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A PERIODICITY PROPERTY OF ITERATED MORPHISMS

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Abstract. Suppose $f : X^* \longrightarrow X^*$ is a morphism and $u, v \in X^*$. For every nonnegative integer n, let z_n be the longest common prefix of $f^n(u)$ and $f^n(v)$, and let $u_n, v_n \in X^*$ be words such that $f^n(u) = z_n u_n$ and $f^n(v) = z_n v_n$. We prove that there is a positive integer q such that for any positive integer p, the prefixes of u_n (resp. v_n) of length p form an ultimately periodic sequence having period q. Further, there is a value of q which works for all words $u, v \in X^*$.

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1. Introduction

In the theory of D0L systems many deep decidability results are proved by showing that certain D0L systems or pairs of D0L systems have some kind of regular behavior. In this paper we investigate a regularity property encountered when we compare the iteration of a morphism on two different words. More precisely, let $f : X^* \longrightarrow X^*$ be a morphism and let $u, v \in X^*$ be words. For every nonnegative integer n, let z_n be the longest common prefix of the words $f^n(u)$ and $f^n(v)$, and let u_n and v_n be words such that $f^n(u) = z_n u_n$ and $f^n(v) = z_n v_n$. Hence, the words u_n and v_n give the words $f^n(u)$ and $f^n(v)$ from the first position where the latter words differ. In [4] it was shown that for any positive integer p the prefixes of length p of u_n (resp. v_n) form an ultimately periodic sequence. This result plays a key role in the solution of the DF0L language equivalence problem in [4]. In this paper we study this periodicity result in more detail. In particular, we prove that there is a period which works simultaneously for prefixes of any length and does not depend on the words u and v.

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We assume that the reader is familiar with the basics concerning iterated morphisms (see [5–7]). Notions and notations that are not defined are taken from these references.

2. Definitions and results

Suppose $X = \{x_1, \ldots, x_m\}$ is an alphabet with $m \geq 1$ letters. If $w \in X^*$, then $\#_{x_i}(w)$ is the number of occurrences of the letter x_i in w. The *Parikh mapping* $\psi : X^* \longrightarrow \mathbf{N}^m$ is defined by

$$\psi(w) = (\#_{x_1}(w), \ldots, \#_{x_m}(w)), \quad w \in X^*.$$

The *length* of a word $w \in X^*$ is denoted by $|w|$. The length of the *empty word* ε equals zero. The first letter of a nonempty word $w \in X^*$ is denoted by $\mathrm{first}(w)$.

If $u, v \in X^*$ are words and there is a word $w \in X^*$ such that $uw = v$, we say that u is a *prefix* of v. If $u \in X^*$ and p is a positive integer, we denote by $\mathrm{pref}_p(u)$ the prefix of u having length p. If $|u| < p$, it is understood that $\mathrm{pref}_p(u) = u$. This notation is extended in a natural way for pairs of words. Hence, if $(u, v) \in X^* \times X^*$ we denote

$$\mathrm{pref}_p(u, v) = (\mathrm{pref}_p(u), \mathrm{pref}_p(v)).$$

Two words $u, v \in X^*$ are called *comparable* (with respect to the prefix order) if u is a prefix of v or vice versa. The longest common prefix of u and v is denoted by $u \wedge v$. Further, we denote

$$u * v = \begin{cases} (\varepsilon, \varepsilon) & \text{if } u \text{ and } v \text{ are comparable,} \\ ((u \wedge v)^{-1}u, (u \wedge v)^{-1}v) & \text{otherwise.} \end{cases}$$

Next, let $(a_n)_{n \geq 0}$ be a sequence and let q be a positive integer. We say that q is a *period* of $(a_n)_{n \geq 0}$ if there is an integer n_0 such that

$$a_{n+q} = a_n \quad \text{whenever } n \geq n_0.$$

A sequence is called *ultimately periodic* if it has a period.

The following theorem gives a basic result concerning periodicities in D0L sequences (see [2, 3]).

