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Partial Differential Equations/Optimal Control

## Uniform analyticity and exponential decay of the semigroup associated with a thermoelastic plate equation with perturbed boundary conditions



*Analyticité et décroissance exponentielle uniformes du semi-groupe lié à une équation de plaque thermo-élastique avec des conditions aux limites perturbées*

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### ABSTRACT

In a bounded domain, we consider an Euler–Bernoulli-type thermoelastic plate equation with perturbed boundary conditions. The boundary conditions are such that when the perturbation parameter goes to infinity, we recover the hinged boundary conditions, while one recovers the clamped boundary conditions when the perturbation parameter goes to zero. Relying on resolvent estimates, we show that the underlying semigroup is uniformly, with respect to the perturbation parameter, analytic and exponentially stable. The main features of our proof are appropriate decompositions of the components of the system and the use of Lions' interpolation inequalities.

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### RÉSUMÉ

Dans un domaine borné, on considère une équation de plaque thermo-élastique de type Euler–Bernoulli avec des conditions aux limites perturbées. Les conditions aux limites utilisées sont telles que l'on retrouve une plaque simplement posée lorsque le paramètre de perturbation tend vers l'infini, alors que l'on retrouve une plaque encastrée quand le paramètre de perturbation tend vers zéro. En nous appuyant sur des estimations de la résolvante, nous montrons que le semi-groupe associé est analytique et exponentiellement stable, uniformément par rapport au paramètre de perturbation. Les éléments principaux de notre démonstration sont des décompositions appropriées des composantes du système et l'utilisation d'inégalités d'interpolation de Lions.

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## Version française abrégée

On considère l'équation de plaque thermo-élastique de type Euler–Bernoulli avec des conditions aux limites perturbées (1). Pour ce système, on démontre le théorème ci-dessous.

**Théorème.** Pour tout  $\gamma > 0$ , l'opérateur  $\mathcal{A}_\gamma$  génère un semi-groupe continu de contractions  $(S_\gamma(t))_{t \geq 0}$ , qui est uniformément analytique par rapport à  $\gamma$ .

De plus, pour tout  $\gamma > 0$ , le semi-groupe  $(S_\gamma(t))_{t \geq 0}$  est uniformément exponentiellement stable ; plus précisément, des constantes strictement positives  $M$  and  $\lambda$ , indépendantes de  $\gamma$ , existent, telles que l'estimation (5) a lieu.

Ce résultat montre que la dissipation induite par la composante parabolique du système est suffisamment robuste ; son action n'est pas altérée par la présence des perturbations frontières considérées. En particulier, on peut retrouver le résultat antérieur sur la stabilité exponentielle du semi-groupe associé à l'équation de plaque thermo-élastique encastrée [5] en laissant tendre le paramètre de perturbation  $\gamma$  vers zéro. De la même manière, on peut retrouver, tout en l'améliorant, le résultat de [15] sur l'analyticité du semi-groupe associé à une plaque thermo-élastique simplement posée en faisant tendre  $\gamma$  vers l'infini, ou encastrée en faisant tendre  $\gamma$  vers zéro.

### 1. Problem formulation and statement of the main result

Let  $\Omega$  be a bounded smooth open connected subset of  $\mathbb{R}^N$ ,  $N \geq 1$ . Let  $\alpha, \beta, \kappa$  and  $\gamma$  be positive constants. Let  $\nu$  denote the unit outward normal vector to the boundary  $\Gamma$  of  $\Omega$ . Consider the Euler–Bernoulli thermoelastic plate equation:

$$\begin{cases} y_{\gamma,tt} + \Delta^2 y_\gamma + \alpha \Delta \theta_\gamma = 0 & \text{in } \Omega \times (0, \infty), \\ \theta_{\gamma,t} - \kappa \Delta \theta_\gamma - \beta \Delta y_{\gamma,t} = 0 & \text{in } \Omega \times (0, \infty), \\ y_\gamma = 0, \quad \gamma \Delta y_\gamma + \partial_\nu y_\gamma = 0, \quad \theta_\gamma = 0 & \text{on } \Sigma = \partial\Omega \times (0, \infty), \\ y_\gamma(0) = y_\gamma^0 \in V, \quad y_{\gamma,t}(0) = y_\gamma^1 \in H, \quad \theta_\gamma(0) = \theta_\gamma^0 \in H, \end{cases} \quad (1)$$

where  $V = H^2(\Omega) \cap H_0^1(\Omega)$ ,  $H = L^2(\Omega)$ ,  $W = H_0^1(\Omega)$ .

