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Partial Differential Equations

Well-posedness of a low Mach number system

Caractère bien posé d'un modèle bas Mach

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A B S T R A C T

This Note deals with a short-time existence result for a system of nonlinear partial differential equations modelling a diphasic flow. The so-called Dlmn system is derived from the compressible Navier–Stokes equations under the assumption that the Mach number is small. A classical solution is obtained by means of a Picard iteration process. The proof of convergence relies on estimates associated to hyperbolic and parabolic equations. This procedure results in conditions on the time of existence of the solution.
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R É S U M É

Cette Note est consacrée à un résultat d'existence en temps court d'une solution classique à un système non-linéaire d'équations aux dérivées partielles. Ce système, appelé Dlmn, correspond à l'ordre 0 dans le développement asymptotique à bas nombre de Mach des équations de Navier–Stokes (adimensionnées). Afin de prouver l'existence d'une solution, on construit une suite de type itérées de Picard, dont la convergence repose sur des estimations associées aux équations hyperboliques et paraboliques présentes dans le système. Il en résulte des contraintes portant sur le temps d'existence de la solution.
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Version française abrégée

Le système Dlmn (1) a été introduit dans [1] dans le cadre de la modélisation d'écoulements à bulles, pour une application aux cœurs de réacteurs nucléaires. Dellacherie [1,2] a construit ce modèle en supposant que le nombre de Mach de l'écoulement était faible et en appliquant le concept de développement asymptotique de Majda [7]. La formulation (1) correspond au cas d'une viscosité constante et d'un domaine périodique $\Omega = \mathbb{T}^d$. L'objectif de cette Note est de présenter un résultat d'existence d'une solution classique à ce système d'EDP pour des données initiales de type Sobolev. L'énoncé de ce résultat est le suivant :

Théorème 0.1. *Soit s un entier tel que $s \geqslant s_0 + 3$ avec $s_0 = E(d/2) + 1$. Supposons que $(Y_0, T_0, p_0) \in \mathrm{H}^s(\mathbb{T}^d)$, que $\boldsymbol{u}_0 \in \mathrm{H}^{s-1}(\mathbb{T}^d)$ et que l'hypothèse de modélisation* (3) *est vérifiée pour $\xi \in \{\rho, c_p, \beta, \kappa, \Gamma\}$ et pour des lois d'état satisfaisant l'hypothèse* 1. *Il existe alors $\mathscr{T} > 0$ pour lequel le système* (1) *admet une unique solution classique $(Y, T, P, \boldsymbol{u}, \nabla\pi)$ sur $[0, \mathscr{T}]$.*

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La preuve de ce résultat repose sur une méthode de type itérées de Picard et sur des estimations d'énergie pour les équations paraboliques (cf. Lemme A.1). On linéarise le modèle selon (5). Une condition suffisante assurant le caractère borné des suites ainsi définies est donnée par une série de quatre contraintes non linéaires (7). Ces contraintes limitent alors le temps d'existence $\mathscr{T}$, lequel est d'autant plus petit que les normes de Sobolev des données initiales sont grandes. Un soin particulier est porté à l'explicitation des différentes constantes (se reporter à [10] pour plus de détails). La convergence des suites provient ensuite d'inégalités de contraction, sans contrainte additionnelle sur le temps d'existence.

Ce résultat s'étend au cas d'une viscosité variable [10] mais le cas de conditions aux limites non périodiques reste ouvert. De même, l'estimation du temps d'existence résulte de conditions suffisantes et peut ne pas être optimale. Une estimation pour un modèle similaire a été prouvée dans [12] mais sa non-optimalité a ensuite été prouvée en dimension 1 dans [11]. Ce résultat constitue toutefois une première approche théorique pour le système bas Mach non linéaire (1).

