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Probability Theory

Comparison theorem for Brownian multidimensional BSDEs via jump processes

Théorème de comparaison pour EDSR multidimensionnelles browniennes par processus à sauts

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ABSTRACT

In this Note, we provide an original proof of the comparison theorem for multidimensional Brownian BSDEs in the case where at each line k the generator depends on the matrix variable Z only through its row k.

RÉSUMÉ

Dans cette Note, nous donnons une preuve originale du théorème de comparaison pour les EDSR multidimensionnelles browniennes dans le cas où chaque ligne k du générateur ne dépend que de la k-ième ligne de l'inconnue Z.

Version française abrégée

Le théorème de comparaison pour les EDS rétrogrades (EDSR en abrégé) est un résultat qui permet de comparer les solutions de deux EDSR dès qu'il est possible de comparer leurs conditions terminales et leurs générateur. Ce résultat a été initialement prouvé dans le cas unidimensionnel brownien par S. Peng [3].

Dans le cas brownien multidimensionnel, Y. Hu et S. Peng [2] ont donné une condition nécessaire et suffisante pour obtenir le théorème de comparaison, en utilisant les résultats de viabilité donné dans [1].

Nous proposons ici une nouvelle méthode pour montrer le théorème de comparaison dans le cas multidimensionnel. Nous établissons un lien entre EDSR multidimensionnelles browniennes et EDSR à sauts et nous utilisons le résultat de comparaison pour ces dernières donné par M. Royer [4]. Nous obtenons le résultat suivant :

Théorème *(Théorème de comparaison). Supposons que* (H1)–(H4) *soient vérifiées et que* f_1 *ou* f_2 *satisfont* (H5)*. Supposons également que* $\mathbb{P}$*-p.s.* $\xi_1 \succeq \xi_2$ *et* $f_1 \succeq f_2$*. Alors si* $(Y_i, Z_i) \in S^2_{\mathbb{F}} \times L^2_{\mathbb{F}}$ *est solution de* (1) *pour* $i = 1, 2$*, nous avons* $Y_1(t) \succeq Y_2(t)$*,* $0 \leqslant t \leqslant T$*,* $\mathbb{P}$*-p.s.*

Les hypothèses (H1)–(H5) et l'équation (1) sont présentées en Section 2.

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1. Introduction

The comparison theorem for Backward SDEs (BSDEs for short) is a result which allows one to compare the solutions of two BSDEs whenever you can compare their terminal conditions and generators. Although the comparison theorem holds true for Lipschitz BSDEs in the Brownian one-dimensional case (see [3]), it needs stronger assumptions in the other cases.

In the Brownian multidimensional case, Hu and Peng [2] give a necessary and sufficient condition for the comparison theorem. Their result is based on the viability property for BSDEs studied in Buckdahn et al. [1] where the authors give a necessary and sufficient condition to ensure that the component Y stays in a given set. However, this approach uses analytical arguments and hence imposes a continuity assumption on the generator w.r.t. the time variable t.

In this Note, we provide a comparison theorem for Lipschitz multidimensional Brownian BSDEs whose generator depends at each line k on the variable Z only through its line k:

$$f^k(t,y,z) = f^k\big(t,y,z^k\big), \quad (t,y) \in [0,T] \times \mathbb{R}^n,\ z = \begin{pmatrix} z^1 \\ \vdots \\ z^n \end{pmatrix} \in \mathbb{R}^{n\times d}.$$

For this, we introduce a random measure and show that the process constructed by choosing the component w.r.t. this random measure is solution to a BSDE with jumps. We then use existing comparison results for BSDES with jumps to state our main result. An important feature is that our result holds without supposing any continuity assumption on the generator w.r.t. the time variable t.

The rest of the Note is organized as follows. In the next section we give the framework and state our result. In Section 3 we recall under which assumptions the comparison theorem for BSDEs with jumps holds. The last section is dedicated to the proof of our main result.

