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Statistics

A nonparametric model check for time series when the random vectors are nonstationary and absolutely regular

Test d'un modèle non paramétrique pour des séries chronologiques lorsque les vecteurs aléatoires sont non stationnaires et absolument réguliers

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ABSTRACT

In this Note, we study some general methods for testing the goodness-of-fit of a parametric model for a real-valued Markovian time series under nonstationarity and absolute regularity. For that, we define a marked empirical process based on residuals, which converges in distribution to a Gaussian process with respect to the Skorohod topology. This method was first introduced by Koul and Stute [1], and then widely developed by Ngatchou-Wandji [2,3] under more general conditions. Applications to general AR-ARCH models are given.

RÉSUMÉ

Dans cette note, nous étudions quelques méthodes générales pour tester un modèle paramétrique associé à une série chronologique markovienne à valeurs réelles lorsque les vecteurs aléatoires sont non stationnaires et absolument réguliers. Notre idée est d'utiliser un processus empirique marqué basé sur les résidus qui converge en loi vers un processus gaussien.

Version française abrégée

Notre but est de tester un modèle de régression hétéroscédastique de la forme

$$X_i = m(X_{i-1}, \ldots, X_{i-d}; \theta) + v(X_{i-1}, \ldots, X_{i-d})\epsilon_i,\ i \geq 1 + d,$$

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en utilisant une approche non paramétrique, et en prenant en compte l'estimation de θ sous l'hypothèse nulle $\mathcal{H}_0$ d'appartenance de la fonction m à un modèle paramétrique $\mathcal{M} = \{m(\cdot;\theta) : \theta \in \Theta\}$.

La fonction ν est inconnue et les bruits ϵ_i sont absolument réguliers.

La suite $\{X_i, \mathbf{X}_{i-1} = (X_{i-1}, \ldots, X_{i-d})\}_{i \geq 1+d}$ est non stationnaire et absolument régulière.

Soit ψ une fonction non décroissante à valeurs réelles telle que $\sup_i E|\psi(X_i - \nu)| < \infty$, pour tout $\nu \in \mathbb{R}$.

On définit la fonction ψ-autorégressive m_ψ sous la condition

$$E\{\psi(X_i - m_\psi(\mathbf{X}_{i-1})) \mid \mathbf{X}_{i-1}\} = 0 \text{ p.s.}$$

Nos statistiques de test sont construites à partir du processus défini par :

$$R^*_{n,\psi}(\mathbf{x}) = n^{-1/2} \sum_{i=d}^{n} \psi(X_i - m(\mathbf{X}_{i-1}; \widetilde{\theta}_n)) \mathbb{1}_{\{\mathbf{X}_{i-1} \leq \mathbf{x}\}}, \ \mathbf{x} \in \mathbb{R}^d$$

où $\widetilde{\theta}_n$ désigne un estimateur convergeant vers le vrai paramètre θ_0 et qui vérifie la Condition 1 ci-dessous.

Nous pouvons en déduire plusieurs statistiques de test possibles : en particulier, un test de type Cramér–von Mises fondé sur

$$\mathcal{T}_n = \int (R^*_{n,\psi}(\mathbf{x}))^2 \omega(\widehat{F}_n(\mathbf{x}))\, \mathrm{d}\widehat{F}_n(\mathbf{x})$$

($\omega(\cdot)$ désigne une fonction poids et $\widehat{F}_n$ est la fonction de répartition empirique de l'échantillon).

D'après le Théorème 1 ci-dessous, le processus $R^*_{n,\psi}$ converge en loi vers un processus gaussien $R^*_{\infty,\psi}$. Par conséquent, sous $\mathcal{H}_0$, $\mathcal{T}_n$ converge en loi vers le processus $\mathcal{T}$ défini ci-dessous.

1. Introduction

The purpose of this Note is to study a general method for testing the goodness-of-fit of a parametric model for a Markovian time series. Now, we define our model.

Let $\{X_i\}_{i\in\mathbb{N}}$ be a sequence of random variables with continuous distribution functions F_i on $\mathbb{R}$. Assume that F_i admits a positive density.