Let us introduce the Hilbert space over the field of complex numbers  $\mathcal{H}_\gamma = V \times H \times H$  equipped with the norm:

$$\|(u, v, w)\|_\gamma^2 = \int_\Omega \left\{ |\Delta u|^2 + |v|^2 + \frac{\alpha}{\beta} |w|^2 \right\} dx + \frac{1}{\gamma} \int_\Gamma |\partial_\nu u|^2 d\Gamma. \quad (2)$$

Setting  $Z = \begin{pmatrix} y \\ y_t \\ \theta \end{pmatrix}$ , System (1) may be recast as:

$$\begin{cases} \dot{Z}_\gamma - \mathcal{A}_\gamma Z_\gamma = 0 & \text{in } (0, \infty), \\ Z_\gamma(0) = \begin{pmatrix} y_\gamma^0 \\ y_\gamma^1 \\ \theta_\gamma^0 \end{pmatrix}, \end{cases} \quad (3)$$

where the unbounded operator  $\mathcal{A}_\gamma$  is given by:

$$\mathcal{A}_\gamma = \begin{pmatrix} 0 & I & 0 \\ -\Delta^2 & 0 & -\alpha \Delta \\ 0 & \beta \Delta & \kappa \Delta \end{pmatrix} \quad (4)$$

with:

$$\begin{aligned} D(\mathcal{A}_\gamma) = & \{(u, v, w) \in V \times V \times W; \Delta^2 u + \alpha \Delta w \in L^2(\Omega), \beta \Delta v + \kappa \Delta w \in L^2(\Omega); \gamma \Delta u + \partial_\nu u = 0 \text{ on } \Gamma\} \\ & = \{(u, v, w) \in (H^4(\Omega) \cap V) \times V \times (H^2(\Omega) \cap W); \gamma \Delta u + \partial_\nu u = 0 \text{ on } \Gamma\}. \end{aligned}$$

Before stating our analyticity and stability result, we note that System (1) is inspired from Lions' work on the controllability of second-order evolution systems with perturbed boundary conditions [13, Chap. 4]. The stabilization of thermoelastic plates has been the subject of extensive research ever since the monograph of Lagnese on the boundary stabilization of thin plates [7]. Although it was known that the dissipation induced by the heat component was sufficient for the strong stability of thermoelastic plates, it was not until the work [5] that it became clear that boundary mechanical dissipation was not needed in order to obtain the exponential stability of thermoelastic plates. Many other works followed suit, e.g., [1,2,16,19,21]. As far as analyticity of semigroups associated with thermoelastic plates is concerned, the first result was established in [15] in the case of hinged or clamped boundary conditions. Later on, other analyticity results involving various

boundary conditions followed, e.g., [8–11,14]. It should be noted that the situation is very different as far as the standard thermoelasticity system is concerned; even strong stability holds in special domains only, e.g., [3,12].

Our main result reads:

**Theorem 1.1.** *For every  $\gamma > 0$ , the linear operator  $\mathcal{A}_\gamma$  generates a  $C_0$ -semigroup of contractions  $(S_\gamma(t))_{t \geq 0}$  that is uniformly, with respect to  $\gamma$ , analytic.*

*Furthermore, for each  $\gamma > 0$ , the semigroup  $(S_\gamma(t))_{t \geq 0}$  is uniformly exponentially stable; more precisely, there exist positive constants  $M$  and  $\lambda$  that are independent of  $\gamma$ , such that:*

$$\|S_\gamma(t)Z^0\|_\gamma \leq M \exp(-\lambda t) \|Z^0\|_\gamma, \quad \forall Z^0 \in \mathcal{H}_\gamma, \quad \forall \gamma > 0. \tag{5}$$

**Remark 1.2.** **Theorem 1.1** shows that the dissipation induced by the heat component of the system is robust enough; its action is not altered by the presence of the perturbation considered. In particular, earlier results about the exponential decay and the analyticity of the semigroup in the case of hinged or clamped boundary conditions [5,15] may be recovered by either letting  $\gamma$  go to infinity or zero; this is feasible because our proof below clearly shows that the constants in our estimates are independent of  $\gamma$ . In particular, we improve on the result in [15] as we work in the usual functional space while those authors worked in an *ad hoc* functional space. We also note that the system considered in [11] is very close to (1); however their approach for proving the analyticity is different from ours; in particular, given that they use the usual norm on  $H^2(\Omega)$ , their constants would explode as  $\gamma$  goes to zero. Besides, as we shall see below, the desire to establish estimates that are uniform in  $\gamma$  involves more technicalities than otherwise.

