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An L^2 extension theorem with optimal estimate

Un théorème d'extension L^2 avec estimation optimale

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ABSTRACT

In this note, we establish an L^2 extension theorem with an optimal estimate for semi-positive vector bundles in the sense of Nakano. This result also implies optimal estimate versions of various L^2 extension theorems. Applications include a solution of the equality case in a conjecture of Suita on logarithmic capacities of open Riemann surface, as well as a solution of the extended Suita conjecture and a confirmation of the so-called L-conjecture.

RÉSUMÉ

Dans cette note, nous présentons un théorème d'extension L^2 avec estimation optimale, pour des fibrés vectoriels holomorphes semi-positifs dans le sens de Nakano. Ce résultat implique aussi des versions optimales pour l'estimation de divers autres théorèmes d'extension L^2. En application, nous obtenons la solution du cas d'égalité dans une conjecture de Suita relative aux capacité logarithmiques de surfaces de Riemann ouvertes, ainsi que la solution de la conjecture de Suita généralisée, et la confirmation d'un énoncé connu sous le nom de L-conjecture.

1. Main result

In [11] and [12], we obtained the L^2 extension theorem with optimal estimate in the frame of [19], and proved a conjecture of Ohsawa posed in [19] as an application. In the present paper, we consider the following positive real function $c_A(t)$ and obtain an L^2 extension theorem with optimal estimate related to the function $c_A(t)$ in a more precise way, which generalizes one of the main theorems in [11] and [12] for smooth plurisubharmonic polar functions and Nakano semi-positive vector bundles by taking $c_A(t) = 1$. Consequently, we prove a conjecture of Suita on the equality condition in Suita's conjecture, the so-called L-conjecture, and the extended Suita conjecture; we also obtain optimal estimate versions of various known L^2-extension theorems by many authors.

Let $c_A(t)$ be a positive smooth function on $(-A, +\infty)$ $(A \in (-\infty, +\infty])$ satisfying $\int_{-A}^{\infty} c_A(t)\mathrm{e}^{-t}\,\mathrm{d}t < \infty$, and

$$\left(\int_{-A}^{t} c_A(t_1)\mathrm{e}^{-t_1}\,\mathrm{d}t_1\right)^2 > c_A(t)\mathrm{e}^{-t}\int_{-A}^{t}\int_{-A}^{t_2} c_A(t_1)\mathrm{e}^{-t_1}\,\mathrm{d}t_1\,\mathrm{d}t_2, \tag{1.1}$$

for any $t \in (-A, +\infty)$.

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It is easy to see that when $c_A(t)e^{-t}$ is decreasing with respect to t and A is finite, inequality (1.1) holds.

Let M be a complex manifold with a continuous volume form dV_M and $\dim M = n$, and S be a closed complex subvariety of M. Let $\Psi - A$ be a negative plurisubharmonic polar function on M, which is smooth on $M \setminus S$ ([18,19], see also [11,12]).

We say that (M, S) satisfies condition (ab) ([19], see also [11] or [12]) if M and S satisfy the following conditions.

There exists a closed subset $X \subset M$ such that:

(a) X is locally negligible with respect to L^2 holomorphic functions, i.e., for any local coordinate neighborhood $U \subset M$ and for any L^2 holomorphic function f on $U \setminus X$, there exists an L^2 holomorphic function $\tilde{f}$ on U such that $\tilde{f}|_{U\setminus X} = f$ with the same L^2 norm.

(b) $M \setminus X$ is a Stein manifold which intersects with every component of S, and X contains S_{sing} (S_{sing} is the singular part of S).

We always assume that (M, S) at least satisfies condition (ab) in the following part of the present note. Let h be a smooth Hermitian metric on a holomorphic vector bundle E on M with rank r which is semi-positive in the sense of Nakano. Recall that an Hermitian metric h on E is said to be semi-positive in the sense of Nakano if Θ_h is semi-positive definite as an Hermitian form on $T_X \otimes E$, i.e. if for every $u \in T_X \otimes E$, $u \neq 0$, we have $\Theta_h(u, u) \geqslant 0$ (see [6]).

