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G-invariant holomorphic Morse inequalities

Inégalités de Morse holomorphes G-invariantes

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A B S T R A C T

Consider an action of a connected compact Lie group on a compact complex manifold M, and two equivariant vector bundles L and E on M, with L of rank 1. In this note, we give holomorphic Morse inequalities in the spirit of Demailly for the invariant part of the Dolbeault cohomology of high tensor powers of L twisted by E, via the induced geometric data on the reduced space.

R É S U M É

Considérons l'action d'un groupe de Lie compact connexe sur une variété complexe compacte, ainsi que deux fibrés vectoriels équivariants L et E sur M, avec L de rang 1. Dans cette note, nous donnons des inégalités de Morse dans l'esprit de Demailly pour la partie invariante de la cohomologie de Dolbeault des grandes puissances tensorielles de L tordues par E, en passant par les données géométriques induites sur l'espace réduit.

Version française abrégée

Soit M une variété complexe compacte munie de deux fibrés vectoriels holomorphes et hermitiens (L, h^L), (E, h^E), avec L de rang 1. On note L^p la p^e puissance tensorielle de L.

Dans [2], Demailly établit des inégalités de Morse asymptotiques pour la cohomologie de Dolbeault associée à $L^p \otimes E$. Les inégalités de Demailly donnent une borne asymptotique sur les sommes de Morse des nombres $\dim H^j(M, L^p \otimes E)$, quand $p \to +\infty$, en termes d'une intégrale de la courbure de Chern R^L de (L, h^L). Par la suite, Bismut [1] donne une nouvelle démonstration de ces inégalités en s'appuyant sur le fait que les sommes de Morse qui nous intéressent peuvent être majorées par des sommes alternées de traces du noyau de la chaleur.

Ces inégalités ont eu de nombreuses applications. Par exemple, Demailly a récemment utilisé ces inégalités dans [3] pour effectuer un premier pas important dans la résolution d'une version généralisée de la conjecture de Green–Griffiths–Lang.

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Soit G groupe de Lie compact connexe agissant pas biholomorphismes sur M. On suppose que cette action se relève en des actions sur (L, h^L) et (E, h^E). Le groupe G agit donc naturellement sur la cohomologie $H^\bullet(M, L^p \otimes E)$ et on peut considérer $H^\bullet(M, L^p \otimes E)^G$, la partie G-invariante de cet espace. Dans cette note, nous donnons des inégalités comparables à celles de Demailly pour les sommes de Morse des nombres $\dim H^j(M, L^p \otimes E)^G$. Pour ce faire, nous nous inspirons de l'approche de Bismut [1] (voir aussi [5]) par le noyau de la chaleur.

Dans notre contexte, de nouveaux phénomènes de concentration et d'asymptotique apparaissent pour la restriction du noyau de la chaleur aux formes G-invariantes, analogues à ceux rencontrés pour le noyau de Bergman G-invariant par Ma et Zhang [6] dans le cas où L est positif. Nous définissons une « application moment » $\mu : M \to \mathrm{Lie}(G)$ par la formule de Kostant, puis la réduction de M sous l'hypothèse que 0 est une valeur régulière de μ. Nos inégalités sont finalement données, sous cette hypothèse, en termes de la courbure du fibré induit par L sur cette réduction.

Les résultats annoncés dans cette note sont démontrés dans [7].

1. Introduction

In [2], Demailly established asymptotic Morse inequalities for the Dolbeault cohomology associated with high tensor powers $L^p := L^{\otimes p}$ of a holomorphic Hermitian line bundle (L, h^L) over a compact complex manifold (M, J). The inequalities of Demailly give asymptotic bounds on the Morse sums of the Betti numbers of $\bar\partial$ on L^p in terms of certain integrals of the Chern curvature R^L of (L, h^L). More precisely, we define $\dot{R}^L \in \mathrm{End}(T^{(1,0)}M)$ by $g^{TM}(\dot{R}^L u, \overline{v}) = R^L(u, \overline{v})$ for $u, v \in T^{(1,0)}M$, where g^{TM} is a J-invariant Riemannian metric on TM. We denote by $M(\le q)$ the set of points where $\dot{R}^L$ is non-degenerate and have at most q negative eigenvalues, and we set $n = \dim_{\mathbb{C}} M$. Then we have for $0 \le q \le n$

$$\sum_{j=0}^{q} (-1)^{q-j} \dim H^j(M, L^p) \le \frac{p^n}{n!} \int_{M(\le q)} (-1)^q \left(\frac{\sqrt{-1}}{2\pi} R^L\right)^n + o(p^n), \tag{1.1}$$

with equality if $q = n$. Here $H^j(M, L^p)$ denotes the Dolbeault cohomology in bidegree $(0, j)$, which is also the j-th group of cohomology of the sheaf of holomorphic sections of L^p.

