decline in selling price, the $TEPj^*$ curves are almost flat for a long range of ISP. "×" marked on the curves denotes the optimal value of ISP $= S_{0j}{}^*|_{\mathrm{ST1}}$ which will provide $TEPj^*|_{\mathrm{ST1}}$. "+" marked on the profit curves indicates the viable value of ISP say

$$S_0j^+ < S_{0\mathrm{j}}{}^*|_{\mathrm{ST1}}$$

such that difference between the optimal profit and profit at $ISP = S_0\mathrm{j}^+$ is tolerable say

$$DIFF_{\mathrm{Profit}} = 100 \times \left(\frac{TEPj^*|_{\mathrm{ST1}} - TEPj^*|_{\mathrm{ISP}=S_{0j}^+}}{TEPj^*|_{\mathrm{ST1}}} \right) \leqslant 10\%$$

but the difference between $S_0^*|_{\mathrm{ST1}}$ and $S_0 j^+$ is significant say

$$DIFF_{\mathrm{ISP}} = 100 \times \left(\frac{S_0 j^*|_{ST1} - S_0 j^+|_{\mathrm{ISP}=S_{0j}^+}}{S_0 j^*|_{ST1}} \right) \geqslant 40\%.$$

For example, in $M4$, $DIFF_{\mathrm{Profit}} = 7.9$ but $S_0 4^+ = 2960$ is approximately 47% lower than $S_{04}^*|_{\mathrm{ST1}} = 5535.4$, which may otherwise be non viable in competitive market. Similarly, for $M2$, $DIFF_{\mathrm{Profit}} = 7.8$ and $DIFF_{\mathrm{ISP}} = 50.1$. When investigated for different values of holding cost and for penalty costs $p = 10$, 30 and 50, $DIFF_{\mathrm{Profit}}$ and $\mathrm{DIFF}_{\mathrm{ISP}}$ for $j = 2$ and 4 are consistent with the above findings. This investigation shows that when there is no decline in selling price, practitioners can identify a viable pricing policy without compromising much on *TEP*. However, for $M1$ and $M3$, in order to bring ISP down to a practical level, one has to compromise around 20% of the profit return obtained from ST1 (data not shown).

4.8. ST3 to identify maximum profitable (*i.e.*, viable target) sales volume

We now consider the situation when a sales target is set for a season and manager can maximize the profit only by optimizing the ISP. Each row in Table 4 shows the values of TEP_j^* and S_{0j}^* calculated for different target values of q. For each value of the holding cost, there exists an upper cap of the profitable sales volume q_{mj} where j represents the market condition (last column of Tab. 4).

Procuring order quantity beyond q_{mj} will lead to loss irrespective of the value of ISP. For example, when $h = 10$, q_{m1} is 160 *i.e.*, procuring more than 160 units will result in loss, making any target above this is not viable in terms of profit. Similarly, $q_{m2} = 410$, $q_{m3} = 210$ and $q_{m4} = 560$. The values of q_{mj} for $h = 15$ and 20 are also provided in the table. This allows managers to identify the viable target sales volumes.

4.9. Target profit: adjusting sales volume for a given S_0 ($<S_0^*$)

If a firm enters a competitive market with a fixed ISP which is less than the theoretical optimal value, then the theoretical optimal profit is not achievable. The firm may then consider a target profit $<TEP^*$ and increase the sales volume above its optimal value to achieve the target. For example, suppose a firm enters $M4$ having parameter values as provided in example 1 and $h = 10$ with fix $S_0 = R$s. 3000 ($<S_0 4^*$). From Table 1, we observe that for $h = 10$, $S_0 4^* = 6043$, $q_4^* = 84$ and $TEP4^* = 225\,796$. Figure 5 – the graph of TEP_j; $j = 1, 2, 3$ & 4 *vs.* q for $h = 10$, $S_0 = 3000$ and other parameters of example 1– shows the range of the values of q $[q_{Lj}, q_{Uj}]$; $q_j^* < q_{Lj} < q_{Uj}$, that need to be sold to achieve a target profit. If the firm sets a target profit $TEP4 = 200\,000$, Figure 5 shows that this target is achievable for $150 \leqslant q \leqslant 190$ *i.e.*, at least 66 units more than q^*4 are required to be sold but not more than 106 units to achieve this target profit.