We establish an L^2 extension theorem with optimal estimate as follows.

Theorem 1.1. *There exists a uniform constant* $\mathbf{C} = 1$ *such that, for any holomorphic section* f *of* $K_M \otimes E|_S$ *on* S *satisfying*:

$$\sum_{k=1}^{n} \frac{\pi^k}{k!} \int_{S_{n-k}} |f|_h^2 \, dV_M[\Psi] < \infty,$$

there exists a holomorphic section F *of* $K_M \otimes E$ *on* M *satisfying* $F = f$ *on* S *and*:

$$\int_M c_A(-\Psi)|F|_h^2 \, dV_M \leqslant \mathbf{C} \int_{-A}^{\infty} c_A(t)\, e^{-t} \, dt \sum_{k=1}^{n} \frac{\pi^k}{k!} \int_{S_{n-k}} |f|_h^2 \, dV_M[\Psi]. \tag{1.2}$$

When E is a holomorphic line bundle and h is a semipositive singular metric, we can obtain the same optimal estimate (1.2) as in Theorem 1.1.

$\mathbf{C} = 1$ is optimal for the case when $\operatorname{codim} S = m$. Taking a product of a ball with a complex manifold X: $\mathbb{B}^m(0, e^{\frac{A}{2m}}) \times X$ for trivial holomorphic line bundle when $S = \{0\} \times X$, and $\Psi = 2m \log|z|$. When $A = +\infty$, $\mathbb{B}^m(0, e^{\frac{A}{2m}}) \times X := \mathbb{C}^m \times X$.

The first proof of the optimal estimate for Ohsawa–Takegoshi's L^2 extension theorem for functions on bounded pseudoconvex domains in $\mathbf{C}^n$ was given in [4], as a continuation of a work towards this direction in [30] (see also [3]).

2. A conjecture of Suita and the *L*-conjecture

2.1. A conjecture of Suita

In this section, we present a corollary of Theorem 1.1, which elucidates the equality case in Suita's conjecture on the comparison between the Bergman kernel and the logarithmic capacity on an open Riemann surface, as also surmised by Suita.

Let Ω be an open Riemann surface which admits a Green function G_Ω. Let z be a local coordinate on a neighborhood V_{z_0} of $z_0 \in \Omega$. Let κ_Ω be the Bergman kernel for holomorphic $(1,0)$ forms on Ω. We define $\kappa_\Omega(z)|_{V_{z_0}} = B_\Omega(z)|dz|^2$, and $\kappa_\Omega(z, \bar{t})|_{V_{z_0}} = B_\Omega(z, \bar{t}) dz \otimes d\bar{t}$. Let $c_\beta(z)$ be the logarithmic capacity which is locally defined by $c_\beta(z) = \exp \lim_{\xi \to z}(G_\Omega(\xi, z) - \log|\xi - z|)$ on Ω (see [21]).

Suita's conjecture in [25] says that on any open Riemann surface Ω as above, $(c_\beta(z_0))^2 \leqslant \pi B_\Omega(z_0)$.

We solved the above conjecture in [10]. The case for bounded planar domains was first proved by Blocki (see [3,4]).

In the same paper [25], Suita also conjectured a necessary and sufficient condition for the equality to hold in his inequality:

A conjecture of Suita. $(c_\beta(z))^2 = \pi B_\Omega(z)$ *for* $z \in \Omega$ *if and only if* Ω *is conformally equivalent to the unit disc less a (possible) closed set of inner capacity zero.*

Note that if the equality holds for one $z \in \Omega$, then the equality holds for any $z \in \Omega$.

As a corollary of Theorem 1.1, we solve the above conjecture positively:

Corollary 2.1. *The above conjecture of Suita holds.*

2.2. L-conjecture

In this subsection, we present the solution of L-conjecture (see [27]) by Corollary 2.1.