2. The comparison theorem for multidimensional BSDEs

Let $(\Omega, \mathcal{A}, \mathbb{P})$ be a probability space endowed with a standard d-dimensional Brownian motion $(W(t))_{t\geqslant 0}$ and let $\mathbb{F} = (\mathcal{F}(t))_{t\geqslant 0}$ be the $\mathbb{P}$-completion of the filtration generated by $(W(t))_{t\geqslant 0}$. We fix a terminal time $T>0$ and an integer $n \geqslant 1$. Throughout this Note, we denote by

- $S^2_{\mathbb{F}}$ the set of $\mathbb{F}$-adapted continuous processes Y valued in $\mathbb{R}^n$ such that
$$\|Y\|_{S^2} := \mathbb{E}\Big[\sup_{t\in[0,T]} \big|Y(t)\big|^2\Big] < \infty,$$
- $L^2_{\mathbb{F}}$ the set of $\mathbb{F}$-predictable processes Z valued in $\mathbb{R}^{n\times d}$ such that
$$\|Z\|_{L^2} := \mathbb{E}\left[\int_0^T \big|Z(t)\big|^2\,\mathrm{d}t\right] < \infty.$$

We then consider two functions f_1 and f_2 from $\Omega \times [0,T] \times \mathbb{R}^n \times \mathbb{R}^{d\times n}$ to $\mathbb{R}^n$, which are $\mathbb{F}$-progressive $\otimes\mathcal{B}(\mathbb{R}^n)\otimes\mathcal{B}(\mathbb{R}^{n\times d})$-measurable, and two random variables ξ_1 and ξ_2 which are $\mathcal{F}_T$-measurable.

We introduce the following assumptions:

(H1) ξ_1 and ξ_2 are square integrable: $\mathbb{E}[|\xi_i|^2] < \infty$, $i = 1,2$,
(H2) $f_1(\cdot,0,0)$ and $f_2(\cdot,0,0)$ are square integrable:
$$\mathbb{E}\left[\int_0^T \big|f_i(t,0,0)\big|^2\,\mathrm{d}t\right] < \infty, \quad i = 1,2,$$
(H3) there exists a constant L such that $\mathbb{P}$-a.s. we have
$$\big|f_i(t,y,z) - f_i\big(t,y',z'\big)\big| \leqslant L\big(\big|y-y'\big| - \big|z-z'\big|\big), \quad i = 1,2,$$
for all $(t,y,y',z,z') \in [0,T] \times [\mathbb{R}^n]^2 \times [\mathbb{R}^{n\times d}]^2$.

For $i = 1,2$, we consider the BSDE

$$Y_i(t) = \xi_i + \int_t^T f_i\big(s, Y_i(s), Z_i(s)\big)\,\mathrm{d}s - \int_t^T Z_i(s)\,\mathrm{d}W(s), \quad 0 \leqslant t \leqslant T. \tag{1}$$

We denote by $\succeq$ the partial ordering relation on $\mathbb{R}^n$: for $y_1, y_2 \in \mathbb{R}^n$, we have $y_1 \succeq y_2$ iff $y_1^k \geqslant y_2^k$, for all $k = 1, \dots, n$. We also need an additional assumption on the form of the dependence of the generator f w.r.t. the unknown Z.

(H4) f_1 and f_2 depend on the last variable z at each line k only through the line k of z:

$$f_i^k(t, y, z) = f_i^k\big(t, y, z^k\big), \quad i = 1, 2, \text{ for all } (t, y) \in [0, T] \times \mathbb{R}^n \text{ and } z = \begin{pmatrix} z^1 \\ \vdots \\ z^n \end{pmatrix} \in \mathbb{R}^{d \times n}.$$

Finally, we introduce an assumption which can be interpreted as a monotonicity condition of the generator w.r.t. the variable y.

(H5) f_i satisfies the following monotonicity condition: there exit two constants $C_2 > C_1 > 0$ and a map δ from $\Omega \times [0, T] \times [\mathbb{R}^n]^2 \times \mathbb{R}^{n\times d}$ to $[C_1, C_2]^k$ which is $\mathbb{F}$-predictable $\otimes\mathcal{B}(\mathbb{R}^n) \otimes \mathcal{B}(\mathbb{R}^n) \otimes \mathcal{B}(\mathbb{R}^{n\times d})$ such that

$$f_i^\ell(t, y, z) - f_i^\ell\big(t, y', z\big) \leqslant \sum_{k\in\mathcal{K}} \delta_k\big(t, y, y', z\big)\big(y^k - y'^k\big), \quad \ell = 1, \dots, n,$$

for all $(t, y, y', z) \in [0, T] \times [\mathbb{R}^n]^2 \times \mathbb{R}^{n\times d}$.

We provide an example of a generator satisfying the previous assumption.