1. Introduction

In order to study bubbly flows, Dellacherie [1,2] introduced the DLMN system which models a diphasic compressible flow in a low Mach number regime. The derivation of this model is based on ideas from Majda [7]. A slightly different formulation has been presented in [12] due to simplifications with the use of the Leray decomposition $\boldsymbol{u} = \boldsymbol{w} + \nabla\phi$, $\nabla \cdot \boldsymbol{w} = 0$. The DLMN system thus reads in the case of constant viscosity μ_0, on a periodic domain Ω:

$$\begin{cases} \partial_t Y + \boldsymbol{u} \cdot \nabla Y = 0, \\ \rho c_p[\partial_t T + \boldsymbol{u} \cdot \nabla T] = \rho c_p \beta T \dfrac{P'(t)}{P(t)} + \nabla \cdot (\kappa \nabla T), \\ P'(t) = \mathcal{H}_\theta(t), \\ \boldsymbol{u} = \boldsymbol{w} + \nabla\phi, \qquad \Delta\phi = \mathcal{G}_\theta(t, \boldsymbol{x}), \qquad \nabla \cdot \boldsymbol{w} = 0, \\ \rho\big[\partial_t \boldsymbol{w} + (\boldsymbol{u} \cdot \nabla)\boldsymbol{w}\big] - \mu_0 \Delta \boldsymbol{w} = -\nabla\pi + \rho \boldsymbol{g} - \rho\big[\partial_t \nabla\phi + (\boldsymbol{u} \cdot \nabla)\nabla\phi\big]. \end{cases} \tag{1}$$

The unknowns in (1) are the velocity $\boldsymbol{u}$ decoupled in a free-divergence part $\boldsymbol{w}$ and a potential part $\nabla\phi$ (which would be zero in the incompressible limit), the mass fraction of the vapour phase Y, the temperature T, the thermodynamic pressure P depending only on t and the dynamic pressure π. The fact that there are two pressure variables is common in the low Mach number approach [7,9]. For the sake of clarity, we introduce the notation $\boldsymbol{\theta} := (Y, T, P)$. From a physical point of view, $\boldsymbol{\theta}$ lies in the set:

$$\Theta := \big\{\boldsymbol{\theta} \in \mathbb{R}^3\colon\ \theta_1 \in [0,1],\ \theta_2 > 0,\ \theta_3 > 0\big\}.$$

Other variables are given by equations of state (ρ, c_p, β, Γ) and constitutive laws (κ). Therefore, they may be considered as functions of $\boldsymbol{\theta}(t, \boldsymbol{x})$.

It is worth mentioning that all variables are global in the sense that a single system of PDEs is considered for both phases (mixture formulation). Other approaches may involve one system for each phase [5].

System (1) is supplemented with initial conditions:

$$\begin{cases} Y(0,\boldsymbol{x}) = Y_0(\boldsymbol{x}),\ T(0,\boldsymbol{x}) = T_0(\boldsymbol{x}),\ P(0) = p_0, \\ \boldsymbol{u}(0,\boldsymbol{x}) = \boldsymbol{u}_0(\boldsymbol{x}) = \boldsymbol{w}_0(\boldsymbol{x}) + \nabla\phi_0(\boldsymbol{x}), \quad \text{with } \nabla \cdot \boldsymbol{w}_0 = 0 \text{ and } \Delta\phi_0 = \mathcal{G}_{\boldsymbol{\theta}_0}. \end{cases} \tag{2}$$

This system is thus nonlinear not only due to the mathematical coupling between equations, but also due to the dependance with respect to $\boldsymbol{\theta}$ of unknowns and functionals like:

$$\mathcal{G}_\theta(t,\boldsymbol{x}) = -\frac{1}{\Gamma(\boldsymbol{\theta})}\frac{P'(t)}{P(t)} + \frac{\beta(\boldsymbol{\theta})\nabla \cdot [\kappa(\boldsymbol{\theta})\nabla T]}{P(t)}, \qquad \mathcal{H}_\theta(t) = \frac{\int_{\mathbb{T}^d} \beta(\boldsymbol{\theta})\nabla \cdot [\kappa(\boldsymbol{\theta})\nabla T]\,\mathrm{d}\boldsymbol{x}}{\int_{\mathbb{T}^d} [\Gamma(\boldsymbol{\theta})]^{-1}\mathrm{d}\boldsymbol{x}}.$$

This approach applies to very general equations of state in each phase. We suppose that the following assumptions hold:

Hypothesis 1.

