C. R. Acad. Sci. Paris, Ser. I 348 (2010) 1039–1043

Contents lists available at ScienceDirect

C. R. Acad. Sci. Paris, Ser. I

www.sciencedirect.com

Numerical Analysis

Estimates of the modeling error for the Kirchhoff–Love plate model

Estimation de l'erreur de modélisation pour le modèle de plaque de Kirchhoff–Love

Sergey Repin [a], Stefan A. Sauter [b]

[a] *St. Petersburg Department of V.A. Steklov Institute of Mathematics, Fontanka 27, 191 023 St. Petersburg, Russia*
[b] *Institut für Mathematik, Universität Zürich, Winterthurerstrasse 190, CH-8057 Zürich, Switzerland*

ARTICLE INFO

Article history:
Received 5 November 2009
Accepted 3 September 2010

Presented by Philippe G. Ciarlet

ABSTRACT

In this Note we consider the Kirchhoff–Love model for approximating problems in linear elasticity on thin plates under certain hypotheses. We will present computable error majorants for the arising modelling error. The majorant for the relative error converges with a rate $O(h^{1/2})$ in the thickness parameter h provided that the KL solution possesses extra reqularity.

RÉSUMÉ

Dans ce travail on considère le modèle de Kirchhoff–Love pour approcher les problèmes de plaques minces sous certaines conditions. Nous présentons des majorants d'erreur calculables. La borne de l'erreur relative converge comme $O(h^{1/2})$ en terme du paramètre d'épaisseur h pourvu que la KL solution ait un peu plus de régularité.

Version française abrégée

Dans ce travail nous nous intéressons à l'erreur commise lorsque l'on remplace un problème tridimenssionnel d'élasticité linéaire pour des objets minces par le modèle de Kirchhoff–Love (voir [7]). Plus précisément soit $\hat{\omega}$ le déplacement scalaire de la surface moyenne ω d'un corps Ω solution d'une équation biharmonique en dimension 2. Dans le modèle $(1,1,2)$ on reconstruit le déplacement tridimensionnel $\mathbf{v}_{\mathrm{KL}}^{112}$ en utilisant $\hat{w}$ et une fonction auxiliaire $\hat{W}$ qui incorpore les différentes couches dans $\mathbf{v}_{\mathrm{KL}}^{112}$. L'erreur peut être estimée par notre nouveau majorant comme suit. Soit $\Omega = \omega \times {]{-h/2}, h/2[} \subset \mathbb{R}^3$, où $\omega \subset \mathbb{R}^2$ est un domaine Lipschitzien et h est supposé petit. Le champ des déplacements est noté $\mathbf{u}$ et le tenseur des déformations par $\boldsymbol{\varepsilon}(\mathbf{u}) := \frac{1}{2}(\nabla\mathbf{u} + (\nabla\mathbf{u})^{\intercal})$. Dans le cas important de l'élasticité linéaire isotrope, la relation entre le tenseur des contraintes et des déformations peut s'écrire

$$\boldsymbol{\sigma} = 2\mu\boldsymbol{\varepsilon} + \lambda(\operatorname{trace}\boldsymbol{\varepsilon})\mathbb{I} \quad \text{et} \quad \boldsymbol{\varepsilon} = \frac{1}{2\mu}\boldsymbol{\sigma} - \frac{\nu}{2\mu(1+\nu)}(\operatorname{trace}\boldsymbol{\sigma})\mathbb{I}, \tag{1}$$

où λ et μ sont les constantes de Lamé, ν est le coefficient de Poisson et $\mathbb{I}$ est l'identité. Soit Γ la partie latérale de Ω et $S_\ominus$, $S_\oplus$ les parties inférieures et supérieures. On recherche $\boldsymbol{\sigma}$, tenseur des contraintes, satisfaisant les conditions d'équilibre

$$\operatorname{Div}\boldsymbol{\sigma} + \mathbf{f} = \mathbf{0} \quad \text{dans } \Omega, \qquad \mathbf{u}|_\Gamma = 0, \qquad \boldsymbol{\sigma}\cdot\boldsymbol{n}|_S = 0. \tag{2}$$

E-mail addresses: repin@pdmi.ras.ru (S. Repin), stas@math.uzh.ch (S.A. Sauter).