These inequalities have found numerous applications. For instance, Demailly used recently these inequalities in [3] to prove a significant step of a generalized version of the Green–Griffiths–Lang conjecture.

Subsequently, Bismut gave in [1] a new proof of the holomorphic Morse inequalities using a heat kernel method. The key point is that we can compare the left-hand side of (1.1) with the alternate trace of the heat kernel of the Kodaira Laplacian $\Box_p$ of L^p on forms of degree $\le q$, i.e., for any $u > 0$,

$$\sum_{j=0}^{q} (-1)^{q-j} \dim H^j(M, L^p) \le \sum_{j=0}^{q} (-1)^{q-j} \mathrm{Tr}^{\Omega^{0,j}(M,L^p)} \left[\exp\left(-\frac{u}{p}\Box_p\right)\right], \tag{1.2}$$

with equality if $q = n$.

In the equivariant case, a connected compact Lie group G acts on M and its action lifts on L. When L is positive, Ma and Zhang [6] have studied the invariant Bergman kernel, i.e., the kernel of the projection from $\mathscr{C}^\infty(M, L^p)$ onto the G-invariant part of $H^0(M, L^p)$. Let μ be the moment map associated with the G-action on M (see (2.2)). Ma and Zhang [6] established that the invariant Bergman kernel concentrate to any neighborhood of $\mu^{-1}(0)$, and that near $\mu^{-1}(0)$, we have a full off-diagonal asymptotic development. They also obtain a fast decay of the invariant Bergman kernel in the normal directions to $\mu^{-1}(0)$, which does not appear in the classical case.

In this note, we give G-invariant holomorphic Morse inequalities under certain natural condition, in the context of Ma–Zhang [6] but without the assumption that L is positive.

More precisely, we consider a holomorphic action of a connected compact Lie group G on a compact complex manifold M and two G-equivariant vector bundles L and E on M, with L of rank 1, and we establish asymptotic holomorphic Morse inequalities similar to (1.1) for the G-invariant part of the Dolbeault cohomology of $L^p \otimes E$ (see Theorem 2.3). To do so, we define a "moment map" $\mu : M \to \mathrm{Lie}(G)$ by the Kostant formula and we define the reduction of M under natural hypothesis on $\mu^{-1}(0)$ (see Assumption 2.1). Our inequalities are then given in term of the curvature of the bundle induced by L on this reduction, and the integral in (1.1) will be over subsets of the reduction.

A new feature in our setting when compared to Demailly's result is the localization near $\mu^{-1}(0)$. We use a heat kernel method inspired by [1] (see also [5, Sect. 1.6-1.7]), the key being that an analogue of (1.2) still holds (see Theorem 3.1) for the Kodaira Laplacian restricted to the space of invariant forms. We show that the heat kernel will concentrate in any neighborhood of $\mu^{-1}(0)$, and then we study the asymptotics of the heat kernel near $\mu^{-1}(0)$. However, as we will have to integrate the heat kernel in the normal directions to $\mu^{-1}(0)$, we need a more precise convergence result that in [5, Sect. 1.6]. Indeed we also need to prove a uniform fast decay of the heat kernel in the normal directions, which is analogous to the decay encountered in [6, Thm. 0.2] for the invariant Bergman kernel. Moreover, we need to compute the limiting model kernel not only at the origin.

2. Statement of the inequalities

Let (M, J) be a connected compact complex manifold. Let $n = \dim_{\mathbb{C}} M$. Let (L, h^L) be a holomorphic Hermitian line bundle on M, and (E, h^E) a holomorphic Hermitian vector bundle on M. We denote the Chern (i.e., holomorphic and Hermitian) connection of L by ∇^L, and its curvatures by $R^L = (\nabla^L)^2$. Let ω be the first Chern form of (L, h^L), i.e., the $(1,1)$-form

$$\omega = \frac{\sqrt{-1}}{2\pi} R^L. \tag{2.1}$$

We *do not assume* that ω is a positive $(1,1)$-form.