In this paper, we will assume that the sequence $\{X_i\}_{i\in\mathbb{N}}$ is absolutely regular with the rate

$$\beta(n) = \mathcal{O}(\tau^n), \ 0 < \tau < 1, \tag{1}$$

where

$$\beta(k) = \sup_{n\in\mathbb{N}} \max_{1\leq j\leq n-k} E\left\{ \sup_{A\in\mathcal{A}^\infty_{j+k}} \left| P(A \mid \mathcal{A}^j_0) - P(A) \right| \right\}$$

with $\mathcal{A}^j_i$ the σ-algebra generated by $X_i, \ldots, X_j$, $i, j \in \mathbb{N} \cup \{\infty\}$.

We also assume that F_i converges to the distribution function F (for the norm of total variation denoted $\|\cdot\|_{TV}$), which admits positive marginal densities. Let $F_{i,j}$ be the distribution function of (X_i, X_j). Furthermore, assume that for any $l > 1$, there exists a continuous distribution function $\widetilde{F}_l$ on $\mathbb{R}^2$ admitting a positive density such that

$$\|F_{i,j} - \widetilde{F}_{j-i}\|_{TV} = \mathcal{O}(\rho_0^i), \ 1 \leq i < j \leq n, \ i, j \in \mathbb{N}, \ 0 < \rho_0 < 1 \tag{2}$$

for which there exists a stationary sequence $\{\widetilde{X}_i\}_{i\in\mathbb{N}}$ that is absolutely regular with rate (1) and such that $(\widetilde{X}_i, \widetilde{X}_j)$ has $\widetilde{F}_{j-i}$ as a distribution function $(i < j + 1)$.

Some literature is concerned with parametric modeling in that m is assumed to belong to a given family

$$\mathcal{M} = \{m(\cdot;\theta) : \theta \in \Theta\}$$

of functions, where Θ is a subset of the q-dimensional Euclidean space $\mathbb{R}^q$.

It is assumed that under $\mathcal{H}_0$, $m_0(x) = m(x;\theta_0)$ for some true value parameter θ_0. We wish to test the null hypothesis $\mathcal{H}_0 : m \in \mathcal{M}$ against the alternative $\mathcal{H}_1 : m \notin \mathcal{M}$.

For this purpose, we consider an empirical process that, under $\mathcal{H}_0$, depends on the unknown but true parameter θ_0. We estimate this parameter by, say $\widetilde{\theta}_n$, and then plug this estimator in the expression of the empirical process. We show then that the resulting empirical process converges in distribution to a noncentered Gaussian process that has the same asymptotic covariance function.

The best known case is the linear model in which $m(\mathbf{x};\theta) = \mathbf{g}^{\mathrm{t}}(\mathbf{x})\theta$ in which $\mathbf{g}$ is a known vector-valued function.

Now, to describe these procedures, let ψ be a known nondecreasing real-valued function such that $\sup_i E|\psi(X_i - \nu)| < \infty$, for each $\nu \in \mathbb{R}$. Define the ψ-autoregressive function m_ψ by the requirement that

$$E\{\psi(X_i - m_\psi(\mathbf{X}_{i-1})) \mid \mathbf{X}_{i-1}\} = 0 \text{ a.s.} \tag{3}$$

Throughout we shall assume that

$$\sup_i E\psi^2(X_i - m_\psi(\mathbf{X}_{i-1})) < \infty. \tag{4}$$

We consider an empirical process such that under $\mathcal{H}_0$ this process depends on a parameter θ_0. First, we start by estimating the parameter and we prove that the empirical process converges in distribution to a centered Gaussian process when the parameter is replaced by its estimator $\widetilde{\theta}_n$. Under $\mathcal{H}_1$, the empirical process converges in distribution to a noncentered Gaussian process which has the same asymptotic covariance function.

Let $\widetilde{\theta}_n$ be a consistent estimator of θ_0 under $\mathcal{H}_0$ based on $\{X_i\}_{i\geq 0}$. Define

$$R^*_{n,\psi}(\mathbf{x}) = n^{-1/2}\sum_{i=1}^{n}\psi(X_i - m(\mathbf{X}_{i-1};\widetilde{\theta}_n))\mathbb{1}_{\{\mathbf{X}_{i-1}\leq \mathbf{x}\}},\ \mathbf{x}\in\mathbb{R}^d. \tag{5}$$

The process $R^*_{n,\psi}$ is a marked empirical process.