1631-073X/$ – see front matter
doi:10.1016/j.crma.2010.09.004

Dans le modèle de Kirchhoff et Love on resout tout d'abord l'équation biharmonique satisfaite par le déplacement de la surface moyenne $\hat{w}$, i.e.,

$$\int_\omega \hat{\Delta}\hat{w}\hat{\Delta}\hat{\varphi}\,\mathrm{d}\hat{\mathbf{x}} = \int_\omega \hat{g}_0\hat{\varphi}\,\mathrm{d}\hat{\mathbf{x}}$$

pour tout $\hat{\varphi} \in H_0^2(\omega)$, où $\hat{g}_0$ désigne une force mise à l'échelle. Ce déplacement est alors utilisé pour reconstruire le champ de déplacement tridimensionnel en définissant

$$\mathbf{v}_{\mathrm{KL}}^{112} := \left(-x_3\hat{w}_{,1}, -x_3\hat{w}_{,2}, \hat{w} + x_3^2\hat{W}\right)^\intercal$$

(appelé modèle 112, voir (11)), où $\hat{W}$ est une fonction de $H_0^1(\omega)$. On pose $\boldsymbol{\tau}_{\mathrm{KL}}^{112} := \mathbb{L}\boldsymbol{\varepsilon}(\mathbf{v}_{\mathrm{KL}}^{112})$, où $\mathbb{L}$ est le tenseur d'élasticité. Une autre reconstruction possible est donnée par

$$\boldsymbol{\tau}_{\mathrm{KL}}^{im}(\hat{\mathbf{q}}) := \begin{bmatrix} -2\mu x_3(\hat{w}_{,11} + \frac{\nu}{1-\nu}\hat{\Delta}\hat{w}) & -2\mu x_3\hat{w}_{,12} & \frac{1}{2}(x_3^2 - \frac{h^2}{4})\hat{q}_1 \\ -2\mu x_3\hat{w}_{,12} & -2\mu x_3(\hat{w}_{,22} + \frac{\nu}{1-\nu}\hat{\Delta}\hat{w}) & \frac{1}{2}(x_3^2 - \frac{h^2}{4})\hat{q}_2 \\ \frac{1}{2}(x_3^2 - \frac{h^2}{4})\hat{q}_1 & \frac{1}{2}(x_3^2 - \frac{h^2}{4})\hat{q}_2 & -(2x_3^3 - \frac{1}{2}x_3h^2)\hat{f}_0 \end{bmatrix}, \tag{3}$$

où

$$\hat{\boldsymbol{q}} \in Q_{\hat{g}_0} := \left\{\hat{\boldsymbol{q}}(\hat{\mathbf{x}}) \in H(\omega, \mathrm{div}) \;\middle|\; \mathrm{div}\,\hat{\boldsymbol{q}} - \frac{2\mu}{1-\nu}\hat{g}_0 = 0,\ \text{p.p. } \hat{x} \in \omega\right\}. \tag{4}$$

L'estimation fondamentale d'erreur sur l'énergie $\boldsymbol{\varepsilon}(\mathbf{u} - \mathbf{v}_{\mathrm{KL}}^{112})$ est la suivante.

Les fonctions $\mathbf{w} \in \mathbf{V}_0 := \{\mathbf{w} \in H^1(\Omega, \mathbb{R}^3) \mid \mathbf{w} = 0 \text{ on } \Gamma\}$, resp. $\boldsymbol{\tau} \in \Sigma := L^2(\Omega, \mathbb{R}_{\mathrm{sym}}^{3\times 3})$, sont munies de la norme $|||\boldsymbol{\varepsilon}(\mathbf{w})|||^2 := (\mathbb{L}\boldsymbol{\varepsilon}(\mathbf{w}), \boldsymbol{\varepsilon}(\mathbf{w}))_\Omega$ et $|||\boldsymbol{\tau}|||_*^2 := (\mathbb{L}^{-1}\boldsymbol{\tau}, \boldsymbol{\tau})_\Omega$, où $(\cdot,\cdot)_\Omega$ désigne le produit scalaire dans L^2 pour les fonctions scalaires, vectorielles ou tensorielles. De plus $\|\cdot\|_\Omega := (\cdot,\cdot)_\Omega^{1/2}$.