Theorem 2.1. *Suppose X is an alphabet, $f : X^* \longrightarrow X^*$ is a morphism and $u \in X^*$. There is a positive integer q such that for any positive integer p, q is a period of the sequence*

$$(\mathit{pref}_p(f^n(u)))_{n \geq 0}.$$

In this paper we study the iteration of a given morphism on two different words. If $f : X^* \longrightarrow X^*$ is a morphism, $u, v \in X^*$ are words and p is a positive integer, we consider the sequence $(\mathrm{pref}_p(f^n(u) * f^n(v)))_{n \geq 0}$. It was shown in [4] that these sequences are ultimately periodic. (In [4] it is assumed that $\psi(u) = \psi(v)$.) In this paper we will study these sequences in more detail. In particular, we will prove the following result.

Theorem 2.2. *Suppose X is an alphabet and $f : X^* \longrightarrow X^*$ is a morphism. There is a positive integer q such that whenever $u, v \in X^*$ are words and p is a positive integer, then q is a period of the sequence*

$$(\mathit{pref}_p(f^n(u) * f^n(v)))_{n\geq 0}. \tag{1}$$

The proof of Theorem 2.2 is given in the following section. The proof uses two lemmas from [4], but otherwise we do not assume familiarity with [4].

Let now $f : X^* \longrightarrow X^*$ be a morphism and let $u, v \in X^*$ be words. For every nonnegative integer n, define $z_n = f^n(u) \wedge f^n(v)$ and let $u_n, v_n \in X^*$ be words such that $f^n(u) = z_n u_n$ and $f^n(v) = z_n v_n$. Then there is a positive integer q such that for any positive integer p, q is a period of the sequences

$$(\mathrm{pref}_p(u_n))_{n\geq 0} \quad \text{and} \quad (\mathrm{pref}_p(v_n))_{n\geq 0}.$$

To deduce the existence of q from Theorem 2.2, choose a new letter \$, extend f by $f(\$) = \$$ and consider the words $u\$$ and $v\$$.

3. Proofs

We will prove Theorem 2.2 in two steps. The first subsection contains the main part of the proof. Without loss of generality we assume that X contains at least two letters.

3.1. Proof of Theorem 2.2 – a special case

If $f : X^* \longrightarrow X^*$ is a morphism, a letter $a \in X$ is called *bounded* (with respect to f) if the length sequence $(|f^n(a)|)_{n\geq 0}$ is bounded above by a constant. If $a \in X$ is not bounded, then a is called *growing.* A word $w \in X^*$ is called *bounded* if no letter of w is growing.

In this subsection we assume that $f : X^* \longrightarrow X^*$ is a morphism which has the following additional properties:

(i) There is a positive integer K such that if $x, y \in X^*$ are nonempty words and $\mathrm{first}(x) \neq \mathrm{first}(y)$ then $|f(x) \wedge f(y)| \leq K$.
(ii) If $a \in X$ is a bounded letter, then $f(a) = a$.
(iii) If $a \in X$ is a growing letter, then $|f(a)| \geq 2$.

In this subsection we prove Theorem 2.2 for morphisms which share properties (i)–(iii). To proceed we fix such a morphism $f : X^* \longrightarrow X^*$.

Let $a \in X$ be a growing letter and let a_n be the first growing letter of $f^n(a)$ for $n \geq 0$. Then the sequence $(a_n)_{n\geq 0}$ is ultimately periodic. Hence we can find a positive integer n_0 such that if $a, b \in X$ are growing letters and $a_n = b_n$ for some $n \geq 0$, then $a_{n_0} = b_{n_0}$. Denote

$$M = \max_{x\in X} |f^{n_0}(x)|.$$

Lemma 3.1. *If $a, b \in X$ are growing letters and $z \in X^*$ is a bounded word with $|z| \geq M$ then $f^n(a) \wedge f^n(zb)$ is a bounded word for all $n \geq 0$.*

For the proof of Lemma 3.1 we refer to [4].