(i) *Functions ρ_i, $c_{p,i}$ and Γ_i do not vanish on a nonempty open set $G_1 \subseteq \Theta$.*
(ii) *Functions ρ_i and $\frac{\kappa_i}{\rho_i c_{p,i}}$ are strictly positive on G_1.*
(iii) *Couple (T_0, p_0) takes values in a bounded convex set G such that $\overline{G} \subset G_1$.*

As G is open, there exists $\delta_G > 0$ such that if $|(T - T_0, P - p_0)|_\infty < \delta_G$, then $(T, P) \in G$.

Handling a diphasic flow implies that most variables are discontinuous at the interface between the two phases. Nevertheless, we are interested in this Note in classical solutions, which means that we shall consider in the sequel smooth initial data. This amounts to introducing a mixture area to ensure smooth transitions. We assume that each variable $\xi \in \{\rho, c_p, \beta, \kappa, \Gamma\}$ is expressed as:

$$\xi(Y, T, P) = \ell_\xi\big(Y, \xi_1(T, P), \xi_2(T, P)\big). \tag{3}$$

ξ_1 and ξ_2 denote the equations of state for the variable ξ in Phase 1 and Phase 2 respectively. The function ℓ_ξ models the shape of the transition between Phases 1 and 2 and satisfies modelling properties (symmetry, homogeneity) as well as mathematical assumptions (smoothness, positivity) [10, Def. 5.1]. For the sake of simplicity, we take the same function $\ell_\xi = \ell$ for each variable without any further difficulty insofar as their expressions are never used in the proof; they only provide sign and regularity specifications.

2. Statement of the existence result

The functional space associated to this problem is:

$$\mathcal{X}_{s,\mathscr{T}}\big(\mathbb{T}^d\big) = \mathscr{C}^0\big(0, \mathscr{T}, \mathrm{L}^2\big(\mathbb{T}^d\big)\big) \cap \mathrm{L}^\infty\big(0, \mathscr{T}, \mathrm{H}^s\big(\mathbb{T}^d\big)\big) \cap \mathrm{L}^2\big(0, \mathscr{T}, \mathrm{H}^{s+1}\big(\mathbb{T}^d\big)\big), \tag{4}$$

which is a Banach space when endowed with the norm

$$|||f|||^2_{s,\mathscr{T}} := \sup_{t\in[0,\mathscr{T}]} \big\|f(t,\cdot)\big\|_s^2 + \int_0^{\mathscr{T}} \big\|\nabla f(t,\cdot)\big\|_s^2 \,\mathrm{d}t.$$

Theorem 2.1. *Let s be an integer such that $s \geqslant s_0 + 3$, $s_0 = E(d/2) + 1$. Assume that $\boldsymbol{\theta}_0 \in \mathrm{H}^s(\mathbb{T}^d)$, $\boldsymbol{u}_0 \in \mathrm{H}^{s-1}(\mathbb{T}^d)$ and that* (3) *holds for $\xi \in \{\rho, c_p, \beta, \kappa, \Gamma\}$. Then, under Hypothesis* 1*, there exists $\mathscr{T} > 0$ for which System* (1) *has a unique classical solution $(Y, T, P, \boldsymbol{u}, \nabla\pi)$:*

- *$Y \in \mathcal{X}_{s-1,\mathscr{T}}(\mathbb{T}^d)$, $(T, P) \in \mathcal{X}_{s,\mathscr{T}}(\mathbb{T}^d)$, $\partial_t\boldsymbol{\theta} \in \mathcal{X}_{s-2,\mathscr{T}}(\mathbb{T}^d)$,*
- *$\boldsymbol{u} \in \mathcal{X}_{s-1,\mathscr{T}}(\mathbb{T}^d)$, $\partial_t\boldsymbol{u} \in \mathcal{X}_{s-3,\mathscr{T}}(\mathbb{T}^d)$,*
- *$\nabla\pi \in \mathcal{X}_{s-3,\mathscr{T}}(\mathbb{T}^d)$.*

Proof. The existence of a solution is shown by means of a Picard iterate method. The proof then splits into two parts (boundedness, contraction) similarly to [3,12].