Théorème. *Pour tout $\beta > 1$ on a*

$$\left|\left|\left|\boldsymbol{\varepsilon}\left(\mathbf{u} - \mathbf{v}_{\mathrm{KL}}^{112}\right)\right|\right|\right|^2 \leqslant \beta\left|\left|\left|\boldsymbol{\tau}_{\mathrm{KL}}^{112} - \boldsymbol{\tau}_{\mathrm{KL}}^{im}\right|\right|\right|_*^2 + \frac{2\beta C_\omega^2}{c_1^2(\beta - 1)}\sum_{i=1}^{2}\left\|r_i\left(\boldsymbol{\tau}_{\mathrm{KL}}^{im}\right)\right\|_\Omega^2, \tag{5}$$

avec $r_i(\tau)$, $i = 1, 2, 3$, defini en (9)*. Si $\hat{\Delta}\hat{w}$ possède des dérivées faibles de carré sommable* (*ce qui est vrai pour des domaines réguliers ou des domaines convexes et polygonaux*)*, alors*

$$\left|\left|\left|\boldsymbol{\varepsilon}\left(\mathbf{u} - \mathbf{v}_{\mathrm{KL}}^{112}\right)\right|\right|\right|^2 \leqslant M_2(\hat{w}, \hat{W}, \hat{\boldsymbol{q}}) := \beta\left(\mathcal{M}_1(\hat{W}, \hat{\boldsymbol{q}}) + \mathcal{M}_2(\hat{w}, \hat{W})\right) + \frac{\beta}{\beta - 1}\mathcal{M}_3(\hat{w}, \hat{\boldsymbol{q}}), \tag{6}$$

pour tout $\hat{W} \in H_0^1(\omega)$ et $\hat{\boldsymbol{q}} \in Q_{\hat{g}_0}(\omega)$, où $\rho(\hat{w}, \hat{W}) := 2\hat{W} - \frac{\nu}{1-\nu}\hat{\Delta}\hat{w}$ et

$$\mathcal{M}_1(\hat{W}, \hat{\boldsymbol{q}}) := \frac{h^5}{40}\int_\omega \left(\frac{\mu}{2}|\hat{\nabla}\hat{W}|^2 + \frac{1}{3}(\hat{W}_{,1}\hat{q}_1 + \hat{W}_{,2}\hat{q}_2) + \frac{1}{3\mu}|\hat{\boldsymbol{q}}|^2\right)\mathrm{d}\hat{\mathbf{x}},$$

$$\mathcal{M}_2(\hat{w}, \hat{W}) := \frac{h^3}{420}\int_\omega \left(70\frac{\mu(1-\nu)}{(1-2\nu)}\rho^2(\hat{w}, \hat{W}) - 14\rho(\hat{w}, \hat{W})\hat{f} + \frac{\hat{f}^2}{\mu(1+\nu)}\right)\mathrm{d}\hat{\mathbf{x}},$$

$$\mathcal{M}_3(\hat{w}, \hat{\boldsymbol{q}}) := \frac{C_\omega^2 h^3}{6c_1^2}\int_\omega \left(\left|\hat{q}_1 - \frac{2\mu}{1-\nu}\hat{\Delta}\hat{w}_{,1}\right|^2 + \left|\hat{q}_2 - \frac{2\mu}{1-\nu}\hat{\Delta}\hat{w}_{,2}\right|^2\right)\mathrm{d}\hat{\mathbf{x}}.$$

1. Statement of the problem

Let $\Omega := \omega \times]-h/2, h/2[\subset \mathbb{R}^3$ where $\omega \subset \mathbb{R}^2$ is a Lipschitz domain and h is small compared to the width of ω. Further geometric quantities are

$$S_0 := \omega \times \{0\}, \qquad S_\oplus := \omega \times \left\{\frac{h}{2}\right\}, \qquad S_\ominus := \omega \times \left\{-\frac{h}{2}\right\}, \qquad \Gamma := \partial\omega \times \left(-\frac{h}{2}, \frac{h}{2}\right)$$

and $\boldsymbol{n}(\hat{\mathbf{x}})$ is the normal vector for $S := S_\oplus \cup S_\ominus$.