TABLE 4. $TEPj^*$ and S_0j^*; $j = 1, 2, 3\&4$ calculated using ST3. q_{mj} is the maximum order quantity beyond which $TEPj^* < 0$. "X" denotes either loss (-ve profit) or non profitable optimal ISP.

h ▼	q ▶	10	60	110	160	210	260	310	360	410	460	510	560	q_{mj}
$h = 10$	$TEP1^*$	82 956	77 754	49 417	14939	-22618	X	X	X	X	X	X	X	160
	S_01^*	25104	7690	5080	3808	3089	X	X	X	X	X	X	X	
	$TEP2^*$	89 296	112 693	109 231	99 191	85 933	70 728	54 203	36 719	18503	−290	X	X	410
	S_02^*	23 635	5889	3661	2721	2189	1843	1598	1413	1269	1153	X	X	
	$TEP3^*$	163 378	173 414	132 201	78 413	18 464	−45 224	X	X	X	X	X	X	210
	S_03^*	32 132	8098	5080	3808	3089	2621	X	X	X	X	X	X	
	$TEP4^*$	171 687	223 249	223 667	211 611	193 499	171 751	147 570	121 648	94 426	66 204	37 197	7566	560
	S_04^*	31 667	7867	4874	3610	2894	2426	2092	1841	1644	1483	1350	1237	
$h = 15$	$TEP1^*$	81 381	67 647	30 516	−12 553	X	X	X	X	X	X	X	X	110
	S_01^*	24 937	7481	5080	3808	X	X	X	X	X	X	X	X	
	$TEP2^*$	87 761	103 568	92 640	75 247	54 749	32 416	8876	−15 509	X	X	X	X	310
	S_02^*	23 473	5723	3491	2549	2015	1666	1418	1232	X	X	X	X	
	$TEP3^*$	160 370	155 357	99 097	30 261	−44 735	X	X	X	X	X	X	X	160
	S_0^*3	32038	8098	5080	3808	3089	X	X	X	X	X	X	X	
	$TEP4^*$	168 702	205 534	191 506	165 297	133 347	98 110	60 831	22 256	−17 108	X	X	X	360
	S_0^*4	31 547	7735	4732	3458	2730	2249	1903	1636	1422	X	X	X	
$h = 20$	$TEP1^*$	79 810	57 631	11 615	−40 045	X	X	X	X	X	X	X	X	110
	S_01^*	24 769	7275	5080	3808	X	X	X	X	X	X	X	X	
	$TEP2^*$	86 230	94 562	76 423	52 078	24 895	−3845	X	X	X	X	X	X	210
	S_02^*	23 310	5553	3316	2367	1828	1475	X	X	X	X	X	X	
	$TEP3^*$	157 368	137 301	65 992	−17 891	X	X	X	X	X	X	X	X	110
	S_03^*	31913	8098	5080	3808	X	X	X	X	X	X	X	X	
	$TEP4^*$	165 723	187 982	159 894	120 202	75 456	28 272	−19 875	X	X	X	X	X	260
	S_04^*	31 425	7596	4575	3280	2529	2020	1639	X	X	X	X	X	

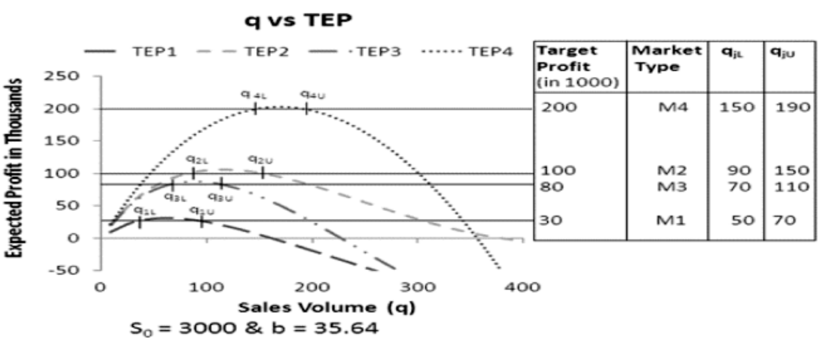

FIGURE 5. The maximum achievable target profit and the range of required sales volume q_{jL} and q_{jU}, where q_{jL}/q_{jU} denotes the minimum/maximum sales volume that need to be sold to achieve the target profit for market Mj.