Next we define a family of mappings needed for the proof of Theorem 2.2. Let M be as above. Denote $N = K(M + K)$ where K is as in (i). First, define the mapping $\tau_1 : X^* \longrightarrow X^*$ as follows. If $w \in X^*$ contains $M + K$ consecutive bounded letters, then $\tau_1(w)$ is the shortest prefix of w ending with $M+K$ bounded letters. Otherwise, $\tau_1(w) = w$. Next, the mapping $\tau_2 : X^* \longrightarrow X^*$ is defined by

$$\tau_2(w) = \mathrm{pref}_N(w), \qquad w \in X^*.$$

Then, define $\tau : X^* \longrightarrow X^*$ as follows. If $w \in X^*$ has a prefix of length M consisting of bounded letters, $\tau(w) = \mathrm{pref}_M(w)$. Otherwise $\tau(w) = \tau_2\tau_1(w)$.

Next, we consider pairs of words and extend the mapping τ in a natural way by setting

$$\tau(u, v) = (\tau(u), \tau(v)), \qquad (u, v) \in X^* \times X^*.$$

Finally, if $(u, v) \in X^* \times X^*$, define the mapping $\rho_{u,v} : \mathbf{N} \longrightarrow X^* \times X^*$ by

$$\rho_{u,v}(n) = \tau(f^n(u) * f^n(v)), \qquad n \geq 0.$$

Denote

$$\Lambda_{u,v} = \{\rho_{u,v}(n) \mid n \in \mathbf{N}\}.$$

Define the set $\mathcal{R} \subseteq X^* \times X^*$ as follows. Let $(u, v) \in X^* \times X^*$ and denote $(u_n, v_n) = f^n(u) * f^n(v)$ for $n \geq 0$. Then $(u, v) \in \mathcal{R}$ if and only if $\psi(u) = \psi(v)$ and for all $n \geq 0$ the word u_n (resp. v_n) contains a growing letter and $|u_n| \geq N$ (resp. $|v_n| \geq N$).

Let $(u, v) \in \mathcal{R}$ and let $n \geq 0$. Write $\rho_{u,v}(n) = (w_1, w_2)$. Then the words w_i, $i = 1, 2$, satisfy the following conditions:

(1) $M \leq |w_i| \leq N$;
(2) if $|w_i| = M$, then w_i is a bounded word;
(3) if $M < |w_i| < N$, then w_i ends with $M + K$ bounded letters;
(4) if $|w_i| = N$, either w_i does not have a bounded factor of length $M + K$ or the suffix of w_i of length $M + K$ is the only bounded factor of length $M + K$ of w_i.

A pair $(u, v) \in X^* \times X^*$ is called *special* if $|u| = M$ or $|v| = M$.

Lemma 3.2. *Suppose $(u, v) \in \mathcal{R}$ and $(x, y) \in \Lambda_{u,v}$. If $w_1, w_2 \in X^*$ and (x, y) is not special, then*

$$\tau(f(x) * f(y)) = \tau(f(x)w_1 * f(y)w_2).$$

For the proof of Lemma 3.2 we again refer to [4].

Lemma 3.3. *Suppose $(u, v) \in \mathcal{R}$ and write $\rho_{u,v}(n) = (x_n, y_n)$ for $n \geq 0$. If none of the pairs (x_n, y_n), $n \geq 0$, is special, then*

$$\rho_{u,v}(i + t) = \tau(f^t(x_i) * f^t(y_i)) \tag{2}$$

for all $i \geq 0$ and $t \geq 0$.

Proof. If $(w_1, w_2) \in X^* \times X^*$, we have $\tau(w_1, w_2) = \tau(\tau(w_1, w_2))$. Hence

$$\rho_{u,v}(i) = \tau(f^i(u) * f^i(v)) = \tau(\tau(f^i(u) * f^i(v))) = \tau(x_i, y_i)$$

for all $i \geq 0$. Consequently, (2) holds for all $i \geq 0$ if $t = 0$.