The displacement field and the strain tensor are denoted by $\mathbf{u}$ and $\boldsymbol{\varepsilon}(\mathbf{u}) := \frac{1}{2}(\nabla\mathbf{u} + (\nabla\mathbf{u})^\intercal)$ and coupled with the stress tensor via the Hooke's law $\boldsymbol{\sigma} := \mathbb{L}\boldsymbol{\varepsilon}$, where the tensor of elasticity constants $\mathbb{L}$ satisfies the standard symmetry conditions, is uniformly positive definite and bounded in the L^∞-norm. For the case of isotropic material the corresponding relation is presented by (1). We consider the problem of finding the stress tensor $\boldsymbol{\sigma}$ such that the equilibrium and boundary conditions

$$\operatorname{Div}\boldsymbol{\sigma} + \mathbf{f} = \mathbf{0} \quad \text{in } \Omega, \qquad \mathbf{u}|_\Gamma = 0, \qquad \boldsymbol{\sigma}\cdot\boldsymbol{n}|_S = 0 \tag{7}$$

are satisfied. For the external force, we assume $\mathbf{f} = (0, 0, \hat f(\hat{\mathbf{x}}))^\intercal$ with $\hat f \in L^2(\omega)$ and, for $\mathbf{x} = (x_1, x_2, x_3)$, we set $\hat{\mathbf{x}} := (x_1, x_2)$.

A function $\mathbf{u} \in \mathbf{V}_0 := \{\mathbf{w} \in H^1(\Omega, \mathbb{R}^3) \mid \mathbf{w} = 0 \text{ on } \Gamma\}$ is a generalized solution of (7) if it satisfies the variational relation

$$\int_\Omega \mathbb{L}\,\boldsymbol{\varepsilon}(\mathbf{u}) : \boldsymbol{\varepsilon}(\mathbf{w})\,\mathrm{d}\mathbf{x} = \int_\Omega \hat f\, w_3(\mathbf{x})\,\mathrm{d}\mathbf{x} \tag{8}$$

for all $\mathbf{w} \in \mathbf{V}_0$. Functions $\mathbf{w} \in \mathbf{V}_0$ resp. $\boldsymbol{\tau} \in \Sigma := L^2(\Omega, \mathbb{R}^{3\times3}_{\mathrm{sym}})$ are equipped with the corresponding energy norms $|||\boldsymbol{\varepsilon}(\mathbf{w})|||^2 := (\mathbb{L}\boldsymbol{\varepsilon}(\mathbf{w}), \boldsymbol{\varepsilon}(\mathbf{w}))_\Omega$ and $|||\boldsymbol{\tau}|||^2 := (\mathbb{L}^{-1}\boldsymbol{\tau}, \boldsymbol{\tau})_\Omega$, where $(\cdot,\cdot)_\Omega$ denotes the L^2-scalar product for scalar functions, vectors, and tensors. Further $\|\cdot\|_\Omega := (\cdot,\cdot)_\Omega^{1/2}$, and we denote by $\|\cdot\|_\omega$, $\|\cdot\|_{S_\oplus}$, $\|\cdot\|_{S_\ominus}$ the L^2-norms on ω, $S_\oplus$, $S_\ominus$.

In the classical theory of the Kirchhoff–Love plates, the above-described 3D model is replaced by an ansatz via the scalar valued deflection $\hat w$ of the middle surface

$$u_1(\mathbf{x}) = -x_3\hat w_{,1}, \qquad u_2(\mathbf{x}) = -x_3\hat w_{,2}, \qquad u_3(\mathbf{x}) = \hat w(\hat{\mathbf{x}}).$$

We introduce the scaled loads $\hat g_0 := \frac{6(1-\nu)}{\mu}\hat f_0$ with $\hat f(h) := h^2\hat f_0$ and deduce $\hat w$ as the solution of the problem: find $\hat w \in H_0^2(\omega)$:

$$\int_\omega \hat\Delta\hat w\,\hat\Delta\hat\varphi\,\mathrm{d}\hat{\mathbf{x}} = \int_\omega \hat g_0\hat\varphi\,\mathrm{d}\hat{\mathbf{x}} \quad \forall\hat\varphi \in H_0^2(\omega).$$

Historically, the subject of error estimation in dimension reduction models was mainly focused on a priori asymptotic error estimates that evaluate the difference between original and reduced models in terms of small (geometric) parameters. In this context, models in the elasticity theory have been studied by different authors (see e.g., [1,3–6,14] and the references therein). Papers [9,10] are devoted to asymptotic consistency of models in the linearized plate theory.