Similarly, range of q for Mj; $j = 1, 2$ and 3 for revised targets are shown in the figure. The practitioner may opt to go for q_{UL} in order to bring market visibility of the products.

5. Conclusion

In this paper, empirical data analysis of price and demand shows the presence of exogenous decline in selling price and stochastic decline in demand potential of products with increase in ISP. Approximating the exogenous price decline by a linear function, we extend an existing newsboy type model considering ISP and order quantity as the decision variables. Profit maximization has been investigated for markets with and without lead time/price decline by considering (i) jointly optimal pricing and ordering policy, (ii) ordering policy for a fixed ISP and (iii) pricing policy for a fixed order quantity. We observe that (i) may not be suitable when lead time and/or price decline is present. Sensitivity of decision variables due to change in holding cost, penalty cost and presence of lead time and price decline have been investigated. For (ii), we have shown that indiscriminately lowering the selling price may not lead to higher optimal sales volume and also identified the minimum value of ISP below which the business will no longer remain profitable. For (iii), we identify the upper limit of target sales volume such that order quantity more than this value will make the business eat red ink. On the other hand, when ISP has to be reduced below the optimal price due to competition, we identify the number of units that need to be sold in addition to the optimal sales volume so as to achieve a target profit. Our work assists a practitioner to set a realistic value of ISP and target profit/sales volume in view of the existing business constraints.

Appendix A1. Proof of Theorem 3.1.

Differentiating (1) with respect to S_0, and equating to 0 and simplifying, we get

$$
\begin{aligned}
&12q - \frac{4(b+h)q^3 S_0^{2a-1}}{K^2 t_1} + 6KpS_0^{-a-1}\varDelta - \frac{3q^2 S_0^a}{Kt_1\varDelta} \\
&\quad \times \left(2t_0 + 3\varDelta - \frac{2a\varDelta\,(p-R+S_0+bt_0+ht_1)}{S_0} + 2\varDelta\,\mathrm{Log}\left[KS_0^{-a}\varDelta/q\right]\right) \\
&= \frac{3aq^2 S_0^{a-1}}{Kt_1\varDelta}\left(A^*\,(2t_0+3\varDelta) + \varDelta\,(3bt_0+2ht_0-2bt_1+ht_1)\right. \\
&\quad \left. +2\varDelta\,(A^*+bt_0+ht_1)\,\mathrm{Log}\left[KS_0^{-a}\varDelta/q\right]\right), \qquad \text{(A1.1)}
\end{aligned}
$$

where $A^* = (p-R+S_0)$

Second derivative of TEP with respect to S_0 is given by,

$$
\begin{aligned}
&\frac{\partial^2 TEP}{\partial S_0^2} = \frac{4aq^3(2a-1)(b+h)}{K^2 t_1 S_0^{2(1-a)}} - \frac{6(a+1)aKp\Delta}{S_0^{a+2}} - \frac{6q^2 S_0^{a-2}a\Delta\left(Q-2S_0^{-1}\right)}{Kt_1\Delta} \\
&-\frac{6aq^2 S_0^{a-1}(2t_0+3\Delta-2\Delta((aQ/S_0)-\mathrm{Log}[K\Delta/(qS_0^a)]))}{Kt_1\Delta} \qquad \text{(A1.2)} \\
&-\frac{3a(a-1)q^2 S_0^{a-2}(A*(2t_0+3\Delta)+\Delta(3bt_0+2ht_0-2bt_1+ht_1)+2\Delta Q\,\mathrm{Log}[K\Delta/(qS_0^a)])}{Kt_1\Delta},
\end{aligned}
$$

where $Q = p - R + S_0 + bt_0 + ht_1$.

Substituting (A1.1) in (A1.2) and simplifying, we get,

$$\frac{\partial^2 TEP}{\partial S_0^2} = \frac{-S_0^{-2-a}}{12K^2 t_1 \Delta} \begin{pmatrix} 4a^2(b+h)q^3 S_0^{3a}\Delta + 6(a+1)Kq^2 S_0^{2a+1}\Delta \\ \times \mathrm{Log}\,[K\Delta/(qS_0^a)] + 12(a-1)K^2 q S_0^{a+1} t_1 \Delta \\ +12a^2K^3 p t_1 \Delta^2 + 3Kq^2 S_0^{2a} \\ \times \left(S_0\left(2t_0(a+1) - \Delta(2a^2+a-3)\right) - 2a^2\Delta B^*\right) \end{pmatrix} \tag{A1.3}$$

where $B^* = (p - R + bt_0 + ht_1)$.