Suppose then that (2) holds for a fixed pair $t \geq 0$, $i \geq 0$. In other words

$$(x_{i+t}, y_{i+t}) = \tau(f^t(x_i) * f^t(y_i)).$$

Hence there exist words $\alpha, \beta_1, \beta_2 \in X^*$ such that

$$f^t(x_i) = \alpha x_{i+t} \beta_1, \quad f^t(y_i) = \alpha y_{i+t} \beta_2.$$

Because $\tau(f^{i+t}(u) * f^{i+t}(v)) = (x_{i+t}, y_{i+t})$, there are words $\gamma, \delta_1, \delta_2 \in X^*$ such that

$$f^{i+t}(u) = \gamma x_{i+t} \delta_1, \quad f^{i+t}(v) = \gamma y_{i+t} \delta_2.$$

Now we get

$$\begin{aligned}
\rho_{u,v}(i+t+1) &= \tau(f^{i+t+1}(u) * f^{i+t+1}(v)) \\
&= \tau(f(\gamma x_{i+t} \delta_1) * f(\gamma y_{i+t} \delta_2)) \\
&= \tau(f(x_{i+t}) * f(y_{i+t})) \\
&= \tau(f(\alpha) f(x_{i+t}) f(\beta_1) * f(\alpha) f(y_{i+t}) f(\beta_2)) \\
&= \tau(f^{t+1}(x_i) * f^{t+1}(y_i)).
\end{aligned}$$

Here the third and fourth equations follow by Lemma 3.2. Consequently, (2) holds for all $i \geq 0$ and $t \geq 0$. □

By Theorem 2.1 there is a positive integer q_1 such that q_1 is a period of the sequence

$$(\mathrm{pref}_p(f^{nt}(a)))_{n \geq 0}$$

for all $p \geq 1$, $a \in X$ and $t < \mathrm{card}(X)^{2N+2}$. Fix such an integer q_1. Then q_1 is also a period of

$$(\mathrm{pref}_p(f^{nt}(w)))_{n \geq 0}$$

for all $p \geq 1$, $w \in X^*$ and $t < \mathrm{card}(X)^{2N+2}$.

Lemma 3.4. *Let $(u, v) \in \mathcal{R}$ and write $\rho_{u,v}(n) = (x_n, y_n)$ for $n \geq 0$. Suppose none of the pairs (x_n, y_n), $n \geq 0$, is special. Then q_1 is a period of (1) for any positive integer p.*

Proof. Because the set $\Lambda_{u,v}$ has less than $\mathrm{card}(X)^{2N+2}$ elements, there are positive integers i and t such that $t < \mathrm{card}(X)^{2N+2}$ and

$$\rho_{u,v}(i) = \rho_{u,v}(i+t).$$

By Lemma 3.3 we have

$$(x_i, y_i) = (x_{i+t}, y_{i+t}) = \tau(f^t(x_i) * f^t(y_i)).$$

Hence there exist words $\alpha, \gamma_1, \gamma_2 \in X^*$ such that

$$f^t(x_i) = \alpha x_i \gamma_1, \quad f^t(y_i) = \alpha y_i \gamma_2.$$

Let $w_1, w_2 \in X^*$ be words such that

$$f^i(u) * f^i(v) = (x_i w_1, y_i w_2).$$

Then

$$\begin{aligned} & f^{i+nt}(u) * f^{i+nt}(v) = f^{nt}(x_i w_1) * f^{nt}(y_i w_2) \\ = \quad & (x_i \gamma_1 f^t(\gamma_1) \dots f^{(n-1)t}(\gamma_1) f^{nt}(w_1), y_i \gamma_2 f^t(\gamma_2) \dots f^{(n-1)t}(\gamma_2) f^{nt}(w_2)) \end{aligned}$$

for $n \geq 1$. This implies that q_1 is a period of

$$(\mathrm{pref}_p(f^{i+nt}(u) * f^{i+nt}(v)))_{n \geq 0}$$

for any $p \geq 1$. Because the morphism f has property (i), it follows that q_1 is also a period of (1) for any $p \geq 1$. □

Next, define

$$\begin{aligned} q_2 \quad = \quad & \max\{1, |\beta| \mid \text{ there are a growing letter } c \in X, \text{ a positive integer} \\ & s \leq \mathrm{card}(X)^2 \text{ and a word } \gamma \in X^* \text{ such that } f^s(c) = \beta c \gamma\} \end{aligned}$$

and define

$$q_3 = q_2!\, q_1.$$

Lemma 3.5. *Let $(u, v) \in \mathcal{R}$ and write $\rho_{u,v}(n) = (x_n, y_n)$ for $n \geq 0$. If there is an integer n such that (x_n, y_n) is special, then q_3 is a period of (1) for any positive integer p.*