Let Ω be an open Riemann surface which admits a Green function G_Ω, and not biholomorphic to unit disc minus a (possible) closed set of inner capacity zero. Assume that $G_\Omega(\cdot, t)$ is an exhaustion function for any $t \in \Omega$. In [27], there is a conjecture on the zero points of the adjoint L-kernel $L_\Omega(z,t) := \frac{2}{\pi}\frac{\partial^2 G_\Omega(z,t)}{\partial z \partial t}$ of the Bergman kernel $\kappa_\Omega(z,\bar{t})$ as follows.

L-Conjecture (LC). *For any $t \in \Omega$, there exists $z \in \Omega$, such that $L_\Omega(z,t) = 0$.*

It is known that, for a finite Riemann surface Ω, $G_\Omega(\cdot,t)$ is an exhaustion function for any $t \in \Omega$ (see [27] or [22]).

By Theorem 6 in [27], L-conjecture for the finite Riemann surfaces can be deduced by Corollary 2.1. Considering exhaustion sub-domains with smooth boundaries of Ω, we obtain the solution of the so-called L-conjecture by Corollary 2.1.

The following remark tells us that the exhaustion assumption of $G_\Omega(\cdot,\bar{t})$ is necessary.

When Ω is an annulus, we have $\#\{z | L_\Omega(z,t) = 0\} = 1$ for suitable $t \in \Omega$ (see [26]). Let $t_1 \in \Omega$ satisfies $\#\{z|L_\Omega(z,t_1) = 0\} = 1$. Assume that $z_1 \in \{z|L_\Omega(z,t_1) = 0\}$. Note that $z_1 \neq t_1$, as $G_{\Omega\setminus\{z_1\}} = G_\Omega|_{\Omega\setminus\{z_1\}}$, then we have $\#\{z|L_{\Omega\setminus\{z_1\}}(z,t_1) = 0\} = 0$.

3. Extended Suita conjecture

Let Ω be an open Riemann surface, which admits a Green function G_Ω. Let $z_0 \in \Omega$, with local coordinate z. Let $p : \Delta \to \Omega$ be the universal covering from unit disc Δ to Ω.

We call the holomorphic function f (resp. holomorphic $(1,0)$ form F) on Δ is a multiplicative function (resp. multiplicative differential (Prym differential) (see [8])) if there is a character χ, which is the representation of the fundamental group of Ω, such that $g^* f = \chi(g) f$ (resp. $g^* F = \chi(g) F$), where $|\chi| = 1$. We denote the set of such kinds of functions f (resp. forms F) by $\mathcal{O}^\chi(\Omega)$ (resp. $\Gamma^\chi(\Omega)$), as in [27].

Remark 3.1. For Green function $G_\Omega(\cdot, z_0)$, one can find a χ_{z_0} and a multiplicative function $f_{z_0} \in \mathcal{O}^{\chi_{z_0}}(\Omega)$, such that $|f_{z_0}| = p^* e^{G_\Omega(\cdot,z_0)}$.

Note that $F \wedge \bar{F}$ is fibre constant respect to p (see [27]); then one can define a multiplicative Bergman kernel $\kappa^\chi(x,\bar{y})$ for $\Gamma^\chi(\Omega)$ on $\Omega \times \Omega$. Let $B^\chi_\Omega(z)|dz|^2 := \kappa^\chi_\Omega(z,\bar{z})$. There is an extended Suita conjecture in [27]:

Extended Suita conjecture. *$c^2_\beta(z_0) \leqslant \pi B^\chi_\Omega(z_0)$, and equality holds if and only if $\chi = \chi_{z_0}$.*

We have proved the inequality part of the above conjecture in [12]. Combining this result in [12] with Theorem 1.1 in the present note, we solve the above extended Suita conjecture completely.

Corollary 3.2. *The extended Suita conjecture is true.*

For a detailed proof, the reader is referred to [12] and [13]. Yamada told us that, for the annulus, the above conjecture was proved in [28].