TEP is concave in S_0 iff

$$\begin{aligned} &- 4a^2(b+h)q^3 S_0^{3a}\Delta - 12(a-1)K^2 q S_0^{a+1} t_1\Delta - 12a^2K^3 p t_1 \Delta^2 \\ &- 6(a+1)\mathrm{Log}\,[K\Delta/(qS_0^a)] \\ &- 3Kq^2 S_0^{2a}\left(S_0\left(2t_0(a+1) - \Delta(2a^2+a-3)\right) - 2a^2\Delta B^*\right) \leqslant 0 \end{aligned} \tag{A1.4}$$

Dividing both side of (A1.4) by $Kq^2 S_0^{2a+1}\Delta$, we get

$$\begin{aligned} &- 4a^2(b+h)\left(qS_0^a/(KS_0)\right) - 12(a-1)\left(Kt_1/q\right) \\ &- 12\left(a^2K^2 p t_1\Delta/(q^2 S_0^{2a+1})\right) \\ &- \frac{3}{\Delta S_0}\left(S_0\left(2t_0(a+1) - \Delta(2a^2+a-3)\right)\right. \\ &\left.-2a^2\Delta B^* - 6(a+1)\mathrm{Log}\,[K\Delta/(qS_0^a)]\right) \leqslant 0 \\ &\Rightarrow -4a^2 q S_0^a\left((b+h)/(3KS_0)\right) - \left(4a^2K^2 p t_1\Delta/(q^2S_0^{2a+1})\right) \\ &+ 2a^2\left(B^*/S_0\right) - 2(a+1)\mathrm{Log}\,[K\Delta/(qS_0^a)] \\ &\leqslant 2t_0(a+1) - \Delta(2a^2+a-3) + 4(a-1)Kt_1/q \\ &\Rightarrow f(S_0) \leqslant 2t_0(a+1) - \Delta(2a^2+a-3) + 4(a-1)Kt_1/q \end{aligned}$$

where

$$\begin{aligned} f(S_0) = &\left(2a^2B^*/S_0\right) - 4a^2\left(\left((b+h)qS_0^a/(3KS_0)\right)\right. \\ &\left.+K^2 p t_1\Delta/(q^2S_0^{2a+1})\right) - 2(a+1)\mathrm{Log}\,[K\Delta/(qS_0^a)] \end{aligned}$$

This completes the proof of Theorem 3.1.

Appendix A2. Proof of Corollary 3.2

Let us denote the negative terms and positive terms in (A1.4) by $y1$ and $y2$ respectively, where

$$\begin{aligned} y1 = &- 4a^2(b+h)q^3 S_0^{3a}\Delta - 12(a-1)K^2 q S_0^{a+1} t_1\Delta \\ &- 12a^2K^3 p t_1\Delta^2 - 6(a+1)Kq^2 S_0^{2a+1}(\Delta\,\mathrm{Log}\,[K\Delta/(qS_0^a)] + t_0) \end{aligned}$$

and

$$y2 = 6a^2Kq^2S_0^{2a}\Delta\,B^* + 3Kq^2S_0^{2a+1}\Delta\left(2a^2 + a - 3\right)$$

TEP is concave in S_0 iff

$$|y1| \geqslant |y2|\,. \tag{A2.1}$$

This is true iff

$$3Kq^2S_0^{2a+1}\Delta(2a^2 + a - 3) - 12(a-1)K^2qS_0^{a+1}t_1\Delta \leqslant 0 \tag{A2.2}$$

and

$$\begin{aligned}6a^2Kq^2S_0^{2a}\Delta\,B^* - 4a^2(b+h)q^3S_0^{3a}\Delta - 12a^2K^3pt_1\Delta^2 - 6(a+1)&\\ \times\,Kq^2S_0^{2a+1}(t_0 + \Delta\,\mathrm{Log}\,[K\Delta/(qS_0^a)]) \leqslant 0&\end{aligned} \tag{A2.3}$$