**2. Ideas for proving Theorem 1.1**

Throughout this section, the quantity  $|u|_2$  denotes the  $L^2(\Omega)$ -norm of  $u$ , and  $C_0$  denotes a generic positive constant independent of  $\gamma$  that may vary from line to line.

The fact that the operator  $\mathcal{A}_\gamma$  generates a  $C_0$ -semigroup of contractions is pretty straightforward thanks to the Lumer–Phillips theorem [18]. We will focus on the uniform analyticity and exponential decay of the semigroup. Since  $D(\mathcal{A}_\gamma)$  is dense in  $\mathcal{H}_\gamma$ , one checks that  $\mathcal{A}_\gamma$  has a compact resolvent; therefore its spectrum is discrete. Further  $\sigma(\mathcal{A}_\gamma) \cap i\mathbb{R} = \emptyset$ . Now, according to Theorem 5.2 in Chap. 2 of [18] for analyticity, and Theorem 3 in [4] or Corollary 4 in [20] for exponential stability, we show that there exists a positive constant  $C_0$  independent of  $\gamma$  such that:

$$\sup\{\|b(ib - \mathcal{A}_\gamma)^{-1}\|_{\mathcal{L}(\mathcal{H}_\gamma)}; b \in \mathbb{R}\} \leq C_0, \quad \sup\{\|(ib - \mathcal{A}_\gamma)^{-1}\|_{\mathcal{L}(\mathcal{H}_\gamma)}; b \in \mathbb{R}\} \leq C_0. \tag{6}$$

If one is not interested in the independence of the constants upon the parameter  $\gamma$ , it is enough to prove the left inequality in (6) for  $|b| > 1$ , and then invoke the continuity of the resolvent for  $|b| \leq 1$ . However, we cannot follow that path, as, though the resolvent is continuous and so is bounded for  $|b| \leq 1$ , we do not know how the boundedness constant depends upon the parameter  $\gamma$ . To prove (6), we will show that there exists  $C_0 > 0$  such that, for every  $U \in \mathcal{H}_\gamma$ , one has:

$$\|b(ib - \mathcal{A}_\gamma)^{-1}U\|_\gamma + \|(ib - \mathcal{A}_\gamma)^{-1}U\|_\gamma \leq C_0 \|U\|_\gamma, \quad \forall b \in \mathbb{R}, \quad \forall \gamma > 0. \tag{7}$$

Let  $b \in \mathbb{R}$ ,  $U = (f, g, h) \in \mathcal{H}_\gamma$ , and let  $Z = (u, v, w) \in D(\mathcal{A}_\gamma)$  such that:

$$(ib - \mathcal{A}_\gamma)Z = U. \tag{8}$$

Multiply both sides of (8) by  $Z$ , then take the real part of the inner product in  $\mathcal{H}_\gamma$  to derive:

$$\frac{\alpha\kappa}{\beta} \int_\Omega |\nabla w|^2 dx = \Re(U, Z) \leq \|U\|_\gamma \|Z\|_\gamma. \tag{9}$$

Eq. (8) may be rewritten:

$$\begin{cases} ibu - v = f, \\ ibv + \Delta^2 u + \alpha \Delta w = g, \\ ibw - \beta \Delta v - \kappa \Delta w = h, \\ u = 0, \quad \gamma \Delta u + \partial_\nu u = 0, \quad v = 0, \quad w = 0 \text{ on } \Gamma. \end{cases} \tag{10}$$

Therefore, (7) will be established if we show the following estimate:

$$(|b| + 1)\|Z\|_\gamma \leq C_0 \|U\|_\gamma, \quad \forall \gamma > 0, \quad \forall b \in \mathbb{R}. \tag{11}$$

Several steps will be needed in the proof of (11).