2. Error bound for plate type elastic bodies

Let $\mathbf{v} \in \mathbf{V}_0$ denote an approximation of the exact solution $\mathbf{u}$ of (8) obtained by some suitable reconstruction of a plate model. In this section, we present different estimates of the modeling error generated by $\mathbf{v}$. Let

$$\Sigma_{\mathrm{Div},\mathbf{n}} := \left\{\boldsymbol{\tau} \in \Sigma \;\middle|\; \operatorname{Div}\boldsymbol{\tau} \in \left(L^2(\Omega)\right)^3;\ \boldsymbol{\tau}\cdot\boldsymbol{n} \in \left(L^2(S)\right)^3\right\}.$$

The following error majorant was introduced in [13] which contains also the proof of the next theorem (see also [11,12]). We use the short notation

$$r_i(\boldsymbol{\tau}) := \operatorname{div}\{\tau_{ij}\}_{j=1}^3 = \tau_{i1,1} + \tau_{i2,2} + \tau_{i3,3}, \quad i = 1, 2, 3. \tag{9}$$

Theorem 2.1. *Assume that $r_3(\boldsymbol{\tau}) + h^2\hat f_0 = 0$ in Ω and $\tau_{i,3}|_{S_\ominus\cup S_\oplus} = 0$ for $i = 1, 2, 3$. Then,*

$$\left|\left|\left|\boldsymbol{\varepsilon}(\mathbf{u}-\mathbf{v})\right|\right|\right|^2 \leqslant M_1(\mathbf{v}, \boldsymbol{\tau}) := \beta\left|\left|\left|\mathbb{L}\boldsymbol{\varepsilon}(\mathbf{v}) - \boldsymbol{\tau}\right|\right|\right|_\star^2 + \frac{2\beta C_\omega^2}{c_1^2(\beta-1)}\sum_{i=1}^2\left\|r_i(\boldsymbol{\tau})\right\|_\Omega^2, \tag{10}$$

for any $\beta > 1$ and where C_ω is the constant in the Friedrichs inequality related to the plain domain ω and $c_1 > 0$ is the lowest eigenvalue of $\mathbb{L}$.

3. Error bound for the KL model

Let $\hat w = \hat w(x_1, x_2)$ be a solution of the Kirchhoff–Love problem. To measure the corresponding modeling error we need to reconstruct 3D displacements and stresses.

We employ here the so-called (112)-model for the dimension reconstruction of the displacements, strains, and stresses (see [1,2,8]) and set

$$\mathbf{v}_{\mathrm{KL}}^{112} := \mathbf{v}_{\mathrm{KL}}^{112}(\hat w, \hat W) := \left(-x_3\hat w_{,1}, -x_3\hat w_{,2}, \hat w + x_3^2\hat W\right)^\intercal, \tag{11}$$

where $\hat W(\hat{\mathbf{x}})$ is a specially selected function. Henceforth, we assume that $\hat W \in H_0^1(\omega)$. In our analysis, we employ two reconstructions of 3D stresses based on $\hat w$ and $\hat W$. The first one reconstructs stresses in accordance with 3D elasticity relations, i.e., $\boldsymbol{\tau}_{\mathrm{KL}}^{112}(\hat{w},\hat{W}) := \mathbb{L}\boldsymbol{\varepsilon}(\mathbf{v}_{\mathrm{KL}}^{112}(\hat{w},\hat{W}))$. Another possible reconstruction (originally introduced in [8]) of the stress tensor uses the KL relations (1) for the components τ_{sk}, $s,k=1,2$, while the components τ_{i3}, $i=1,2,3$, are not zero (as in the classical KL theory) and are defined with the help of a correction functions $\hat{\boldsymbol{q}}$. This *improved* reconstruction has the form (3), where we assume that $\hat{\boldsymbol{q}} \in Q_{\hat{g}_0}$ (cf. (4)). This implies that the conditions of Theorem 2.1 are satisfied.