(A2.2) holds true if

$$-3KqS_0^{a+1}\Delta\left(4(a-1)Kt_1 - qS_0^a(2a^2 + a - 3)\right) \leqslant 0$$

$$i.e.,\text{ iff } 4(a-1)Kt_1 \geqslant\ qS_0^a(2a^2 + a - 3)$$

$$i.e.,\text{ iff } \left(\frac{4(a-1)Kt_1}{q(2a^2 + a - 3)}\right)^{1/a} \geqslant\ S_0 \tag{A2.4}$$

Now, from (A2.3), compare the two components $6a^2Kq^2S_0^{2a}\Delta p$ and $12a^2 K^3pt_1\Delta^2$. The inequality

$$6a^2Kq^2S_0^{2a}\Delta p - 12a^2K^3pt_1\Delta^2 \leqslant 0$$

$i.e.,\ q^2S_0^{2a+1} - 2K^2t_1\Delta \leqslant 0$

is always true since $qS_0^a \leqslant K\Delta \leqslant Kt_1$. Thus, (A2.3) holds true if

$$\begin{aligned}6a^2Kq^2S_0^{2a}\Delta(bt_0 + ht_1) - 4a^2(b+h)q^3S_0^{3a}\Delta&\\ - 6(a+1)Kq^2S_0^{2a+1}\left(t_0 + \Delta\,\mathrm{Log}\,[K\Delta/(qS_0^a)]\right) \leqslant 0&\end{aligned}$$

The above inequality always holds true if

$$6a^2Kq^2S_0^{2a}\Delta(bt_0 + ht_1) - 4a^2(b+h)q^3S_0^{3a}\Delta \leqslant 0$$

$$i.e.,\text{ iff } 3K(bt_0 + ht_1) \leqslant 2(b+h)qS_0^a$$

$$i.e.,\text{ iff } \left(\frac{3K(bt_0 + ht_1)}{2(b+h)q}\right)^{1/a} \leqslant S_0 \tag{A2.5}$$

From (A2.4) and (A2.5), it is proved that if *TEP* is always a concave function of S_0 in the range

$$\left(\frac{3K(bt_0 + ht_1)}{2(b+h)q}\right)^{1/a} \leqslant S_0\ \left(\frac{4(a-1)Kt_1}{q(2a^2 + a - 3)}\right)^{1/a}.$$

Appendix B1. Proof of Theorem 3.3

Differentiating Equation (3.2) with respect to S_0 and equating to 0, we get

$$2(p-R)+t_1\left(b+3h+2(b+h)\mathrm{Log}\left[Kt_1/(qS_0^a)\right]\right)+2S_0=$$
$$\left(4Kt_1/(aq^2S_0^{a-1})\right)\left(q+\left(aKpt_1S_0^{-a-1}/2\right)-\left(q^2S_0^a a(b+h)/2K\right)\right)\quad \text{(B1.1)}$$

Now, the second derivative of $TEP_{L=0}$ with respect to S_0 is calculated as follows

$$\frac{\partial TEP_{L=0}}{\partial S_0^2}=\frac{aS_0^{-2-a}}{4K}\begin{pmatrix}-2(a+1)K^2pt_1-4q^2S_0^a\left(at_1(b+h)-S_0\right)/t_1\\ \left((a-1)q^2S_0^a\left(2(p-R)\right.\right.\\ \left.\left.+t_1\left(b+3h+2(b+h)\mathrm{Log}\left[Kt_1/(qS_0^a)\right]\right)+2S_0\right)\right)/2\end{pmatrix}$$

Substituting the rhs of (B1.1) in (B1.2) and simplifying, we get

$$\frac{\partial TEP_{L=0}}{\partial S_0^2}=\frac{aS_0^{-2-a}}{2Kt_1}\left(a^2(b+h)t_1q^2S_0^{2a}-2a^2K^2pt_1^2\right.$$
$$\left.-2(a-1)Kt_1qS_0^{a+1}-(a+1)q^2S_0^{2a+1}\right)\quad \text{(B1.2)}$$

$TEP_{L=0}$ is a concave function of S_0 iff

$$a^2(b+h)t_1q^2S_0^{2a}-2a^2K^2pt_1^2-2(a-1)Kt_1qS_0^{a+1}-(a+1)q^2S_0^{2a+1}\leqslant 0\quad \text{(B1.3)}$$