**Step 1.** In this step, we are going to show that, for every  $\varepsilon > 0$ , there exists a positive constant  $C_\varepsilon$ , independent of  $\gamma$  and  $b$  such that:

$$\|Z\|_\gamma \leq \varepsilon |b| \|Z\|_\gamma + C_\varepsilon \|U\|_\gamma. \quad (12)$$

To this end, multiply the first equation in (10) by  $\bar{u}$ , apply Green's formula, take the real parts, then use Hölder's inequality and (9) to derive:

$$|\Delta u|_2^2 + \frac{1}{\gamma} \int_\Gamma |\partial_\nu u|^2 d\Gamma = \Re \int_\Omega \{v(\bar{v} + \bar{f}) - \alpha w \Delta \bar{u} + g \bar{u}\} dx \leq |v|_2^2 + C_0 (\|Z\|_\gamma \|U\|_\gamma + \|U\|_\gamma^{\frac{1}{2}} \|Z\|_\gamma^{\frac{3}{2}}). \quad (13)$$

If  $G$  denotes the inverse of the operator  $-\Delta$  with Dirichlet boundary conditions, multiply the third equation in (10) by  $G\bar{v}$ , apply Green's formula, take the real parts, then use the Hölder and Young inequalities and (9) to obtain:

$$2|v|_2^2 = \Re \int_\Omega \{-ib\bar{v}Gw - \kappa w\bar{v} + \bar{v}Gh\} dx \leq \varepsilon^2 |b|^2 |v|_2^2 + C_\varepsilon (\|Z\|_\gamma \|U\|_\gamma + \|U\|_\gamma^{\frac{1}{2}} \|Z\|_\gamma^{\frac{3}{2}}), \quad \forall \varepsilon > 0, \quad (14)$$

where and in the sequel  $C_\varepsilon$  is a generic positive constant independent of  $\gamma$  and  $b$ .

Adding (9), (13) and (14) side by side, then applying the Poincaré and Young inequalities, (12) easily follows.

**Step 2.** Here, we will show that the following estimate holds:

$$|b|(|w|_2 + |v|_2) \leq \varepsilon |b| \|Z\|_\gamma + C_\varepsilon \|U\|_\gamma, \quad \forall b \neq 0. \quad (15)$$

To this end, we shall borrow some ideas from [15]. Set  $w = w_1 + w_2$ , where  $w_1 \in W$  and  $w_2 \in H$ , with:

$$ibw_1 - \Delta w_1 = h, \quad ibw_2 = \kappa \Delta w + \beta \Delta v - \Delta w_1. \quad (16)$$

Proceeding as above, one easily derives from the left equation in (16):

$$|b| \|w_1\|_2 + |b|^{\frac{1}{2}} \|w_1\|_W + |\Delta w_1|_2 \leq C_0 \|U\|_\gamma. \quad (17)$$

On the other hand, it follows from the right equation in (16):

$$|b| \|w_2\|_{H^{-2}(\Omega)} \leq C_0 (|w|_2 + |v|_2 + |w_1|_2) \leq C_0 (\|Z\|_\gamma + |b|^{-1} \|U\|_\gamma). \quad (18)$$

Now, by Lions' interpolation inequality as well as (17), (18), the fact that  $\|w_2\|_W \leq \|w\|_W + \|w_1\|_W$ , and (9), we derive:

$$|b| \|w_2\|_2 \leq C_0 |b| \|w_2\|_{H^{-2}(\Omega)}^{\frac{1}{3}} \|w_2\|_{H^1(\Omega)}^{\frac{2}{3}} \leq C_0 b^{\frac{2}{3}} (\|Z\|_\gamma + |b|^{-1} \|U\|_\gamma)^{\frac{1}{3}} (\|U\|_\gamma^{\frac{1}{2}} \|Z\|_\gamma^{\frac{1}{2}} + |b|^{-\frac{1}{2}} \|U\|_\gamma)^{\frac{2}{3}}. \quad (19)$$