$$\begin{cases} \partial_t Y^{(k+1)} + \boldsymbol{u}^{(k)} \cdot \nabla Y^{(k+1)} = 0, \\ \partial_t T^{(k+1)} + \boldsymbol{u}^{(k)} \cdot \nabla T^{(k+1)} - \dfrac{\kappa}{\rho c_p}\big(\boldsymbol{\theta}^{(k)}\big)\Delta T^{(k+1)} = \beta\big(\boldsymbol{\theta}^{(k)}\big)T^{(k)}\dfrac{[P^{(k)}]'(t)}{P^{(k)}(t)} + \dfrac{\nabla[\kappa(\boldsymbol{\theta}^{(k)})]\cdot\nabla T^{(k)}}{[\rho c_p](\boldsymbol{\theta}^{(k)})}, \\ \partial_t P^{(k+1)} + \boldsymbol{u}^{(k)} \cdot \nabla P^{(k+1)} - \Delta P^{(k+1)} = \mathcal{H}_{\theta^{(k)}}(t), \\ \boldsymbol{u}^{(k+1)} = \boldsymbol{w}^{(k+1)} + \nabla\phi^{(k+1)}, \qquad \Delta\phi^{(k+1)} = \mathcal{G}_{\theta^{(k+1)}}(t, \boldsymbol{x}), \qquad \nabla\cdot\boldsymbol{w}^{(k+1)} = 0, \\ \partial_t \boldsymbol{w}^{(k+1)} + \big(\boldsymbol{u}^{(k)}\cdot\nabla\big)\boldsymbol{w}^{(k+1)} - \dfrac{\mu_0}{\rho(\boldsymbol{\theta}^{(k)})}\Delta\boldsymbol{w}^{(k+1)} = -\dfrac{\nabla\pi^{(k+1)}}{\rho(\boldsymbol{\theta}^{(k)})} + \boldsymbol{g} - \partial_t\nabla\phi^{(k+1)} - \big(\boldsymbol{u}^{(k)}\cdot\nabla\big)\nabla\phi^{(k+1)}. \end{cases} \tag{5}$$

We first show that the sequences are bounded in $\mathcal{X}_{s,\mathscr{T}}(\mathbb{T}^d)$:

$$\big|\big|\big|Y^{(k)}\big|\big|\big|_{s-1,\mathscr{T}} \leqslant R_\theta, \qquad \big|\big|\big|\big(T^{(k)}, P^{(k)}\big)\big|\big|\big|_{s,\mathscr{T}} \leqslant R_\theta, \qquad \big|\big|\big|\boldsymbol{u}^{(k)}\big|\big|\big|_{s-1,\mathscr{T}} \leqslant R_u.$$

These boundedness inequalities are satisfied provided:

$$\begin{cases} \|Y_0\|_{s-1} < R_\theta, \qquad \big\|(T_0, p_0)\big\|_s < R_\theta, & \text{(6a)} \\ \|\boldsymbol{u}_0\|^2_{s-1} + C_5^\theta < R_u^2, & \text{(6b)} \end{cases}$$

and

$$\begin{cases} \|Y_0\|_s^2(1+\mathscr{T})e^{C_1 R_u\sqrt{\mathscr{T}}} \leqslant R_\theta^2, \\ \exp\big[C_2\mathscr{T}\big(C_1^\theta\big\{R_\theta^2 + e^{C_1 R_u\sqrt{\mathscr{T}}}\|Y_0\|_s^2\big\} + R_u^2 + 1\big)\big] \\ \quad\times\big[\big\|(T_0, p_0)\big\|_s^2 + C_3\mathscr{T}\big(C_2^\theta + C_3^\theta\big\{R_\theta^2 + e^{C_1 R_u\sqrt{\mathscr{T}}}\|Y_0\|_s^2\big\}\big)\big] \leqslant R_\theta^2, \\ C_4\mathscr{T}e^{C_1^{\theta,u}\mathscr{T}}\big(C_4^\theta + C_2^{\theta,u}\big\|(T_0, p_0)\big\|_s^2\big) \leqslant \delta_G^2, \\ e^{C_3^{\theta,u}\mathscr{T}}\big(\|\boldsymbol{u}_0\|^2_{s-1} + C_4^{\theta,u}\mathscr{T} + C_5^\theta\big) \leqslant R_u^2. \end{cases} \tag{7}$$