Example 1. Suppose that f_i admits partial derivatives w.r.t. y_k, $k = 1, \dots, n$, such that

$$\frac{\partial f_i^\ell}{\partial y^k}(t, y, z) \in [C_1, C_2], \quad k, \ell = 1, \dots, n, \tag{2}$$

for all $(t, y, z) \in [0, T] \times \mathbb{R}^n \times \mathbb{R}^{n\times d}$, then f_i satisfy (H5). Indeed, we have

$$f_i^\ell(t, y, z) - f_i^\ell\big(t, y', z\big) = \sum_{k=1}^n f_i^\ell\big(t, y'^1, \dots, y'^{k-1}, y^k, \dots, y^n, z\big) - f_i^\ell\big(t, y'^1, \dots, y'^k, y^{k+1}, \dots, y^n, z\big).$$

Then using the mean value theorem and (2), we get

$$f_i^\ell(t, y, z) - f_i^\ell\big(t, y', z\big) = \sum_{k=1}^n \delta^k\big(t, y, y', z\big)\big(y^k - y^{k'}\big),$$

with $\delta^k(t, y, y', z) \in [C_1, C_2]$. Hence (H5) is satisfied.

Example 2. Notice that we can extend the previous example to the case where the generator f is less regular. To this end we replace condition (2) by the following one

$$\frac{f_i^\ell(t, y^1, \dots, y^{k-1}, x, y^{k+1}, \dots, y^n, z) - f_i^\ell(t, y^1, \dots, y^{k-1}, x', y^{k+1}, \dots, y^n, z)}{x - x'} \in [C_1, C_2], \tag{3}$$

for all $x, x' \in \mathbb{R}$ such that $x \neq x'$.

If $f_i^\ell(t, y^1, \dots, y^{k-1}, \cdot, y^{k+1}, \dots, y^n, z)$ satisfy (3) for all $(t, y^1, \dots, y^{k-1}, y^{k+1}, \dots, y^n, z) \in [0, T] \times \mathbb{R}^{n-1} \times \mathbb{R}^{n\times d}$, and all $\ell = 1, \dots, n$, then f_i satisfy (H5). Indeed, we write as previously

$$f_i^\ell(t, y, z) - f_i^\ell\big(t, y', z\big) = \sum_{k=1}^n f_i^\ell\big(t, y'^1, \dots, y'^{k-1}, y^k, \dots, y^n, z\big) - f_i^\ell\big(t, y'^1, \dots, y'^k, y^{k+1}, \dots, y^n, z\big).$$

Then from (3) we have

$$f_i^\ell\big(t, y'^1, \dots, y'^{k-1}, y^k, \dots, y^n, z\big) - f_i^\ell\big(t, y'^1, \dots, y'^k, y^{k+1}, \dots, y^n, z\big) \leqslant (C_1 \mathbb{1}_{y^k < y'^k} + C_2 \mathbb{1}_{y'^k \leqslant y^k})\big(y^k - y'^k\big).$$

Taking $\delta^k(t, y, y', z) = C_1 \mathbb{1}_{y^k < y'^k} + C_2 \mathbb{1}_{y'^k \leqslant y^k}$ we get (H5).

We now state the main result of this Note:

Theorem 2.1 *(Comparison theorem). Suppose that* (H1)–(H4) *hold and that f_1 or f_2 satisfies* (H5)*. Suppose also that $\mathbb{P}$-a.s. $\xi_1 \succeq \xi_2$ and $f_1 \succeq f_2$. Then if $(Y_i, Z_i) \in S^2_{\mathbb{F}} \times L^2_{\mathbb{F}}$ is solution of* (1) *for $i = 1, 2$, we have $Y_1(t) \succeq Y_2(t)$, $0 \leqslant t \leqslant T$, $\mathbb{P}$-a.s.*

Remark 1. An important feature of our result is that it relaxes the continuity assumption of the generator w.r.t. the time variable t used in [2].

Remark 2. Under assumptions (H4) and (H5), the necessary and sufficient condition given by Theorem 2.1 in [2] is satisfied. Indeed, suppose w.l.o.g. that (H5) holds true for f_1. Since $f_1 \succeq f_2$, we have

$$\begin{aligned}-4\langle y^-, f_1(t, y^+ + y', z) - f_2(t, y', z')\rangle &\leqslant -4\langle y^-, f_1(t, y^+ + y', z) - f_1(t, y', z')\rangle \\ &\leqslant -4\sum_{k=1}^n [y^k]^- \big(f_1^k(t, y^+ + y', z^k) - f_1^k(t, y', z^k)\big) - 4\sum_{k=1}^n [y^k]^- \big(f_1^k(t, y', z^k) - f_1^k(t, y', z'^k)\big).\end{aligned}$$