Let G be a connected compact Lie group with Lie algebra $\mathfrak{g}$. Let $d = \dim_{\mathbb{R}} G$. We assume that G acts holomorphically on (M, J), and that the action lifts to a holomorphic action on L and E. We assume that h^L and h^E are preserved by the G-action. Then R^L and ω are G-invariant.

In the sequel, if F is any G-representation, we denote by F^G the space of elements of F invariant under the action of G. The infinitesimal action of $K \in \mathfrak{g}$ will be denoted by $\mathcal{L}_K$.

For $K \in \mathfrak{g}$, let K^M be the Killing vector field on M induced by K. We can define a map $\mu\colon M \to \mathfrak{g}^*$ by the Kostant formula

$$\mu(K) = \frac{1}{2\sqrt{-1}\pi}\left(\nabla^L_{K^M} - \mathcal{L}_K\right). \tag{2.2}$$

Then for any $K \in \mathfrak{g}$,

$$\mathrm{d}\mu(K) = i_{K^M}\omega. \tag{2.3}$$

We thus call μ an moment map, even though ω may be degenerate. Moreover the set defined by

$$P := \mu^{-1}(0) \tag{2.4}$$

is stable by G.

We make the following assumption:

Assumption 2.1. 0 is a regular value of μ.

Under Assumption 2.1, P is a submanifold of M and G acts locally freely on P, so that the quotient $M_G = P/G$ is an orbifold, which we call the *reduction* of M. The projection $P \to M_G$ is denoted by π.

The following analogue of the classical Kähler reduction holds.

Theorem 2.2. *The complex structure J on M induces a complex structure J_G on M_G, for which the orbifold bundles L_G, E_G induced by L, E on M_G are holomorphic. Moreover, the form ω descends to a form ω_G on M_G and if R^{L_G} is the Chern curvature of L_G for the metric h^{L_G} induced by h^L, then*

$$\omega_G = \frac{\sqrt{-1}}{2\pi} R^{L_G}. \tag{2.5}$$

Finally, for $x \in M_G$, π_* induces an isomorphism $(\ker\omega_G)_x \simeq (\ker\omega)|_{\pi^{-1}(x)}$.

We denote by TY the tangent bundle of the G-orbits in P. As G acts locally freely on P, we know that $TY = \mathrm{Span}(K^M, K \in \mathfrak{g})$ and that it is a vector bundle on P.

Let b^L be the bilinear form on TM

$$b^L(\cdot,\cdot) = \frac{\sqrt{-1}}{2\pi} R^L(\cdot, J\cdot) = \omega(\cdot, J\cdot). \tag{2.6}$$

Then we prove that when restricted to $TY \times TY$, the bilinear form b^L is non-degenerate on P. In particular, the signature of $b^L|_{TY\times TY}$ is constant on P. We denote by $(r, d-r)$ this signature, i.e., in any orthogonal (with respect to b^L) basis of $TY|_P$, the matrix of b^L will have r negative diagonal elements and $d-r$ positive diagonal elements.

We define $\dot{R}^{L_G} \in \mathrm{End}(T^{(1,0)}M_G)$ by

$$g(\dot{R}^{L_G}u, \overline{v}) = R^{L_G}(u, \overline{v}) \tag{2.7}$$

for $u, v \in T^{(1,0)}M_G$, where g is a J_G-invariant Riemannian metric on the orbifold tangent bundle TM_G. We denote by $M_G(q)$ the set of $x \in M_G$ such that $\dot{R}^{L_G}_x$ is invertible and has exactly q negative eigenvalues, with the convention that if $q \notin \{0, \dots, n-d\}$, then $M_G(q) = \emptyset$. Set $M_G(\leq q) = \cup_{i\leq q} M_G(i)$. Note that $M_G(q)$ does not depend on the metric g.

Consider the finite normal subgroup of G given by

$$G^0 := \{g \in G \,:\, g \cdot x = x \text{ for any } x \in M\}. \tag{2.8}$$

In fact, we also have $G^0 = \{g \in G \,:\, g \cdot x = x$ for any $x \in P\}$.

As G preserves every structure we are given, it acts naturally on the Dolbeault cohomology $H^\bullet(M, L^p \otimes E)$. The following theorem is the main result of this note.