3.1. Error estimate

From (10) we deduce the following theorem (see [13]):

Theorem 3.1. *For any $\beta > 1$ it holds*

$$\left|\!\left|\!\left|\boldsymbol{\varepsilon}\left(\mathbf{u}-\mathbf{v}_{\mathrm{KL}}^{112}\right)\right|\!\right|\!\right|^2 \leqslant \beta \left|\!\left|\!\left|\boldsymbol{\tau}_{\mathrm{KL}}^{112}-\boldsymbol{\tau}_{\mathrm{KL}}^{im}\right|\!\right|\!\right|_*^2 + \frac{2\beta C_\omega^2}{c_1^2(\beta-1)} \sum_{i=1}^{2} \left\| r_i\left(\boldsymbol{\tau}_{\mathrm{KL}}^{im}\right)\right\|_\Omega^2. \tag{12}$$

If $\hat{\Delta}\hat{w}$ possesses square summable generalized derivatives (which is true for domains with smooth boundaries or for convex polygonal domains) then $|||\boldsymbol{\varepsilon}(\mathbf{u}-\mathbf{v}_{\mathrm{KL}}^{112})|||^2 \leqslant M_2(\hat{w},\hat{W},\hat{\boldsymbol{q}})$ for all $\hat{W} \in H_0^1(\omega)$ and $\hat{\boldsymbol{q}} \in Q_{\hat{g}_0}(\omega)$, where M_2 is as in (6).

Note that $C_\omega \leqslant \frac{1}{\pi}\frac{ab}{\sqrt{a^2+b^2}}$ holds if ω is contained in a rectangle with side lengths a and b. Thus, $M_2(\hat{w},\hat{W},\hat{\boldsymbol{q}})$ contains only known functions and constants. If $\hat{w}$ is known, then the majorant is directly computable. By selecting $\hat{\boldsymbol{q}}$ and $\hat{W}$, we can (approximately) minimize the value of the majorant. The number obtained presents a *guaranteed* upper bound of the modeling error encompassed in $\hat{w}$.

4. Asymptotic behavior of the error majorant

In this section, we will choose $\hat{W}$ and $\hat{\mathbf{q}}$ in a concrete way and will get a simplified version of the majorant which will allow us to detect the asymptotic behavior of the modelling error. By estimating the products $\langle\hat{\nabla}\hat{W},\hat{\mathbf{q}}\rangle$ and $\rho(\hat{w},\hat{W})\hat{f}$ appearing in the definition of M_1, M_2 by Young's inequalities we obtain an upper bound of M_i, $i=1,2$, which is minimal by choosing $\hat{W}$ as the minimizer of the functional

$$J : H_0^1(\omega) \to \mathbb{R}, \qquad J(\hat{Z}) := h^2\|\hat{\nabla}\hat{Z}\|_\omega^2 + \frac{80(1-\nu)}{3(1-2\nu)}\left\|\rho(\hat{w},\hat{Z})\right\|_\omega^2.$$

The function $\hat{\boldsymbol{q}}$ is selected by $\hat{\boldsymbol{q}} := \frac{2\mu}{1-\nu}\hat{\nabla}\hat{\Delta}\hat{w} \in Q_{\hat{g}_0}$; we assume here that $\hat{w}$ is sufficiently regular. The proof of the following theorem is in [13] and uses some regularity results for the minimizer $\hat{W}$ (see [1,2]):