Dividing (B1.4) by $a^2q^2S_0^{2\mathrm{a}}>1$ and simplifying, we get

$$2p\left(Kt_1/(qS_0^a)\right)^2+2a^{-2}(a-1)S_0\left(Kt_1/(qS_0^a)\right)+a^{-2}(a+1)S_0\geqslant(b+h)t_1$$

which implies that $TEP_{L=0}$ is a concave function of S_0 iff $g(S_0)\geqslant(b+h)\ t_1$ where

$$g(S_0)=2pK_1^2+S_0\,a^{-2}\left(2(a-1)K_1+a+1\right),$$

such that

$$K_1=Kt_1/(qS_0^a)\geqslant 1$$

since

$$qS_0^{\mathrm{a}}\leqslant Kt_1.$$

Appendix C1. Proof of Theorem 3.4

Differentiating Equation 3.2 with respect to q, we get

$$\frac{\partial TEP_{L=0}}{\partial q}=S_0+p-c+\frac{qS_0^a(b+h)}{2K}-\frac{qS_0^a}{2Kt_1}$$
$$\times\left(2(p-R)+t_1\left(b+3h+2(b+h)\mathrm{Log}\left[Kt_1/(qS_0^a)\right]\right)+2S_0\right)\quad \text{(C1.1)}$$

Equating (C1.1) to 0 and simplifying, we get

$$\left(S_0 + p - c + \frac{qS_0^a(b+h)}{2K}\right)\frac{2Kt_1}{qS_0^a} = 2(p-R) + t_1\left(b + 3h + 2(b+h)\mathrm{Log}\left[Kt_1/(qS_0^a)\right]\right) + 2S_0 \quad \text{(C1.2)}$$

Differentiating (C1.1) with respect to q further, we get

$$\frac{\partial^2 TEP_{L=0}}{\partial q^2} = \frac{3(b+h)S_0^a}{2K} - \frac{S_0^a\left(2(p-R) + t_1\left(b + 3h + 2(b+h)\mathrm{Log}\left[Kt_1/(qS_0^a)\right]\right) + 2S_0\right)}{2Kt_1} \quad \text{(C1.3)}$$

Substituting the equality of (C1.2) in (C1.3) and simplifying, we get

$$\frac{\partial^2 TEP_{L=0}}{\partial q^2} = \frac{K(c-p) - KS_0 + (b+h)qS_0^a}{Kq} \quad \text{(C1.4)}$$

$TEP_{L=0}$ is a concave function of q iff

$$K(c-p) - KS_0 + (b+h)qS_0^a \leqslant 0$$

$$i.e., \text{ iff } q \leqslant \frac{K(S_0 - c + p)}{(b+h)S_0^a} \quad \text{(C1.5)}$$

Hence proved.

Appendix C2. Proof of Corollary 3.5

We assume that $q \leqslant Kt_1S_0^{-a}$. Thus (C1.5) holds true if

$$\frac{S_0 - c + p}{(b+h)} \geqslant t_1$$

This implies that $TEP_{L=0}$ is concave in q if $S_0 \geqslant c - p + (b+h)t_1$. Hence proved.

Appendix D1: Proof of Theorem 3.6

Differentiating (C1.1) with respect to S_0 and substituting (C1.2) we get,

$$\frac{\partial^2 TEP_{L=0}}{\partial q S_0} = \frac{aK(c-p)t_1 - (a-1)Kt_1S_0 + a(b+h)t_1qS_0^a - qS_0^{a+1}}{Kt_1S_0} \quad \text{(D1.1)}$$

Since we have already proved that $TEP_{L=0}$ is a concave function in q and S_0, it is a concave function in $(q,\ S_0)$ iff the principal minor of the Hessian matrix (HSN) is positive. Here,

$$HSN = \frac{\partial^2 TEP_{L=0}}{\partial q^2} \times \frac{\partial^2 TEP_{L=0}}{\partial S_0^2} - \left(\frac{\partial^2 TEP_{L=0}}{\partial q S_0}\right)^2$$

Substituting (B1.3), (C1.4) and (D1.1) in the above expression and simplifying, we get

$$HSN = C^* \Bigg(\frac{Kt_1(c-p) - Kt_1 S_0 + (b+h)t_1 q S_0^a}{q S_0^a} C^{\#} \\ - 2\left(Kt_1\left(a(c-p) - (a-1)S_0\right) + qS_0^a(a(b+h)t_1 - S_0)\right)^2 \Bigg) \qquad \text{(D1.2)}$$