From the Lipschitz property of f_1 and the inequality $ab \leqslant \frac{1}{2}a^2 + 2b^2$ for $a, b \in \mathbb{R}$, we get

$$-4\sum_{k=1}^n [y^k]^- \big(f_1^k(t, y', z^k) - f_1^k(t, y', z'^k)\big) \leqslant 2\sum_{k=1}^n \mathbb{1}_{y^k<0}\big|z^k - z'^k\big|^2 + 8\big|y^-\big|^2. \tag{4}$$

Then using assumption (H5) we have

$$-4\langle y^-, f_1(t, y^+ + y', z) - f_1(t, y', z)\rangle \leqslant 0. \tag{5}$$

Combining (4) and (5) we get

$$-4\langle y^-, f_1(t, y^+ + y', z) - f_2(t, y', z')\rangle \leqslant 2\sum_{k=1}^n \mathbb{1}_{y^k<0}\big|z^k - z'^k\big|^2 + 8\big|y^-\big|^2,$$

which is the necessary and sufficient condition for comparison given in [2].

3. Comparison for BSDEs with jumps

We consider a Poisson random measure μ on $\mathbb{R}_+ \times \mathcal{K}$ with $\mathcal{K} := \{1, \dots, n\}$. We assume that this random measure μ is independent of $(W(t))_{t\geqslant 0}$. We denote by $\mathbb{G} = (\mathcal{G}(t))_{t\geqslant 0}$ the $\mathbb{P}$-augmentation of the filtration generated by $\mathbb{F}$ and μ. We suppose that μ is of the form

$$\mu([0,t] \times B) = \sum_{\ell\geqslant 1} \mathbb{1}_{[0,t]\times B}(\tau_\ell, \zeta_\ell), \quad t \geqslant 0,\ B \subset \mathcal{K}, \tag{6}$$

where $(\tau_\ell)_{\ell\geqslant 1}$ is nondecreasing sequence of $\mathbb{G}$-stopping times and ζ_ℓ is a $\mathcal{G}(\tau_\ell)$-measurable random variable for each $\ell \geqslant 1$.

We assume that μ admits a compensator of the form $\lambda(k)\,\mathrm{d}t$ with $\lambda(k) > 0$ for all $k \in \mathcal{K}$ and we denote by $\tilde{\mu}$ the compensated measure associated to μ:

$$\tilde{\mu}([0,t] \times B) = \mu([0,t] \times B) - t\sum_{k\in\mathcal{K}} \lambda(k)\mathbb{1}_B(k), \quad t \geqslant 0,\ B \subset \mathcal{K}.$$

We then consider the following spaces:

- $S^2_{\mathbb{G}}$ the set of $\mathbb{G}$-adapted *càdlàg* processes Y valued in $\mathbb{R}$ such that $\|Y\|_{S^2} < \infty$;
- $L^2_{\mathbb{G}}$ the set of $\mathbb{G}$-predictable processes Z valued in $\mathbb{R}^d$ such that $\|Z\|_{L^2} < \infty$;
- $L^2_{\mathbb{G},\lambda}$ the set of processes U from $\Omega \times [0,T] \times \mathcal{K}$ to $\mathbb{R}$ such that $U(k,.)$ is $\mathbb{G}$-predictable for all $k \in \mathcal{K}$ and

$$\|U\|_{L^2_\lambda} := \mathbb{E}\left[\int_0^T \sum_{k\in\mathcal{K}} \big|U(t,k)\big|^2 \lambda(k)\,\mathrm{d}t\right] < \infty.$$

We consider two functions g_1 and g_2 from $\Omega \times [0,T] \times \mathbb{R} \times \mathbb{R}^d \times \mathbb{R}^n$ to $\mathbb{R}^n$, which are $\mathbb{G}$-progressive $\otimes\mathcal{B}(\mathbb{R})\otimes\mathcal{B}(\mathbb{R}^d)\otimes\mathcal{B}(\mathbb{R}^n)$-measurable, and two random variables η_1 and η_2 which are $\mathcal{G}_T$-measurable.