Theorem 2.3. *As $p \to +\infty$, the following strong Morse inequalities hold for $q \in \{1, \dots, n\}$*

$$\sum_{j=0}^{q} (-1)^{q-j} \dim H^j(M, L^p \otimes E)^G$$
$$\leq \operatorname{rk}(L^p \otimes E)^{G^0} \frac{p^{n-d}}{(n-d)!} \int_{M_G(\leq q-r)} (-1)^{q-r} \omega_G^{n-d} + o(p^{n-d}), \tag{2.9}$$

with equality for $q = n$.

In particular, we get the weak Morse inequalities

$$\dim H^q(M, L^p \otimes E)^G \leq \operatorname{rk}(L^p \otimes E)^{G^0} \frac{p^{n-d}}{(n-d)!} \int_{M_G(q-r)} (-1)^{q-r} \omega_G^{n-d} + o(p^{n-d}). \tag{2.10}$$

Remark 2.4. The integer $\operatorname{rk}(L^p \otimes E)^{G^0}$ depends on p. However, as G^0 is finite and acts by rotations on L, there exists $k \in \mathbb{N}$ (a divisor of the cardinal of G^0) such that G^0 acts trivially on L^k. In particular, we have $\dim(L^{kp} \otimes E)^{G^0} = \dim E^{G^0}$.

Moreover, if G acts effectively on M, then $G^0 = \{1\}$ and $\operatorname{rk}(L^p \otimes E)^{G^0} = \operatorname{rk}(E)$.

Remark 2.5. In the terminology introduced by Guillemin and Sternberg [4], our inequalities can be seen as a version of the fact that reduction commutes with quantization for the holomorphic Morse inequalities. We refer to Vergne's Bourbaki seminar [8] for a survey on the Guillemin–Sternberg geometric quantization conjecture.

3. Main steps of the proof

Let g^{TM} be a J- and G-invariant metric on TM. Let dv_M be the corresponding Riemannian volume on M, and let ∇^{TM} be the Levi-Civita connection on (TM, g^{TM}). Let $\bar\partial^{L^p \otimes E}$ be the Dolbeault operator acting on $\Omega^{0,\bullet}(M, L^p \otimes E)$. Let $\bar\partial^{L^p \otimes E,*}$ be its dual with respect to the L^2 product induced by g^{TM}, h^L and h^E on $\Omega^{0,\bullet}(M, L^p \otimes E)$. We set $D_p = \sqrt{2}\left(\bar\partial^{L^p \otimes E} + \bar\partial^{L^p \otimes E,*}\right)$, and we denote by $e^{-uD_p^2}$ the associated heat kernel.

We denote P_G the orthogonal projection from $\Omega^{0,\bullet}(M, L^p \otimes E)$ onto $\Omega^{0,\bullet}(M, L^p \otimes E)^G$. Let $\left(P_G e^{-\frac{u}{p} D_p^2} P_G\right)(m, m')$ be the Schwartz kernel of $P_G e^{-\frac{u}{p} D_p^2} P_G$ with respect to $dv_M(m')$.

Note that the operator D_p^2 acts on $\Omega^{0,\bullet}(M, L^p \otimes E)^G$ (i.e., commutes with P_G) and preserves the $\mathbb{Z}$-grading. We denote by $\operatorname{Tr}_q[P_G e^{-\frac{u}{p} D_p^2} P_G]$ the trace of $P_G e^{-\frac{u}{p} D_p^2} P_G$ acting on $\Omega^{0,q}(M, L^p \otimes E)$. We then have an analogue of (1.2):

Theorem 3.1. *For any $u > 0$, $p \in \mathbb{N}^*$ and $0 \leq q \leq n$, we have*

$$\sum_{j=0}^{q} (-1)^{q-j} \dim H^j(M, L^p \otimes E)^G \leq \sum_{j=0}^{q} (-1)^{q-j} \operatorname{Tr}_j\left[P_G e^{-\frac{u}{p} D_p^2} P_G\right], \tag{3.1}$$

with equality for $q = n$.

We now give the estimates on $P_G e^{-\frac{u}{p} D_p^2} P_G$ to treat the right-hand side of (3.1).

Let U be a small open G-invariant neighborhood of P, such that G acts locally freely on its closure $\overline{U}$. First, we have, away from P, the following theorem.