Theorem 4.1. *For any $\gamma > 1$, it holds*

$$\left|\!\left|\!\left|\boldsymbol{\varepsilon}\left(\mathbf{u}-\mathbf{v}_{\mathrm{KL}}^{112}\right)\right|\!\right|\!\right|^2 \leqslant M_3(\hat{w},\hat{W},\hat{\boldsymbol{q}}_0) := \frac{\mu\gamma}{80}h^5\|\hat{\nabla}\hat{W}\|_\omega^2 + \frac{\mu(1-\nu)}{3(1-2\nu)}h^3\left\|\rho(\hat{w},\hat{W})\right\|_\omega^2 + h^5\mathcal{R}(\hat{w},\hat{f}_0), \tag{13}$$

where

$$\mathcal{R}(\hat{w},\hat{f}_0) = C_{\mu,\nu,\gamma}^{\mathrm{I}}\|\hat{\nabla}\hat{\Delta}\hat{w}\|_\omega^2 + C_{\mu,\nu,\gamma}^{\mathrm{II}}h^2\|\hat{f}_0\|_\omega^2$$

and the $C_{\mu,\nu,\gamma}^{\mathrm{I,II}}$ are positive constants depending only on μ,ν,γ.

By using this theorem the following asymptotic behavior of the error majorant can be derived (see [13])

$$\frac{\sqrt{M_3(\hat{w},\hat{W},\hat{\boldsymbol{q}}_0)}}{|||\boldsymbol{\varepsilon}(\mathbf{v}_{\mathrm{KL}}^{112})|||} \leqslant ch^{1/2} \quad \text{and} \quad \frac{\sqrt{M_3(\hat{w},\hat{W},\hat{\boldsymbol{q}}_0)}}{|||\boldsymbol{\varepsilon}(\mathbf{u})|||} \leqslant ch^{1/2}.$$

Acknowledgements

The authors are grateful to Swiss National Science Foundation for supporting this research under the grant 200021_119809/1.

References

[1] A.L. Alessandrini, D.N. Arnold, R.S. Falk, A.L. Madureira, Derivation and justification of plate models by variational methods, in: M. Fortin (Ed.), Plates and Shells, Quebec, 1996, in: CRM Proceedings and Lecture Notes, vol. 21, American Mathematical Society, Providence, RI, 1996, pp. 1–20.
[2] D. Braess, S. Sauter, C. Schwab, On the justification of plate models, Journal of Elasticity, in press.
[3] P.G. Ciarlet, Mathematical Elasticity, vol. II: Theory of Plates, North-Holland, Amsterdam, 1997.
[4] P.G. Ciarlet, P. Destuynder, A justification of a nonlinear model in plate theory, Comput. Methods Appl. Mech. Engrg. 17/18 (1979) 227–258.
[5] M. Dauge, I. Gruais, Asymptotics of arbitrary order in thin elastic plates and optimal estimates for the Kirchhoff–Love model, Asymptot. Anal. 13 (1996) 167–197.
[6] P. Destuynder, Comparaison entre les modeles tridimensionnels et bidimensionnels de plaque en élasticité, RAIRO Anal. Numer. 15 (1981) 331–369.
[7] G. Kirchhoff, Über das Gleichgewicht und die Bewegung einer elastischen Scheibe, J. Reine Angew. Math. 40 (1850) 51–58.
[8] D. Morgenstern, Herleitung der Plattentheorie aus der dreidimensionalen Elastizitätstheorie, Arch. Ration. Mech. Anal. 4 (1959) 145–152.
[9] J.C. Paumier, A. Raoult, Asymptotic consistency of the polynomial approximation in the linearized plate theory. Application to the Reissner–Mindlin model, in: Élasticité, viscoélasticité et contrôle optimal, Lyon, 1995, in: ESAIM Proc., vol. 2, Soc. Math. Appl. Indust., Paris, 1997, pp. 203–213.
[10] A. Raoult, Construction d'un modèle d'évolution de plaques avec terme d'inertie de rotation, Ann. Mat. Pura Appl. CXXXIX (1985) 361–400.
[11] S. Repin, Estimates for errors in two-dimensional models of elasticity theory, J. Math. Sci. (N. Y.) 106 (2001) 3027–3041.
[12] S. Repin, A Posteriori Estimates for Partial Differential Equations, Walter de Gruyter, Berlin, 2008.
[13] S. Repin, S. Sauter, Computable estimates of the modeling error related to Kirchhoff–Love plate model, Preprint Series, No. 11-2009, Inst. für Mathematik, University of Zurich, 2009.
[14] B.A. Shoikhet, On asymptotically exact equations of thin plates of complex structure, J. Appl. Math. Mech. 37 (1973) (1974) 867–877.