Constants C_i depend on s and d; C_i^θ on s, d and R_θ; $C_i^{\theta,u}$ on s, d, R_θ and R_u – see [10, Ch. 5] for the expressions of all constants. We first choose R_θ satisfying (6a), then R_u such that (6b) holds. We finally take $\mathscr{T}$ small enough to simultaneously satisfy (7). These estimates are based on product [8] and composition inequalities [4,6] and on Lemma A.1. We must underline that the four nonlinear constraints (7) are trivially verified for $\mathscr{T}=0$. The time interval is thus prescribed by the first inequality to become an equality for increasing $\mathscr{T}>0$.

The second part of the proof consists in showing a contraction inequality. Indeed, we have for $t\in[0,\mathscr{T}]$:

$$\mathcal{N}_k(t)\leqslant C\int_0^t \mathcal{N}_{k-1}(\tau)\,\mathrm{d}\tau + C^2\int_0^t \mathcal{N}_{k-2}(\tau)\,\mathrm{d}\tau,$$

for some constant $C>0$, where

$$\mathcal{N}_k(t)=\big|\!\big|\!\big|Y^{(k+1)}-Y^{(k)}\big|\!\big|\!\big|_{2,t}^2+\big|\!\big|\!\big|T^{(k+1)}-T^{(k)}\big|\!\big|\!\big|_{3,t}^2+\big|\!\big|\!\big|P^{(k+1)}-P^{(k)}\big|\!\big|\!\big|_{3,t}^2+\big|\!\big|\!\big|\boldsymbol{u}^{(k+1)}-\boldsymbol{u}^{(k)}\big|\!\big|\!\big|_{2,t}^2.$$

This inequality holds due to the fact that the exponents given by Lemma A.1 for each difference term may be bounded by a common term. We infer from Lemma B.1 that each of $(Y^{(k)})$, $(T^{(k)})$, $(P^{(k)})$ and $(\boldsymbol{u}^{(k)})$ are Cauchy sequences. As $\mathcal{X}_{3,\mathscr{T}}(\mathbb{T}^d)$ and $\mathcal{X}_{2,\mathscr{T}}(\mathbb{T}^d)$ are Banach spaces, we deduce the convergence of the previous sequences in the latter spaces.

On the one hand, by the interpolation inequality [8], $(Y^{(k)})$ converges in each $\mathcal{X}_{s',\mathscr{T}}(\mathbb{T}^d)$, $s'\in\{2,\dots,s-2\}$ towards $\tilde{Y}\in\mathcal{X}_{s-2,\mathscr{T}}(\mathbb{T}^d)$. On the other hand, the boundedness property and the weak compactness of $\mathcal{X}_{s-1,\mathscr{T}}(\mathbb{T}^d)$ show that $(Y^{(k)})$ converges weakly-$\star$ to $Y\in\mathcal{X}_{s-1,\mathscr{T}}(\mathbb{T}^d)$. As the weak-$\star$ convergence also holds in $\mathcal{X}_{s',\mathscr{T}}(\mathbb{T}^d)$ and the limit being unique, we deduce that $Y=\tilde{Y}\in\mathcal{X}_{s-1,\mathscr{T}}(\mathbb{T}^d)$. The same reasoning also applies to the other sequences.

In the last step, we show that these limits are solutions to the DLMN system (1). This is a consequence of the passing to the limit as $k\to+\infty$ in the integral form of the iterative scheme (5). □

3. Conclusion

The existence result presented in this Note corresponds to a first step in the theoretical study of the DLMN system. Although the outline of the proof is classical, it is applied to a highly nonlinear system of coupled PDEs and required the combination of several a priori estimates and functional inequalities. A special care has been taken to constants appearing in these inequalities as well as to the constraints involving the time of existence. Time $\mathscr{T}$ prescribed by inequalities (7) is not said to be optimal but provides a first estimate of the interval on which a solution does exist. Indeed, the similar estimate shown in [12] has been proven not to be optimal in 1D in [11]. However, it must be underlined that unlike other similar results, the contraction step does not provide any additional constraint on $\mathscr{T}$.