Theorem 3.2. *For any fixed $u > 0$ and $k, \ell \in \mathbb{N}$, there exists $C > 0$ such that for any $p \in \mathbb{N}^*$ and $m, m' \in M$ with $m, m' \in M \setminus U$,*

$$\left| P_G e^{-\frac{u}{p} D_p^2} P_G(m, m') \right|_{\mathscr{C}^\ell} \leq C p^{-k}, \tag{3.2}$$

where $|\cdot|_{\mathscr{C}^\ell}$ is the $\mathscr{C}^\ell$-norm induced by ∇^L, ∇^E, ∇^{TM}, h^L, h^E and g^{TM}.

We now turn to the "near-P" asymptotics of the heat kernel. To explain simply this asymptotics, we assume now that G acts freely on P. We can thus also assume that G acts freely on $\overline{U}$. Let $B = U/G$. Then M_G and B are here genuine manifolds. We explain in [7] how to adapt the proof of Theorems 2.3 to the case of a locally free action.

In the sequel, if F is a bundle on M, we will denote by F_B the bundle induced on B, and if $\pi(y) = x$ we will keep the same notation for an element in F_y and the corresponding element in $F_{B,x}$.

Set $T^H P = TP \cap JTP$. Then we prove that $TP = TY \oplus T^H P$ and $TU = JTY \oplus TY \oplus T^H P$. Let $g^{T^H P}$ be a G-invariant and J-invariant metric on $T^H P$. Let g^{TY} be a G-invariant metric on TY and let g^{JTY} be the G-invariant metric on JTY induced by J and g^{TY}. We will chose in the rest of this section the metric g^{TM} on TM so that on P:

$$g^{TM}|_P = g^{JTY}|_P \oplus g^{TY}|_P \oplus g^{T^H P}. \tag{3.3}$$

We denote by g^{TB} the metric on TB induced by g^{TM} and the horizontal space $T^H U := JTY \oplus T^H P$.

Suppose that U is small enough so that it can be identified with a ε-neighborhood, $\varepsilon > 0$, of the zero section of the normal bundle N of P in U via exponential map. We denote the corresponding coordinate by $m = (y, Z^\perp) \in U$ with $y \in P$ and $Z^\perp \in N_y$. Note that N_y can be identified with JTY_y and that moreover, the normal bundle N_G of M_G in B can be identified with the bundle $(JTY)_B|_{M_G}$. In particular, if $\pi(y) = x$, we keep the same notation for an element of N_y and the corresponding element in $N_{G,x}$.

Let $\mathbf{J} \in \operatorname{End}(TM|_P)$ be such that on P, $\omega = g^{TM}(\mathbf{J}\cdot, \cdot)$. We can prove that $\mathbf{J}$ intertwines the bundles TY and JTY. This implies that $\mathbf{J^2}$ induces an endomorphism of N_G and if $\{a_1^\perp, \dots, a_d^\perp\} = -2\sqrt{-1}\pi \operatorname{Sp}(\mathbf{J}|_{(TY\oplus JTY)^{(1,0)}})$ $(a_j^\perp \in \mathbb{R}^*)$, then

$$\operatorname{Sp}(\mathbf{J^2}|_{N_G}) = -\frac{1}{4\pi^2}\{a_1^{\perp,2}, \dots, a_d^{\perp,2}\}. \tag{3.4}$$

Let g^{N_G} be the metric induced on N_G by g^{TB} and dv_{N_G} the corresponding volume form. For $x \in M_G$, let $\{e_i^\perp\}_{i=1}^d$ be an orthonormal basis of $N_{G,x}$ such that $\mathbf{J}_x^2 e_i^\perp = -\frac{1}{4\pi^2} a_i^{\perp,2}(x) e_i^\perp$. We can then identify $\mathbb{R}^d \simeq N_{G,x}$ via this basis.

We now define the harmonic oscillator $\mathscr{L}_x^\perp$ acting on $N_{G,x} \simeq \mathbb{R}^d$ by

$$\mathscr{L}_x^\perp = -\sum_{i=1}^d \left((\nabla_{e_i^\perp})^2 - |a_i^\perp Z_i^\perp|^2 \right) - \sum_{j=1}^d a_j^\perp \tag{3.5}$$

where ∇_U denotes the ordinary differentiation operator on $\mathbb{R}^d$ in the direction U. We denote by $e^{-u\mathscr{L}_x^\perp}(Z^\perp, Z'^\perp)$ the heat kernel of $\mathscr{L}_x^\perp$ with respect to $dv_{N_{G,x}}(Z'^\perp)$. There is an explicit formula for $e^{-u\mathscr{L}_x^\perp}(Z^\perp, Z'^\perp)$ (Mehler's formula, see [5, Appendix E]), but we do not give it to have a simpler asymptotic formula for the heat kernel.