The main results will be to prove the weak convergence of the process $R^*_{n,\psi}$ with respect to the Skorohod topology under some reasonable conditions and to investigate the power of tests based on $R^*_{n,\psi}$. Our results are more general than the results of Koul and Stute [1] with a larger set of applications. A testing procedure and applications to AR-ARCH model will be given in Section 3 and Section 4, respectively.

2. Conditions and weak convergence of the marked empirical process

For simplicity, we now assume $d = 1$. We know that the process defined in (5) takes its values in the Skorohod space $D(-\infty,\infty)$ and the convergence in this space is equivalent to the weak convergence on compacts. This excludes the possibility of handling goodness-of-fit statistics such as $\sup_{x\in\mathbb{R}}|R^*_{n,\psi}(x)|$.

To also deal with such statistics, we continuously extend $R^*_{n,\psi}$ to $-\infty$ and ∞ by setting: $R^*_{n,\psi}(-\infty) = 0$, $R^*_{n,\psi}(x)$ is defined by (5) for $x\in\mathbb{R}$ and $R^*_{n,\psi}(\infty) = n^{-1/2}\sum_{i=d}^{n}\psi(X_i - m(\mathbf{X}_{i-1};\widetilde{\theta}_n))$.

Then $R^*_{n,\psi}$ becomes a process in $D[-\infty,\infty]$, which, modulo a continuous transformation, is the same as $D[0,1]$.

Consider the sequence of distribution functions $\{\overline{F}_n\}_{n\geq 1}$ defined by $\overline{F}_n = \frac{1}{n}\sum_{i=1}^{n} F_i$.

For the behavior of the process $R^*_{n,\psi}$ defined in (5), some regularity assumptions on the estimator $\widetilde{\theta}_n$ will be needed. These conditions are similar to those of Koul and Stute [1], but our sequence X_i is nonstationary and geometrically absolutely regular, rather than being i.i.d.

(A1) Under $\mathcal{H}_0$, $\widetilde{\theta}_n$ admits an expansion:
$n^{1/2}(\widetilde{\theta}_n - \theta_0) = n^{-1/2}\sum_{i=1}^{n} l(X_i, X_{i-1};\theta_0) + o_p(1)$ for some vector-valued function l such that
 (a) $\sup_i E\{l(X_i, X_{i-1};\theta_0)|X_{i-1}\} = 0$ for any $i\geq 1$,
 (b) $L_{i,j} = E\{l(X_i, X_{i-1};\theta_0)l^{\text{t}}(X_j, X_{j-1};\theta_0)\}$ exists for all $i, j\geq 1$.

(A2) (a) $m(x;\theta)$ is continuously differentiable for each θ in the interior set Θ^0 of Θ. Put

$$\mathbf{g}(x;\theta) = \frac{\partial m(x;\theta)}{\partial\theta} = \left(g_1(x;\theta),\ldots,g_q(x;\theta)\right)^{\text{t}} \tag{6}$$

 (b) there exists an $\{F_i\}_{i\geq 1}$ and F-integrable function $M(x)$ such that

$$\left|g_j(x;\theta)\right| \leq M(x),\ \text{for all}\ \theta\in\Theta\ \text{and}\ 1\leq j\leq q. \tag{7}$$

(A3) There exists a function m from $\mathbb{R}\times\Theta$ to $\mathbb{R}^q$ such that $m(.;\theta_0)$ is measurable and satisfies the following:
(a) for all $k<\infty$,

$$\sup_{1\leq i\leq n}\ \sup_{n^{1/2}\|t-\theta_0\|\leq k} n^{1/2}|m(X_{i-1};t) - m(X_{i-1};\theta_0) - (t-\theta_0)^{\text{t}} m'(X_{i-1};\theta_0)| = o_p(1) \tag{8}$$

and

$$\sup_i E\|m'(X_{i-1};\theta_0)\|^{2+\delta} < \infty,\ \text{for some}\ \ \delta > 0. \tag{9}$$

(b) (Smooth ψ). The function ψ is absolutely continuous with its almost everywhere derivative ψ' bounded and having right and left limits.
(c) (Nonsmooth ψ). The function ψ is nondecreasing, right continuous, bounded and such that the function $x\mapsto \sup_i E\{\psi(X_i - m(X_{i-1};\theta_0)+x) - \psi(X_i - m(X_{i-1};\theta_0))\}^{2+\delta}$, for some $\delta>0$ is continuous at 0.