For the sequel, we also need to estimate  $|b| \|w_2\|_{H^{-1}(\Omega)}$ . Applying Lions' interpolation inequality once more and proceeding as above, one gets:

$$|b| \|w_2\|_{H^{-1}(\Omega)} \leq C_0 |b| \|w_2\|_{H^{-2}(\Omega)}^{\frac{2}{3}} \|w_2\|_{H^1(\Omega)}^{\frac{1}{3}} \leq C_0 |b|^{\frac{1}{3}} (\|Z\|_\gamma + |b|^{-1} \|U\|_\gamma)^{\frac{2}{3}} (\|U\|_\gamma^{\frac{1}{2}} \|Z\|_\gamma^{\frac{1}{2}} + |b|^{-\frac{1}{2}} \|U\|_\gamma)^{\frac{1}{3}}. \quad (20)$$

Set  $v = v_1 + v_2$ , where  $v_1 \in V$  and  $v_2 \in H$ , with:

$$ibv_1 - \Delta v_1 = g, \quad ibv_2 = -\Delta^2 u - \alpha \Delta w - \Delta v_1. \quad (21)$$

One checks the following estimates:

$$|b| \|v_1\|_2 + |b|^{\frac{1}{2}} \|v_1\|_W + |\Delta v_1|_2 \leq C_0 \|U\|_\gamma, \quad (22)$$

and

$$|b| \|v_2\|_{H^{-2}(\Omega)} \leq C_0 (|\Delta u|_2 + |w|_2 + |v_1|_2) \leq C_0 (\|Z\|_\gamma + |b|^{-1} \|U\|_\gamma). \quad (23)$$

Now, using the third equation in (10), one can show that:

$$\|v\|_W \leq C_0 \|U\|_\gamma^{\frac{1}{2}} \|Z\|_\gamma^{\frac{1}{2}} + C_0 |b| \|w_2\|_{H^{-1}(\Omega)}. \quad (24)$$

Applying Lions' interpolation inequality once more, and using the fact that  $\|v_2\|_W \leq \|v_1\|_W + \|v\|_W$ , we find:

$$|b| \|v_2\|_2 \leq C_0 |b| \|v_2\|_{H^{-2}(\Omega)}^{\frac{1}{3}} \|v_2\|_{H^1(\Omega)}^{\frac{2}{3}} \leq C_0 b^{\frac{2}{3}} (\|Z\|_\gamma + |b|^{-1} \|U\|_\gamma)^{\frac{1}{3}} (|b|^{-\frac{1}{2}} \|U\|_\gamma + \|v\|_W)^{\frac{2}{3}}. \quad (25)$$

The combination of (20) and (25) shows:

$$\begin{aligned}
 |b| \|v_2\|_2 \leq C_0 & (|b|^{\frac{1}{3}} \|Z\|_{\gamma}^{\frac{1}{3}} \|U\|_{\gamma}^{\frac{2}{3}} + |b|^{\frac{2}{3}} \|Z\|_{\gamma}^{\frac{2}{3}} \|U\|_{\gamma}^{\frac{1}{3}} + |b|^{\frac{8}{9}} \|Z\|_{\gamma}^{\frac{8}{9}} \|U\|_{\gamma}^{\frac{1}{9}} + |b|^{\frac{7}{9}} \|Z\|_{\gamma}^{\frac{7}{9}} \|U\|_{\gamma}^{\frac{2}{9}} \\
 & + |b|^{\frac{5}{9}} \|Z\|_{\gamma}^{\frac{5}{9}} \|U\|_{\gamma}^{\frac{4}{9}} + |b|^{\frac{4}{9}} \|Z\|_{\gamma}^{\frac{4}{9}} \|U\|_{\gamma}^{\frac{5}{9}} + |b|^{\frac{1}{9}} \|Z\|_{\gamma}^{\frac{1}{9}} \|U\|_{\gamma}^{\frac{8}{9}} + \|U\|_{\gamma}).
 \end{aligned}
 \tag{26}$$

Gathering (17), (19), (22), (26), and applying Young’s inequality, one gets (15).