4. Optimal estimates of various L^2 extension theorems

In this section, we give optimal estimates of various well-known L^2 extension theorems, using our Theorem 1.1 by taking different $c_A(t)$.

Assume that a pair (M,S) satisfies condition (ab), where $\dim M = n$ and $\operatorname{codim} S = m$. Let (E,h) be an Hermitian holomorphic vector bundle, which is semi-positive in the sense of Nakano. We denote $|u|^2_h dV_M$ by $\{u,u\}_h$ for any continuous section of $K_M \otimes E$. Define $\{f,f\}_h := \langle e,e\rangle_h c_{n-m} f_1 \wedge \bar{f}_1$ for any continuous section f of $K_{S_{\mathrm{reg}}} \otimes E|_{S_{\mathrm{reg}}}$ (S_{reg} is the regular part of S), where $c_k = (\sqrt{-1})^{k^2}$ and $f = f_1 \otimes e$ locally (see [7]). It is clear that $\{f,f\}_h$ is well defined.

Let $c_\infty(t) := (1 + e^{-\frac{t}{m}})^{-m-\varepsilon}$, where ε be a positive constant. One can check that

$$\int_{-\infty}^{\infty} c_\infty(t) e^{-t}\, dt = m \sum_{j=0}^{m-1} C^j_{m-1} (-1)^{m-1-j} \frac{1}{m-1-j+\varepsilon} < \infty,$$

and inequality (1.1) holds for any $t \in (-\infty, +\infty)$.

Let $\Psi = m \log(|g_1|^2 + \cdots + |g_m|^2)$, where $S = \{g_1 = \cdots = g_m = 0\}$, g_i are holomorphic functions on M and satisfy $\bigwedge_{j=1}^m dg_j|_{S_{\mathrm{reg}}} \neq 0$.

By Theorem 1.1, we obtain an optimal estimate version of the main result in [16] as follows:

Corollary 4.1. *For any holomorphic section f of $K_{S_{\mathrm{reg}}} \otimes E|_{S_{\mathrm{reg}}}$ on S_{reg} satisfying*:

$$\frac{\pi^m}{m!}\int_{S_{\mathrm{reg}}} \{f, f\}_h < \infty,$$

there exists a holomorphic section F of $K_M \otimes E$ on M satisfying $F = f \wedge \bigwedge_{k=1}^{m} \mathrm{d}g_k$ on S_{reg} and:

$$\int_M \left(1 + \left(|g_1|^2 + \cdots + |g_m|^2\right)^m\right)^{-1-\varepsilon} \{F, F\}_h \leqslant \mathbf{C}\left(m\sum_{j=0}^{m-1} C_{m-1}^{j}(-1)^{m-1-j}\frac{1}{m-1-j+\varepsilon}\right)\frac{(2\pi)^m}{m!}\int_{S_{\mathrm{reg}}} \{f, f\}_h,$$

with the uniform constant $\mathbf{C} = 1$ which is optimal for any m.

Let $c_{-1}(t) := \mathrm{e}^t t^{-2}$. Then $\int_1^{\infty} c_{-1}(t)\,\mathrm{e}^{-t}\,\mathrm{d}t = 1 < \infty$ and $c_{-1}(t)\,\mathrm{e}^{-t}$ is decreasing with respect to t, where $t \in (1, +\infty)$.

Let $\Psi = m\log(|g_1|^2 + \cdots + |g_m|^2) < 1$, where $S = \{g_1 = \cdots = g_m = 0\}$, g_i are holomorphic functions on M and satisfy $\bigwedge_{j=1}^{m} \mathrm{d}g_j|_{S_{\mathrm{reg}}} \neq 0$.