Remark. The assumption (b) in (A3) covers many interesting ψ's including the least-square score $\psi(x) \equiv x$ and the Huber score $\psi(x) \equiv x\mathbb{1}_{\{|x|\le c\}} + c\ \mathrm{sign}(x)\mathbb{1}_{\{|x|>c\}}$, where c is a real constant, while (c) in (A3) covers the α-quantile score $\psi(x) \equiv \mathbb{1}_{\{x>0\}} - (1-\alpha)$.

Theorem 1. *Under $\mathcal{H}_0$, assume that for any $u \in [0,1]$, $\sup_{i\ge1} E\big(|\psi(X_i - m(X_{i-1}))|^{2+\gamma_0} \mid U_{i-1} = u\big) < CE\big(|\psi(\widetilde{X}_1 - m(\widetilde{X}_0))|^{2+\gamma_0} \mid \widetilde{U} = u\big) < \infty$, $\gamma_0 > 0$, where $U_{i-1} = \overline{F}_n(X_{i-1})$, $1 \le i \le n$, $\widetilde{U} = F(\widetilde{X})$, C is some positive constant and the conditions (1) and (2) hold and let (A1)–(A3) be satisfied, then $R^*_{n,\psi} \to R^*_{\infty,\psi}$ in distribution in the Skorohod space $D[-\infty,\infty]$, where $R^*_{\infty,\psi}$ is a centered Gaussian process with covariance function*

$$K^*_\psi(x,y) = K_\psi(x,y) + \mathbf{G}^{\mathrm{t}}(x;\theta_0)\Big(L_{1,1}(\theta_0) + 2\sum_{k=2}^{\infty} L_{1,k}(\theta_0)\Big)\mathbf{G}(y;\theta_0)$$
$$- \mathbf{G}^{\mathrm{t}}(x;\theta_0)\sum_{k=0}^{\infty} E\big(\mathbb{1}_{\{\widetilde{X}_0\le x\}}\psi(\widetilde{X}_1 - m(\widetilde{X}_0;\theta_0)) l(\widetilde{X}_{k+1},\widetilde{X}_k;\theta_0)\big)$$
$$- \mathbf{G}^{\mathrm{t}}(y;\theta_0)\sum_{k=0}^{\infty} E\big(\mathbb{1}_{\{\widetilde{X}_0\le y\}}\psi(\widetilde{X}_1 - m(\widetilde{X}_0;\theta_0)) l(\widetilde{X}_{k+1},\widetilde{X}_k;\theta_0)\big), \quad (10)$$

where

$$\mathbf{G}(x;\theta) = \big(\mathbf{G}_0(x;\theta),\ldots,\mathbf{G}_q(x;\theta)\big)^{\mathrm{t}},\ \mathbf{G}_j(x;\theta) = \int_{\infty}^{x} g_j(u;\theta)\,\mathrm{d}F(u),\ 0\le j\le q$$

and

$$K_\psi(x,y) = F(x\wedge y)Var(\psi(\widetilde{X}_1)) + 2\sum_{k=1}^{\infty} Cov(\psi(\widetilde{X}_1),\psi(\widetilde{X}_{1+k}))\widetilde{F}_k(x,y),\ x,y\in\mathbb{R}. \quad (11)$$

3. The testing procedure

From the results obtained in Theorem 1, some testing procedure can be derived. We can consider the Cramér–von Mises type test defined by

$$\mathcal{T}_n = \int (R^*_{n,\psi}(\mathbf{x}))^2 \omega(\widehat{F}_n(\mathbf{x}))\mathrm{d}\widehat{F}_n(\mathbf{x}), \quad (12)$$

where ω is a weight function and $\widehat{F}_n$ is the empirical distribution function of the random vectors $\mathbf{X}_d,\ldots,\mathbf{X}_n$.