**Step 3.** This step is devoted to showing the estimate:

$$b^2 |\Delta u|_2^2 + \frac{b^2}{\gamma} \int_{\Gamma} |\partial_\nu u|^2 d\Gamma \leq \varepsilon^2 b^2 \|Z\|_{\gamma}^2 + C_\varepsilon \|U\|_{\gamma}^2.
 \tag{27}$$

For this purpose, set  $u = u_1 + u_2$  with:

$$ib u_1 = v_1 + f, \quad ib u_2 = v_2 = -\frac{\Delta^2 u + \alpha w + \Delta v_1}{ib}.
 \tag{28}$$

Notice that the left equation in (28) may be recast as:

$$b^2 \Delta^2 u = b^4 u_2 - \alpha b^2 \Delta w - b^2 \Delta v_1.
 \tag{29}$$

Multiplying Eq. (29) by  $\bar{u}$  and using Green’s formula, one gets:

$$b^2 |\Delta u|_2^2 + \frac{b^2}{\gamma} \int_{\Gamma} |\partial_\nu u|^2 d\Gamma = b^4 |u_2|_2^2 + \Re \int_{\Omega} \{b^4 u_2 \bar{u}_1 - b^2 \alpha w \Delta \bar{u} - b^2 v_1 \Delta \bar{u}\} dx.
 \tag{30}$$

Now, one checks:

$$\begin{aligned}
 b^4 \int_{\Omega} u_2 \bar{u}_1 dx &= \int_{\Omega} (ib \bar{v}_1 + ib \bar{f})(-ib v_2) dx = b^2 \int_{\Omega} \bar{v}_1 v_2 dx + ib \int_{\Omega} \bar{f} (\Delta^2 u + \alpha \Delta w + \Delta v_1) dx \\
 &= b^2 \int_{\Omega} \bar{v}_1 v_2 dx + ib \int_{\Omega} \Delta u \Delta \bar{f} dx + \frac{ib}{\gamma} \int_{\Gamma} \partial_\nu u \partial_\nu \bar{f} d\Gamma + ib \int_{\Omega} (\alpha w + v_1) \Delta \bar{f} dx.
 \end{aligned}
 \tag{31}$$

Thanks to the Hölder and Young inequalities, one derives from (30)–(31):

$$b^2 |\Delta u|_2^2 + \frac{b^2}{\gamma} \int_{\Gamma} |\partial_\nu u|^2 d\Gamma \leq C_0 b^2 (|v_1|_2^2 + |v_2|_2^2 + |w|_2^2) + C_0 \|U\|_{\gamma}^2.
 \tag{32}$$

With the help of Young’s inequality, the claimed estimate (27) then follows from (17), (19), (22) and (26).

Thanks to Young’s inequality, it follows from Steps 2 and 3:

$$|b| \|Z\|_{\gamma} \leq \varepsilon |b| \|Z\|_{\gamma} + C_\varepsilon \|U\|_{\gamma}.
 \tag{33}$$

Gathering (12) and (33), and choosing  $\varepsilon = 1/4$ , one gets (11) for all nonzero  $b$  and all positive  $\gamma$ . The case  $b = 0$  is pretty straightforward.

### 3. Final remarks

We have used the resolvent estimate method to prove both the uniform analyticity and exponential decay of the semigroup  $(S_\gamma(t))_{t \geq 0}$ . Though that approach is best suited to the proof of analyticity, it is seldom used in the proof of exponential stability; this might be due to the fact that it does not adapt well to nonlinear systems, and many interesting distributed systems are nonlinear. Oftentimes, in the literature, the proof of exponential stability relies on energy integral estimates, e.g., [6], or on differential estimates of a perturbed energy, e.g., [17,19]. One of the advantages of the energy integral or differential estimate method is that either one works well for both linear and nonlinear systems. It is then natural to wonder whether, say, the perturbed energy method could be used to prove the uniform exponential decay of the energy of System (1). It can be attested beyond the shadow of a doubt that the perturbed energy method would work in the case  $\gamma \nearrow \infty$ . However, at the present time, the author of this note does not know whether it would work in the most interesting case  $\gamma \searrow 0$ ; the reason why the case  $\gamma \searrow 0$  is trickier than the case  $\gamma \nearrow \infty$  is due to two facts:

- (i) the limit system is a clamped plate, leading to a change of functional setting in the limit process,
- (ii) in solving the exponential stability problem for a clamped plate, one gets a boundary integral (this does not occur in the case of a hinged plate) that must be absorbed in order to conclude, e.g., [1,5,19]. The first-order multiplier that is used in this absorption process does not apply to the case of the perturbed system (1).

In conclusion, if the perturbed energy method were to work in the case  $\gamma \searrow 0$ , it would require the introduction of a new functional, which is beyond the scope of this note.

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