We then introduce the following assumptions:

(H′1) η_1 and η_2 are square integrable: $\mathbb{E}[|\eta_i|^2] < \infty$, $i = 1, 2$,
(H′2) $g_1(\cdot,0,0,0)$ and $g_2(\cdot,0,0,0)$ are square integrable:

$$\mathbb{E}\left[\int_0^T \big|g_i(t,0,0,0)\big|^2\,\mathrm{d}t\right] < \infty, \quad i = 1, 2,$$

(H′3) there exists a constant L such that $\mathbb{P}$-a.s. we have

$$\left|g_i(t,y,z,u)-g_i\big(t,y',z',u'\big)\right|\leqslant L\big(\left|y-y'\right|+\left|z-z'\right|+\left|u-u'\right|\big),\quad i=1,2,$$

for all $(y,y',z,z',u,u')\in[\mathbb{R}]^2\times[\mathbb{R}^d]^2\times[\mathbb{R}^n]^2$.

For $i=1,2$, we consider the BSDE with jumps

$$\mathcal{Y}_i(t)=\eta_i+\int_t^T g_i\big(s,\mathcal{Y}_i(s),\mathcal{Z}_i(s),\mathcal{U}_i(s,\cdot)\big)\,\mathrm{d}s-\int_t^T\mathcal{Z}(s)\,\mathrm{d}W(s)-\int_t^T\int_{\mathcal{K}}\mathcal{U}(s,k)\tilde{\mu}(\mathrm{d}k,\mathrm{d}s),\quad 0\leqslant t\leqslant T. \tag{7}$$

We finally introduce a monotonic assumption of the generator used by Royer [4] to get comparison result for BSDEs with jumps.

(H′4) There exist two constants $C_4\geqslant C_3>-1$ and a map γ from $\Omega\times[0,T]\times\mathcal{K}\times\mathbb{R}\times\mathbb{R}^d\times[\mathbb{R}^n]^2$ to $[C_3,C_4]$ which is $\mathbb{G}$-predicable $\otimes\mathcal{B}(\mathbb{R})\otimes\mathcal{B}(\mathbb{R}^d)\otimes\mathcal{B}(\mathbb{R}^n)\otimes\mathcal{B}(\mathbb{R}^n)$-measurable such that $\mathbb{P}$-a.s.

$$g_i(t,y,z,u)-g_i\big(t,y,z,u'\big)\leqslant\int_{\mathcal{K}}\gamma^{y,z,u,u'}(t,k)\big(u(k)-u'(k)\big)\lambda(\mathrm{d}k),$$

for all $(t,y,z,u,u')\in[0,T]\times\mathcal{I}\times\mathbb{R}\times\mathbb{R}^d\times[\mathbb{R}^n]^2$.

Under the previous assumptions we have the following comparison theorem (see Theorem 2.5 in [4]):

Theorem 3.1. *Suppose that* (H′1)–(H′3) *hold and that* g_1 *or* g_2 *satisfies* (H′4)*. Suppose also that* $\mathbb{P}$*-a.s.* $\eta_1\geqslant\eta_2$ *and* $g_1\geqslant g_2$*. Then if* $(\mathcal{Y}_i,\mathcal{Z}_i,\mathcal{U}_i)\in S^2_{\mathbb{G}}\times L^2_{\mathbb{G}}\times L^2_{\mathbb{G},\lambda}$ *is solution of* (7) *for* $i=1,2$*, we have*

$$\mathcal{Y}_1(t)\geqslant\mathcal{Y}_2(t),\quad 0\leqslant t\leqslant T,\ \mathbb{P}\text{-a.s.}$$

4. Proof of Theorem 2.1

Let $(Y_i,Z_i)\in S^2_{\mathbb{F}}\times L^2_{\mathbb{F}}$ be solution to (1) for $i=1,2$ and $k\in\mathcal{K}$. To prove that ${}^k_1(t)\geqslant Y^k_2(t)$, $0\leqslant t\leqslant T$, $\mathbb{P}$-a.s. we proceed in three steps.

Step 1. *Construction of jump processes.* Take μ a Poisson random measure on $\mathbb{R}_+\times\mathcal{K}$ of the form (6), independent of $(W(t))_{t\geqslant0}$ with intensity λ defined by $\lambda(k)=2L$ for all $k\in\mathcal{K}$, where L is the Lipschitz constant of f_i, $i=1,2$. Consider the process $(N(t))_{t\geqslant0}$ defined by

$$N(t)=k+\int_0^t\int_{\mathcal{K}}\big(k-N(t-)\big)\mu(\mathrm{d}k,\mathrm{d}t),\quad 0\leqslant t\leqslant T.$$