We easily deduce that under the conditions of Theorem 1, $\mathcal{T}_n$ converges in law to

$$\mathcal{T} = \int (R^*_{\infty,\psi}(F^{-1}(u)))^2\omega(u)\mathrm{d}(u).$$

We remark that $\mathcal{T}_n$ can be also written as

$$\mathcal{T}_n = \frac{1}{n}\sum_{i=d}^{n}\omega(\widehat{F}_n(\mathbf{X}_{i-1}))\left\{\sum_{j=d}^{n}\psi(X_j - m(\mathbf{X}_{j-1};\widetilde{\theta}_n))\mathbb{1}_{\{\mathbf{X}_{j-1}\le\mathbf{X}_{i-1}\}}\right\}^2.$$

The tails probability of the limiting distribution of the Cramér–von Mises test statistics would be very difficult to compute. That is why it is necessary to proceed to a discretization of $\mathcal{T}$ like in Ngatchou-Wandji [2], Ngatchou-Wandji and Harel [4].

As in Ngatchou-Wandji [2], the discretization that we can propose, follows from the Karhunen–Loève expansion of the processes $\mathcal{T}$.

Denote by $W(\cdot) = R^*_{\infty,\psi}(F^{-1}(\cdot))$ the process defined on $[0,1]^d$. Its Karhunen–Loève expansion can be written as

$$W = \sum_{j=d}^{\infty}\lambda_j^{1/2}W_j f_j, \quad (13)$$

where $\lambda_d \ge \lambda_{d+1} \ge \ldots$ are the eigenvalues of the covariance operator $B_\psi(\cdot) = K^*_\psi\big(F^{-1}(\cdot),F^{-1}(\cdot)\big)$, which are assumed strictly positive, the sequence of functions $f_d, f_{d+1},\ldots$ is a complete orthonormal base for $L^2[0,1]^d$ of eigenvectors of the operator B_ψ and the random variables

$$W_j = \lambda_j^{-1/2} \int_0^1 W(\mathbf{v}) f_j(\mathbf{v}) \mathrm{d}(\mathbf{v})$$

are independent $\mathcal{N}(0,1)$ under $\mathcal{H}_0$.

Then it is possible to choose a test statistic on the form $\mathcal{T}_n^J = \sum_{j=d}^J W_{n,j}^2$, where $J > 1$ is the number of the more informative terms in the development (13) and for any $j \geq 1$

$$W_{n,j} = \lambda_j^{-1/2} n^{-1} \sum_{i=d}^{n} R_{n,\psi}^*(\mathbf{X}_{i-1}) \omega(\widehat{F}_n(\mathbf{X}_{i-1})) f_j(\widehat{F}_n(\mathbf{X}_{i-1})).$$

Under $\mathcal{H}_0$, $\mathcal{T}_n^J$ converges in law to $\mathcal{T}^J = \sum_{j=d}^J W_j^2$ which has asymptotically a chi-square distribution with J degrees of freedom. However, the λ_j's and f_j's are difficult to compute in practice. A way to overcome this difficulty was suggested by Ngatchou-Wandji [2,3] by approximating the integrals by discretization.

4. Applications to the AR-ARCH model

Now we apply the results of Section 3 to test an AR-ARCH model against an other AR-ARCH model.

Consider a model that can be written in the form

$$X_i = m(X_{i-1}, \ldots, X_{i-d}; \theta) + v(X_{i-1}, \ldots, X_{i-d})\epsilon_i, \ i \geq 1 + d, \tag{14}$$

where $\theta \in \Theta \subset \mathbb{R}^q$ a parameter set, $m(\cdot)$ satisfying (A2) and (A3) and $v(\cdot)$ is continuous and unknown.

Let $\{\mathbf{X}_{i-1}\}_{i\geq 1+d}$ be the random sequence of vectors in $\mathbb{R}^d$ defined by

$$\mathbf{X}_{i-1} = (X_{i-1}, \ldots, X_{i-d})^{\mathrm{t}}, \ i \geq 1 + d.$$

We assume that the sequence $\{X_i\}_{i\in\mathbb{N}}$ satisfies the conditions (1) and (2) of the introduction and $\{\epsilon_i\}_{i\geq 1+d}$ is sequence of absolutely regular random variables satisfying (1).