Theorem 2.1 easily extends to the variable viscosity case where higher order terms are added (see [10, § 5.3.5] for further details). The next step will consist in dealing with weak solutions insofar as the underlying physical framework includes nonsmooth (bounded) solutions. This will however require another strategy to prove the existence of solutions. It also remains to show the continuous dependance of the solution with respect to initial data in order to achieve the well-posedness of the system.

Appendix A. Advection–diffusion equations

Lemma A.1. *(See [10, p. 47].) Let* $\boldsymbol{w}_0\in \mathrm{H}^s(\mathbb{T}^d)$, $\boldsymbol{u}\in\mathcal{X}_{s-1,\mathscr{T}}(\mathbb{T}^d)$, $\boldsymbol{f}\in\mathcal{X}_{s-2,\mathscr{T}}(\mathbb{T}^d)$, $a\in\mathcal{X}_{s-1,\mathscr{T}}(\mathbb{T}^d)$ *and* $a_1\in\,]0,1[$ *such that*: $\forall(t,\boldsymbol{x})\in\,]0,+\infty[\,\times\mathbb{T}^d$, $a_1\leqslant a(t,\boldsymbol{x})\leqslant a_1^{-1}$. *Assume* $s\geqslant s_0+2$, $s_0=E(d/2)+1$. *Then, the Cauchy problem*:

$$\begin{cases}\dfrac{\partial\boldsymbol{w}}{\partial t}+(\boldsymbol{u}\cdot\nabla)\boldsymbol{w}-a\Delta\boldsymbol{w}=\boldsymbol{f},\\ \boldsymbol{w}(t=0,\cdot)=\boldsymbol{w}_0,\end{cases}$$

has a unique classical solution $\boldsymbol{w}\in\mathcal{X}_{s,\mathscr{T}}(\mathbb{T}^d)$ *satisfying the energy estimate for all* $r\in\{0,\dots,s\}$:

$$\|\boldsymbol{w}\|_r^2+e^{\zeta_r(\mathscr{T})}\int_0^{\mathscr{T}} e^{-\zeta_r(\tau)}\big\|\nabla\boldsymbol{w}(\tau,\cdot)\big\|_r^2\,\mathrm{d}\tau\leqslant e^{\zeta_r(\mathscr{T})}\left(\|\boldsymbol{w}_0\|_r^2+C_*(d,a_1)\int_0^{\mathscr{T}} e^{-\zeta_r(\tau)}\big\|\boldsymbol{f}(\tau,\cdot)\big\|_{r-1}^2\,\mathrm{d}\tau\right)$$

and every function ζ_r *such that*:

$$\zeta_r'(t)\geqslant\bar{\zeta}_r'(t)=C_{**}(r,d,a_1)\big(\big\|\boldsymbol{u}(t,\cdot)\big\|_{\max(r-2,s_0)+1}^2+\big\|\nabla a(t,\cdot)\big\|_{\max(r-1,s_0)}^2+1\big).$$

Appendix B. Induction inequalities

Lemma B.1. *Let* $(\mathcal{N}_k)$ *be a sequence such that*: $\mathcal{N}_k(t)\leqslant C\int_0^t\mathcal{N}_{k-1}(\tau)\,\mathrm{d}\tau+C^2\int_0^t\mathcal{N}_{k-2}(\tau)\,\mathrm{d}\tau$. *Then*:

$$\mathcal{N}_k(t)\leqslant C^{k-1}\mathrm{Q}_k(t)+C^k\int_0^t \mathrm{Q}_{k-1}(t-\tau)\,\mathrm{d}\tau,\qquad \mathrm{Q}_k=\sum_{j=E(k/2)}^{k-1}\frac{X^j}{(k-j-1)!(2j-k+1)!},$$

and the series $\sum\mathcal{N}_k$ *is convergent.*

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