Notice that since μ has the form (6), N is the process taking the value ζ_ℓ on $[\tau_\ell,\tau_{\ell+1})$. Define also, for $i=1,2$, the processes $\mathcal{Y}_i$, $\mathcal{Z}_i$ and $\mathcal{U}_i$ by

$$\mathcal{Y}_i(t)=Y_i^{N(t)}(t),\qquad\mathcal{Z}_i(t)=Z_i^{N(t-)}(t),\qquad\mathcal{U}_i(t,k)=\big(Y_i^k(t)-Y_i^{N(t-)}(t-)\big)_{k\in\mathcal{K}},\quad 0\leqslant t\leqslant T \tag{8}$$

(at time t, the component is chosen as the value given by the process N). Since $(Y_i,Z_i)\in S^2_{\mathbb{F}}\times L^2_{\mathbb{F}}$, we easily check that $(\mathcal{Y}_i,\mathcal{Z}_i,\mathcal{U}_i)\in S^2_{\mathbb{G}}\times L^2_{\mathbb{G}}\times L^2_{\mathbb{G},\lambda}$ for $i=1,2$. A straightforward computation shows that, for $i=1,2$, $(\mathcal{Y}_i,\mathcal{Z}_i,\mathcal{U}_i)$ satisfies (7) with $\eta_i=\xi_i^{N(T)}$ and

$$g_i(t,y,z,u)=f_i^{N(t)}\big(t,y+u(1),\dots,y+u(n),z\big)-\sum_{k\in\mathcal{K}}u(k)\lambda(k), \tag{9}$$

for $(t,y,z,u)\in[0,T]\times\mathbb{R}\times\mathbb{R}^d\times\mathbb{R}^n$.

Step 2. *Comparison of the jump processes.* Notice that since $\xi_1\succeq\xi_2$ and $f_1\succeq f_2$ we have $\eta_1\geqslant\eta_2$ and $g_1\geqslant g_2$. Notice also that since f_i satisfy (H1)–(H3), g_i satisfy (H′1)–(H′3) for $i=1,2$. We prove that g_i defined by (9), satisfy (H′4). Indeed, for $(t,y,z,u,u')\in[0,T]\times\mathbb{R}\times\mathbb{R}^d\times[\mathbb{R}^n]^2$ we have from (H5) and (9)

$$g_i(t,y,z,u) - g_i\big(t,y,z,u'\big) \leqslant \sum_{k\in\mathcal{K}} \delta^k\big(t, y+u, y+u', z\big)\big(u(k)-u'(k)\big) - \sum_{k\in\mathcal{K}}\big(u(k)-u'(k)\big)\lambda(k)$$
$$\leqslant \sum_{k=1}^{n} \gamma^{y,z,u,u'}(t,k)\big(u(k)-u'(k)\big)\lambda(k)$$

with $\gamma^{y,z,u,u'}(t,k) = \frac{\delta^k(t,y+u,y+u',z)}{\lambda(k)} - 1 \in [C_3, C_4]$ with $C_4 = \frac{C_1}{\min_k \lambda(k)} - 1$ and $C_3 = \frac{C_1}{\max_k \lambda(k)} - 1 > -1$. Hence g_i satisfy (H′4) and we have

$$\mathcal{Y}_1(t) \geqslant \mathcal{Y}_2(t), \quad 0 \leqslant t \leqslant T,\ \mathbb{P}\text{-a.s.} \tag{10}$$

Step 3. *Back to the continuous processes.* Recall that the random measure is of the form

$$\mu\big([0,t]\times B\big) = \sum_{\ell\geqslant 1} \mathbb{1}_{[0,t]\times B}(\tau_\ell, \zeta_\ell), \quad t \geqslant 0,\ B \subset \mathcal{K},$$

where $(\tau_\ell)_\ell$ is an increasing sequence of $\mathbb{G}$-stopping times and ζ_ℓ in a $\mathcal{G}_{\tau_\ell}$-measurable r.v. valued in $\mathcal{K}$. Using (10), we have

$$Y_1^k(t) = \mathbb{E}\big[\mathcal{Y}_1(t)\big|\mathcal{F}(t) \vee \big\{N(t)=k\big\}\big] \geqslant \mathbb{E}\big[\mathcal{Y}_2(t)\big|\mathcal{F}(t) \vee \big\{N(t)=k\big\}\big] = Y_2^k(t), \quad \mathbb{P}\text{-a.s.}$$

for all $t \in [0,T]$. Since the processes Y_i, $i = 1, 2$, are continuous and k is arbitrary fixed we get the result.

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