Formula (14) can be written as

$$X_i = m(\mathbf{X}_{i-1}; \theta) + v(\mathbf{X}_{i-1})\epsilon_i, \ i \geq 1 + d. \tag{15}$$

We will use the results of Section 3 to test the null hypothesis $\mathcal{H}_0 : m(\cdot;\theta) \in \mathcal{M}$ against the local alternatives $\mathcal{H}_{1,n}$: $m_n(\cdot;\theta) = m(\cdot;\theta) + n^{-1/2} r(\cdot)$, $\theta \in \Theta$, where r is a function that has the same properties as v and $E(r(\widetilde{X}_1)) \neq 0$, where $\{\widetilde{X}_i\}_{i\geq 1+d}$ is the stationary sequence associated with the sequence $\{X_i\}_{i\geq 1+d}$.

Theorem 2. *Assume that* $\sup_{i\geq 1+d} E\big(|v(\mathbf{X}_{i-1})\epsilon_i|^{2+\gamma_0}\big) < \infty$ *and* $E\big(|v(\widetilde{\mathbf{X}}_d)\epsilon_{1+d}|^{2+\gamma_0}\big) < \infty$, $\gamma_0 > 0$ *hold and that (A1)–(A3) also hold. Then under* $\mathcal{H}_0$, $R_{n,\psi}^* \to R_{\infty,\psi}^*$ *in distribution in the Skorohod space* $\mathbf{D}_d[-\infty,\infty]$, *where* $R_{\infty,\psi}^*$ *is a centered Gaussian process with covariance function*

$$\begin{aligned} K_\psi^*(\mathbf{x},\mathbf{y}) &= K_\psi(\mathbf{x},\mathbf{y}) + \mathbf{G}^{\mathrm{t}}(\mathbf{x};\theta_0)\Big(L_{1,1}(\theta_0) + 2\sum_{k=d}^{\infty} L_{1,k}(\theta_0)\Big)\mathbf{G}(\mathbf{y};\theta_0) \\ &\quad - \mathbf{G}^{\mathrm{t}}(\mathbf{x};\theta_0) \sum_{k=d-1}^{\infty} E\Big(v(\widetilde{\mathbf{X}}_d)\epsilon_{1+d} l(\widetilde{\mathbf{X}}_k, \widetilde{X}_{k+1};\theta_0)\Big) \\ &\quad - \mathbf{G}^{\mathrm{t}}(\mathbf{y};\theta_0) \sum_{k=d-1}^{\infty} E\Big(v(\widetilde{\mathbf{X}}_d)\epsilon_{1+d} l(\widetilde{\mathbf{X}}_k, \widetilde{X}_{k+1};\theta_0)\Big), \ \mathbf{x},\mathbf{y} \in \mathbb{R}^d, \end{aligned} \tag{16}$$

where $\psi(\mathbf{x}) = \mathbf{x}$.

Corollary. *Under* $H_{1,n}$, *and the conditions of Theorem 1,* $R_{n,\psi}^* \to R_{\infty,\psi}^*$ *in distribution in the Skorohod space* $\mathbf{D}_d[-\infty,\infty]$ *where* $R_{\infty,\psi}^*$ *is a Gaussian process with mean* $s(\mathbf{x})$ *and covariance function* $K_\psi^*(\mathbf{x},\mathbf{y})$ *defined in (16), where*

$$s(\mathbf{x}) = \int_{\mathbf{h}\leq\mathbf{x}} r(\mathbf{h})\,\mathrm{d}F(\mathbf{h}) - \mathbf{G}(\mathbf{x};\theta_0) \int_{\mathbf{h}\leq\mathbf{x}} \int_{\mathbb{R}} \frac{r(\mathbf{h})}{v(\mathbf{h})} l(\mathbf{h}, y;\theta_0)\,\mathrm{d}\widetilde{F}(\mathbf{h}, y)$$

and $\widetilde{F}$ *is the distribution function of* $(\widetilde{X}_i, \widetilde{X}_{i-1}, \ldots, \widetilde{X}_{i-d})$.

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