Where

$$C^* = 1/(2K^2 t_1^2 S_0^2)$$

and

$$C^{\#} = \left(\left(a^2(b+h)t_1 - (a+1)\right) q^2 S_0^{2a} - 2Kt_1(a^2 pKt_1 - (a-1)qS_0^{a+1})\right)$$

Simplifying (D1.2) further, we get, $HSN>0$ iff

$$\begin{aligned} &2a^2(S_0 - c + p)(Kt_1)^3 + (qS_0^a)^3 \\ &\times \left((3a-1)(b+h)t_1 S_0 - (a(b+h)t_1)^2 - 2S_0^2\right) \\ &+ Kt_1\,(qS_0^a)^2\left(-3a^2(b+h)(c-p)t_1\right. \\ &\left.+S_0\left((3a-1)(c-p) + (3a^2-6a+2)(b+h)t_1\right) - (3a-5)S_0^2\right) \\ &+ 2\,(Kt_1)^2\, qS_0^a\left(-a^2\left((c-p)^2 + p(b+h)t_1\right)\right. \\ &\left.+S_0(2a^2-3a+1)(c-p) - S_0^2(a^2+3a-2)\right) \geqslant 0 \end{aligned} \qquad \text{(D1.3)}$$

(D1.3) still holds true if it is divided by $q^3 {S_0}^{3a} > 0$. That is, $HSN>0$ iff

$$\begin{aligned} &2a^2(S_0 - c + p)K_1^3 + \left((3a-1)(b+h)t_1 S_0 - (a(b+h)t_1)^2 - 2S_0^2\right) \\ &+ K_1\left(-3a^2(b+h)(c-p)t_1\right. \\ &+S_0\left((3a-1)(c-p) + (3a^2-6a+2)(b+h)t_1\right) \\ &\left.-(3a-5)S_0^2\right) + 2K_1^2\left(-a^2\left((c-p)^2 + p(b+h)t_1\right)\right. \\ &\left.+S_0(2a^2-3a+1)(c-p) - S_0^2(a^2+3a-2)\right) \geqslant 0 \end{aligned}$$

where $K_1 = Kt_1/(qS_0^a) \geqslant 1$ since $qS_0^{\mathrm{a}} \leqslant Kt_1$.

Dividing the above inequality by $S_0^2 > 0$, *i.e.*, $HSN > 0$ iff $g(S_0, K_1) \geqslant 2$ where

$$
\begin{aligned}
&g\left(S_0, K_1\right) = 2a^2(S_0 - c + p)K_1^3 S_0^{-2} \\
&+ S_0^{-2}\left((3a-1)(b+h)t_1 S_0 - (a(b+h)t_1)^2\right) \\
&+ K_1\, S_0^{-2}\left(S_0\left((3a-1)(c-p)\right.\right. \\
&\left.\left. +(3a^2-6a+2)(b+h)t_1\right) - (3a-5)S_0^2 - 3a^2(b+h)(c-p)t_1\right) \\
&+ 2K_1^2\, S_0^{-2}\left(p(b+h)t_1 - a^2\left((c-p)^2\right)\right. \\
&\left. + S_0(2a^2-3a+1)(c-p) - S_0^2(a^2+3a-2)\right)
\end{aligned}
$$

Hence proved.

Appendix E. Mathematical expressions to calculate total expected profits

TEP = Selling Price (SP)−(Holding Cost (HC)+Penalty Cost (SC)+cq).