Proof. Suppose that k is a nonnegative integer such that $\rho_{u,v}(k) = (x_k, y_k)$ is special. Without loss of generality assume that $|x_k| = M$. Hence x_k is a bounded word. Let

$$f^k(u) = \alpha \beta_1 a \gamma_1, \quad f^k(v) = \alpha \beta_2 b \gamma_2$$

where $\beta_1, \beta_2 \in X^*$ are bounded words, $a, b \in X$ are growing letters and $\alpha, \gamma_1, \gamma_2 \in X^*$. Furthermore, x_k is a prefix of β_1 and y_k is a prefix of $\beta_2 b \gamma_2$.

If β_2 is not empty, then

$$f^{k+n}(u) * f^{k+n}(v) = f^n(\beta_1 a \gamma_1) * f^n(\beta_2 b \gamma_2) = (\beta_1 f^n(a \gamma_1), \beta_2 f^n(b \gamma_2))$$

for $n \geq 0$. Hence q_1 and q_3 are periods of (1).

To proceed assume that β_2 is the empty word. Then Lemma 3.1 implies that $f^n(\beta_1 a) \wedge f^n(b)$ is a bounded word for all $n \geq 0$. Now, there exist integers $k_1 \geq 0$, $k_2 \geq 1$, growing letters $c, d \in X$, bounded words $\beta_3, \beta_4, \beta_5, \beta_6 \in X^*$ and words $\gamma_3, \gamma_4, \gamma_5, \gamma_6 \in X^*$ such that $k_2 \leq \mathrm{card}(X)^2$ and

$$f^{k_1}(a) = \beta_3 c \gamma_3, f^{k_1}(b) = \beta_4 d \gamma_4, f^{k_2}(c) = \beta_5 c \gamma_5, f^{k_2}(d) = \beta_6 d \gamma_6.$$

(Here we may have $c = d$.) Hence

$$\begin{aligned} &f^{k+k_1+nk_2}(u) * f^{k+k_1+nk_2}(v) = \\ &\quad \beta_1\beta_3\beta_5^n c\gamma_5 f^{k_2}(\gamma_5)\ldots f^{nk_2-k_2}(\gamma_5) f^{nk_2}(\gamma_3) f^{k_1+nk_2}(\gamma_1) \\ &\quad * \beta_4\beta_6^n d\gamma_6 f^{k_2}(\gamma_6)\ldots f^{nk_2-k_2}(\gamma_6) f^{nk_2}(\gamma_4) f^{k_1+nk_2}(\gamma_2) \end{aligned}$$

for $n \geq 1$. Furthermore, the words $\beta_1\beta_3\beta_5^n c$ and $\beta_4\beta_6^n d$ are not comparable. Consequently the sequence

$$(\mathrm{pref}_p(f^{k+k_1+nk_2}(u) * f^{k+k_1+nk_2}(v)))_{n\geq 0} \tag{3}$$

has period q_3. In fact, if there is an integer n such that $\beta_1\beta_3\beta_5^n$ and $\beta_4\beta_6^n$ are not comparable, (3) has period q_3. Otherwise, without loss of generality assume that $\beta_1\beta_3\beta_5^n$ is a prefix of $\beta_4\beta_6^n$ for all large n, say, if $n \geq n_0$. Then the sequence

$$(\mathrm{pref}_p((\beta_1\beta_3\beta_5^n)^{-1}\beta_4\beta_6^n))_{n\geq n_0}$$

has period $q_2!$ for any $p \geq 1$. Because the sequences

$$(\mathrm{pref}_p(c\gamma_5 f^{k_2}(\gamma_5)\ldots f^{nk_2-k_2}(\gamma_5) f^{nk_2}(\gamma_3) f^{k_1+nk_2}(\gamma_1)))_{n\geq 0}$$

and

$$(\mathrm{pref}_p(d\gamma_6 f^{k_2}(\gamma_6)\ldots f^{nk_2-k_2}(\gamma_6) f^{nk_2}(\gamma_4) f^{k_1+nk_2}(\gamma_2)))_{n\geq 0}$$

have period q_1 for any $p \geq 1$, it follows that (3) has period q_3. Because f has property (i) it again follows that (1) has period q_3. □

We have now shown that if f satisfies conditions (i)–(iii) then q_3 is a period of (1) for all $p \geq 1$ and for all $(u, v) \in \mathcal{R}$.