Let g^{TM_G} be the metric on M_G induced by g^{TM} and $T^H P$ and dv_{M_G} the corresponding volume form. We define $\dot{R}^{L_G} \in \operatorname{End}(T^{(1,0)}M_G)$ as in (2.7) with the metric g^{TM_G}.

Let $\{w_j\}$ be a local orthonormal frame of $T^{(1,0)}M$ with dual frame $\{w^j\}$. Set

$$\omega_d = -\sum_{i,j} R^L(w_i, \overline{w}_j)\overline{w}^j \wedge i_{\overline{w}_i}. \tag{3.6}$$

Let h be the G-invariant function on M given by $h(x) = \sqrt{\operatorname{vol}(G.x)}$, and let $\kappa \in \mathscr{C}^\infty(TB|_{M_G})$ be the function defined by $\kappa|_{M_G} = 1$ and $dv_B = \kappa\, dv_{M_G} dv_{N_G}$.

Theorem 3.3. *Assume that G acts freely on P. For any fixed $u > 0$ and $m \in \mathbb{N}$, we have the following convergence as $p \to +\infty$ for $|Z^\perp| < \varepsilon$:*

$$\begin{aligned} & h(y, Z^\perp)^2 \left(P_G e^{-\frac{u}{p}D_p^2} P_G\right)\left((y, Z^\perp), (y, Z^\perp)\right) \\ &= \frac{\kappa^{-1}(x, Z^\perp)}{(2\pi)^{n-d}} \frac{\det(\dot{R}_x^{L_G}) e^{2u\omega_d(x)}}{\det\left(1 - \exp(-2u\dot{R}_x^{L_G})\right)} e^{-u\mathscr{L}_x^\perp}(\sqrt{p}Z^\perp, \sqrt{p}Z^\perp) \otimes \operatorname{Id}_E p^{n-d/2} \\ &\quad + O\left(p^{n-d/2-1/2}(1 + \sqrt{p}|Z^\perp|)^{-m}\right), \end{aligned} \tag{3.7}$$

where $x = \pi(y) \in M_G$ and the term $O(\cdot)$ is uniform. The convergence is in the $\mathscr{C}^\infty$-topology in $y \in P$. Here, we use the convention that if an eigenvalue of $\dot{R}_{x_0}^{L_G}$ is zero, then its contribution to $\frac{\det(\dot{R}_{x_0}^{L_G})}{\det\left(1-\exp(-u\dot{R}_{x_0}^{L_G})\right)}$ is $\frac{1}{2u}$.

From Theorems 3.1, 3.2 and 3.3, we find Theorem 2.3 by integrating on M and then taking the limit $u \to +\infty$.

For more details and the proofs of the results announced here, we refer the reader to [7].

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References

[1] J.-M. Bismut, Demailly's asymptotic Morse inequalities: a heat equation proof, J. Funct. Anal. 72 (2) (1987) 263–278.
[2] J.-P. Demailly, Champs magnétiques et inégalités de Morse pour la d''-cohomologie, Ann. Inst. Fourier (Grenoble) 35 (1985) 189–229.
[3] J.-P. Demailly, Holomorphic Morse inequalities and the Green–Griffiths–Lang conjecture, Pure Appl. Math. Q. 7 (4) (2011) 1165–1207. Special Issue: In Memory of Eckart Viehweg.
[4] V. Guillemin, S. Sternberg, Geometric quantization and multiplicities of group representations, Invent. Math. 67 (1982) 515–538.
[5] X. Ma, G. Marinescu, Holomorphic Morse Inequalities and Bergman Kernels, Progress in Mathematics, vol. 254, Birkhäuser Verlag, Basel, Switzerland, 2007.
[6] X. Ma, W. Zhang, Bergman kernels and symplectic reduction, Astérisque 318 (2008) viii+154.
[7] M. Puchol, G-invariant holomorphic Morse inequalities, J. Differ. Geom. (2015), arXiv:1506.04526, in press.
[8] M. Vergne, Quantification géométrique et réduction symplectique, In: Séminaire Bourbaki, No. 888, 2000–2001.