For the above case, by Theorem 1.1, we obtain the optimal estimate version of the main result in [15] and [5] (see also [7]):

Corollary 4.2. *For any holomorphic section f of $K_{S_{\mathrm{reg}}} \otimes E|_{S_{\mathrm{reg}}}$ on S_{reg} satisfying*:

$$\frac{\pi^m}{m!}\int_{S_{\mathrm{reg}}} \{f, f\}_h < \infty,$$

there exists a holomorphic section F of $K_M \otimes E$ on M satisfying $F = f \wedge \bigwedge_{k=1}^{m} \mathrm{d}g_k$ on S_{reg} and

$$\int_M \frac{\{F, F\}_h}{(|g_1|^2 + \cdots + |g_m|^2)^m(-m\log(|g_1|^2 + \cdots + |g_m|^2))^2} \leqslant \mathbf{C}\frac{(2\pi)^m}{m!}\int_{S_{\mathrm{reg}}} \{f, f\}_h,$$

with the uniform constant $\mathbf{C} = 1$ which is optimal for any m.

Let $c_0(t) := 1$. Then $\int_0^{\infty} c_0(t)\,\mathrm{e}^{-t}\,\mathrm{d}t = 1 < \infty$ and $c_0(t)\,\mathrm{e}^{-t}$ is decreasing with respect to t. So we obtain that inequality (1.1) holds for any $t \in (-\infty, +\infty)$.

Let M be a Stein manifold, and S be a closed analytic hypersurface on M, which is locally defined by $\{w_j\}$ on $U_j \subset M$, where $\{U_j\}_{j=1,2,\ldots}$ is an open covering of M, and functions $\{w_j\}_{j=1,2,\ldots}$ together gives a nonzero holomorphic section w of the holomorphic line bundle $[S]$ associated with the hypersurface S (see [9]).

Let $|\cdot|$ be an Hermitian metric on $[S]$, satisfying that $|\cdot|_{e^{\psi}}$ is semi-negative, where ψ is an upper-semicontinuous function on M. Assume that $\log(|w|^2) + \psi < 0$ on M. Let φ be a plurisubharmonic function.

Let $\Psi := \log(|w|^2) + \psi$. Note that Ψ is plurisubharmonic, and

$$|F|^2\,\mathrm{d}V_M[\Psi] = 2\frac{c_{n-1}\frac{F}{\mathrm{d}w} \wedge \frac{\bar{F}}{\mathrm{d}w}}{|\mathrm{d}w|^2}\,\mathrm{e}^{-\psi},$$

for any continuous $(n, 0)$ form F on M, where $|\mathrm{d}w|$ is the Hermitian metric on $[-S]|_{S_{\mathrm{reg}}}$ induced by the Hermitian metric $|\cdot|$ on $[S]|_{S_{\mathrm{reg}}}$.

By Theorem 1.1, we obtain another proof of the following result in [10], which is an optimal estimate version of the main results in [17,14] and [30], etc. Note that in [10], S is assumed to be defined globally by a holomorphic function w on M, while the following corollary says that this assumption is not necessary.

Corollary 4.3. *(See [10].) For any holomorphic section f of $K_{S_{\mathrm{reg}}}$ on S_{reg} satisfying*:

$$c_{n-1}\int_{S_{\mathrm{reg}}} \frac{f \wedge \bar{f}}{|\mathrm{d}w|^2}\mathrm{e}^{-\varphi-\psi} < \infty,$$

there exists a holomorphic section F of K_M on M satisfying $F = f \wedge \mathrm{d}w$ on S_{reg} and:

$$c_n\int_M F \wedge \bar{F}\mathrm{e}^{-\varphi} \leqslant 2\pi\mathbf{C}c_{n-1}\int_{S_{\mathrm{reg}}} \frac{f \wedge \bar{f}}{|\mathrm{d}w|^2}\mathrm{e}^{-\varphi-\psi},$$

with the uniform constant $\mathbf{C} = 1$, which is optimal.

When $\psi = 0$, and w is a global holomorphic function on M, the special case of the above statement appears to be an optimal estimate version of Ohsawa–Takegoshi L^2 extension theorem in [20,23,1,24,2,3], etc. For a survey on L^2 extension theorem, the reader is referred to [29]

Details of the proofs of the results in the present note will appear in [13].

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