To proceed, let \$ be a new letter. Define $X_1 = X \cup \{\$\}$ and let $f_1 : X_1^* \longrightarrow X_1^*$ be the extension of f defined by $f_1(\$) = \$$. Then f_1 satisfies (i)–(iii) and we know that there are positive integers q and N_1 such that q is a period of

$$(\mathrm{pref}_p(f_1^n(u_1) * f_1^n(v_1)))_{n\geq 0} \tag{4}$$

for all $p \geq 1$ and for all $u_1, v_1 \in X_1^*$ such that $\psi(u_1) = \psi(v_1)$ and each component of $f_1^n(u_1) * f_1^n(v_1)$ contains a growing letter and has length at least N_1 for all $n \geq 0$.

Let now $u, v \in X^*$. If there is an integer n such that $f^n(u)$ and $f^n(v)$ are comparable, then (1) has period one for all $p \geq 1$. Also, if neither u nor v contains a growing letter, (1) has period one for all $p \geq 1$. Assume that $f^n(u)$ and $f^n(v)$

are incomparable for all $n \geq 0$ and assume that u or v contains a growing letter. Now, define

$$u_1 = u\$v(uv)^{N_1}, \quad v_1 = v\$u(vu)^{N_1}.$$

Then q is a period of (4) for all $p \geq 1$. Consequently (1) has period q for all $p \geq 1$.

This concludes the proof of Theorem 2.2 for a morphism f which satisfies conditions (i)–(iii).

3.2. Proof of Theorem 2.2 – the general case

We assume that the reader is familiar with elementary morphisms (see [1, 5]). In particular, recall that an elementary morphism permutes the set of bounded letters.

Let $f : X^* \longrightarrow X^*$ be a morphism. Then there exist a positive integer j_1, an alphabet Y and morphisms $g_1 : X^* \longrightarrow Y^*$, $g_2 : Y^* \longrightarrow X^*$ such that

$$f^{j_1} = g_2 g_1$$

and the morphisms g_2 and $g_1 g_2$ are elementary (see Th. III 2.2 in [5]). Consider the morphism $g_1 g_2 : Y^* \longrightarrow Y^*$. There exists a positive integer j_2 such that

$$(g_1 g_2)^{j_2}(y) = y$$

if $y \in Y$ is a bounded letter with respect to $g_1 g_2$ and

$$|(g_1 g_2)^{j_2}(y)| \geq 2$$

if $y \in Y$ is a growing letter with respect to $g_1 g_2$. Define $g = (g_1 g_2)^{j_2}$ and $j = j_1 j_2$. Then

$$f^{nj+j_1} = g_2 g^n g_1$$

for $n \geq 0$.

Because g satisfies conditions (i)-(iii) of the previous subsection, there is a positive integer q such that q is a period of

$$(\mathrm{pref}_p(g^n g_1 f^i(u) * g^n g_1 f^i(v)))_{n \geq 0}$$

for all $p \geq 1$, $i = 0, 1, \ldots, j-1$ and $u, v \in X^*$. Because g_2 is elementary, q is also a period of

$$(\mathrm{pref}_p(g_2 g^n g_1 f^i(u) * g_2 g^n g_1 f^i(v)))_{n \geq 0}$$

for all $p \geq 1$, $i = 0, 1, \ldots, j-1$ and $u, v \in X^*$. In other words, q is also a period of

$$(\mathrm{pref}_p(f^{nj+j_1+i}(u) * f^{nj+j_1+i}(v)))_{n \geq 0}$$

for all $p \geq 1$, $i = 0, 1, \ldots, j-1$ and $u, v \in X^*$. Hence jq is a period of (1) for all $p \geq 1$ and $u, v \in X^*$.

This concludes the proof of Theorem 2.2 in the general case.

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