From the time inventory graph (Fig. 3),

$$
SP = \frac{1}{\mathrm{D}t_1}\left(\begin{array}{r}
\int\limits_0^{t_0}\left(\int\limits_0^{q/\Delta}\int\limits_{t_0}^{t_1} x\,S\left(t\right)\mathrm{d}t\mathrm{d}x + \int\limits_0^{q/\Delta} q_1 R\,\mathrm{d}x + \int\limits_{q/\Delta}^{D}\int\limits_{t_0}^{t_0^*} x\,S\left(t\right)\mathrm{d}t\mathrm{d}x\right)\mathrm{d}L + \\
\int\limits_{t_0}^{\tau}\left(\int\limits_0^{q/\Delta_1}\int\limits_{L}^{t_1} x\,S\left(t\right)\mathrm{d}t\mathrm{d}x + \int\limits_0^{q/\Delta_1} q_2 R\,\mathrm{d}x + \int\limits_{q/\Delta_1}^{D}\int\limits_{L}^{t_1^*} x\,S\left(t\right)\mathrm{d}t\mathrm{d}x\right)\mathrm{d}L \\
+\int\limits_{\tau}^{t_1}\left(\int\limits_0^{D}\int\limits_{L}^{t_1} x\,S\left(t\right)\mathrm{d}t\mathrm{d}x + \int\limits_0^{D} q_2 R\,\mathrm{d}x\right)\mathrm{d}L
\end{array}\right)
$$

$$
HC = \frac{h}{2\mathrm{D}t_1} \times \left(\begin{array}{l}
\int\limits_0^{t_0}\left(2Dq(t_0 - L) + \int\limits_0^{q/\Delta} \Delta\left(q+q_1\right)\mathrm{d}x + \int\limits_{q/\Delta}^{D} q\left(t_0^* - t_0\right)\mathrm{d}x\right)\mathrm{d}L \\
+\int\limits_{t_0}^{\tau}\left(\int\limits_0^{q/\Delta_1} \Delta_1\left(q+q_2\right)\mathrm{d}x + \int\limits_{q/\Delta_1}^{D} q\left(t_1^* - L\right)\mathrm{d}x\right)\mathrm{d}L + \int\limits_{\tau}^{t_1}\int\limits_0^{D} \Delta_1\left(q+q_2\right)\mathrm{d}x\mathrm{d}L
\end{array}\right)
$$

$$
SC = \frac{p}{\mathrm{D}t_1}\left(\int\limits_0^{t_0}\int\limits_{q/\Delta}^{D} x\left(t_1 - t_0^*\right)\mathrm{d}x\mathrm{d}L + \int\limits_{t_0}^{t_1}\int\limits_0^{D} x\left(L - t_0\right)\mathrm{d}x\mathrm{d}L + \int\limits_{t_0}^{\tau}\int\limits_{q/\Delta_1}^{D} x\left(t_1 - t_1^*\right)\mathrm{d}x\mathrm{d}L\right)
$$

On simplification, we get

$$TEP = \begin{pmatrix} qS_0 - \left((b+h)q^3/(6D^2t_1)\right) - (Dp\Delta/2) \\ \qquad - \left(q(ht_0^2 + 2ct_1 - 2pt_1 + b\Delta^2)/2t_1\right) \\ -q^2\,(4\mathrm{d}t_1\Delta)^{-1}\left((p-R+S_0)\,(2t_0+3\Delta) + \Delta\,(3bt_0\right. \\ \left. +2ht_0 - 2bt_1 + ht_1) + 2\Delta\,(p-R+S_0+bt_0+ht_1)\,\mathrm{Log}\left[KS_0^{-a}\Delta/q\right]\right) \end{pmatrix} \tag{E.1}$$

For the case $L = 0$

$$TEP_{L=0} = \frac{1}{D}\left(\left(\int_0^{q/t_1}\int_{t_0}^{t_0} x\,S\,(t)\,\mathrm{d}t\mathrm{d}x + \int_0^{q/t_1}(q-xt_1)\,R\,\mathrm{d}x + \int_{q/t_1}^{D}\int_{t_0}^{t_0^*} x\,\left(S_0 - bt_1\right)\mathrm{d}t\mathrm{d}x\right) - \frac{h}{2}\left(\int_0^{q/t_1} t_1\,(q+q_1)\,\mathrm{d}x + \int_{q/t_1}^{D}\frac{q^2}{x}\mathrm{d}x\right) - p\left(\int_{q/t_1}^{D}(xt_1-q)\mathrm{d}x\right)\right) - cq$$

On simplification, we get

$$TEP_{L=0} = q(S_0 - c + p) - \frac{Dpt_1}{2} + \frac{q^2}{4\mathrm{d}t_1}\left(-2(S_0 - R + p) - (b+3h)t_1 - 2t_1(b+h)\mathrm{Log}\left[\mathrm{d}t_1/q\right]\right) \tag{E.2}$$

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