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FOREWORD

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When I started my PhD on the collisional kinetic theory of gases, about two decades ago, the status of optimal transport theory was nowhere near what it is now. The subject was thought to be complicated, there was no general introduction textbook available, and nobody was sure about precise theorems and assumptions. True, there was a new wave transforming the subject, with a taste of partial differential equations; but the specialists could be counted on just a bit more than the fingers of one hand: Brenier, Caffarelli, Cullen, Gangbo, McCann, Urbas. I was lucky enough to profit by the advice of one of them, Yann Brenier, who was tutoring me in École Normale Supérieure in Paris. At that time I started to realize that there was much more to the subject than just a convenient contracting distance for solutions of a particular case of the Boltzmann equation...

Now optimal transport theory has been enriched by hundreds and hundreds of papers, and it keeps dozens of researchers busy. I spent a nonnegligible portion of these past years writing what have become the two most popular textbooks in the field – *Topics in Optimal Transportation*, more as an introduction; and *Optimal Transport, old and new*, more as a reference source. Both were attempting to present, survey, order, explain the amazing wealth of subjects touched upon by optimal transport. Incompressible fluid mechanics, Gaussian inequalities, gradient flows, logarithmic Sobolev inequalities, image processing, nonlinear diffusion equations, meteorology, fully nonlinear elliptic partial differential equations, concentration theory, isoperimetry, Sobolev inequalities, Riemannian geometry, Aubry–Mather theory, geometry of the space of probability measures... Like an epidemic, optimal transport contaminated subject after subject and revisited them, triggering wonder from mathematical analysts.

For me, the most amazing bit outcome of optimal transport has probably been the synthetic theory of Ricci curvature. It is in the mid-nineties that I learnt from Eric Carlen about Ricci curvature bounds and their use in the theory of logarithmic Sobolev inequalities; less than ten years later, I was among those who suggested a new definition of Ricci curvature bounds, based on optimal transport. This has led to the solution of a few outstanding geometric problems, but also a new way to understand Ricci bounds.

But progress in the case of optimal transport has not only affected theories at a fundamental level: it also came with the progress of applications, modelling, numerical analysis, fast computing. At the same time as the power and efficiency of optimal transport has been proven, more and more variants and situations of applications were arising.

In this volume a number of contributions will be presented in this spirit; some will take the form of a survey and some of a research contribution. All of them will deal with areas which are not mainstream optimal transport, but contribute to enrich the theory. Taken together, they provide a faithful photograph of these fast-paced incoming developments. Thus we shall encounter such diverse subjects as the modelling of pits (Ekeland & Queyranne) or traffic (Buttazzo *et al.*), the applications to image processing (Maas *et al.*; Hug *et al.*) and physics (De Pascale; Lombardi & Maitre). We shall explore the ties to partial differential equations arising in hydrodynamics (Brenier) and nonlinear dissipative dynamics (Blanchet *et al.*). We shall also consider theoretical variants of the optimal transport problem, such as the multi-marginal optimal transport (Pass) or ramified optimal transport (Xia), as well as new numerical schemes (Carlier *et al.*; Bouhargane *et al.*; Levy).

With such a diverse and far-reaching panorama, optimal transport once more demonstrates its vitality, flexibility and versatility. Now entering maturity, the field continues to widely inspire and to absorb new applications while remaining true to its fundamental principles.

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