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Numerical Analysis

A time discretization scheme of a characteristics method for a fluid–rigid system with discontinuous density

Discrétisation en temps d'une méthode de caractéristiques pour un système d'interaction fluide–rigide avec densité discontinue

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ABSTRACT

We propose a new characteristics method for the time discretization of a fluid–rigid system in the case when the densities of the fluid and the solid are different. This method is based on a global weak formulation involving only terms defined on the whole fluid–rigid domain. The main idea is to construct a characteristic function which preserves the rigidity of the solid at the discrete time levels. A convergence result for this semi-discrete scheme is then given.

RÉSUMÉ

Nous présentons un schéma de semi-discrétisation en temps d'une méthode de caractéristiques pour un problème fluide–rigide dans le cas où les densités du fluide et du solide sont différentes. Cette méthode est basée sur une formulation faible globale faisant intervenir uniquement des termes définis sur tout le domaine fluide–rigide. L'idée principale est de construire une fonction caractéristique qui préserve la rigidité du solide d'une itération en temps à l'autre. Le résultat principal porte sur la convergence du schéma semi-discrétisé en temps.

Version française abrégée

On considère un système d'interaction fluide–rigide qui occupe un domaine borné régulier $\mathcal{O} \subset \mathbb{R}^2$. A l'instant t, la position du solide rigide dans le fluide est donnée par $B(t) \subset \mathcal{O}$ et le fluide visqueux incompressible occupe le domaine $\Omega(t) = \mathcal{O} \setminus B(t)$. Le système complet est donné par les équations (3)–(9) où les équations de Navier–Stokes sont couplées avec les lois de Newton pour prendre en compte la dynamique du solide. On suppose que le solide est un boule de rayon 1.

On s'intéresse au cas où la densité ρ_s du solide est différente de la densité ρ_f du fluide. La fonction densité définie sur tout le domaine $\mathcal{O}$ est alors discontinue. Nous proposons un schéma de semi-discrétisation en temps obtenu à partir d'une formulation mixte globale de (3)–(9) qui fait uniquement intervenir des termes définis sur tout le domaine $\mathcal{O}$. Dans cette

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formulation la dérivée particulaire est prise en compte par une fonction caractéristique. Dans le cas où les densités sont égales ($\rho_f = \rho_s$), un résultat de convergence d'une méthode de Lagrange–Galerkin a déjà été établi dans [7], en utilisant une fonction caractéristique standard (i.e. classique pour les équations de Navier–Stokes). Pour le cas considéré dans cette Note ($\rho_f \neq \rho_s$), la fonction caractéristique standard ne permet pas de prouver la convergence de la méthode numérique résultant. Pour cette raison, on a du définir la nouvelle fonction caractéristique $\overline{\mathbf{X}}^k$ par (17)–(18). Cette fonction caractéristique modifiée n'applique pas le domaine $\mathcal{O}$ dans lui-même, i.e. $\overline{\mathbf{X}}^k(\mathcal{O}) \nsubseteq \mathcal{O}$ mais elle possède les propriétés suivantes :

Lemme 0.1. *Pour tout $k \in \{0, \dots, N\}$, la fonction caractéristique $\overline{\mathbf{X}}^k$ définie par* (17)–(18) *vérifie les propriétés suivantes* :

i) $\overline{\mathbf{X}}^k(B(\boldsymbol{\zeta}^{k+1})) = B(\boldsymbol{\zeta}^k)$.
ii) *Si la densité ρ^k définie par* (21) *est étendue à ρ_f en dehors de $\mathcal{O}$, alors on a $\rho^{k+1} = \rho^k \circ \overline{\mathbf{X}}^k$.*
iii) *Pour toute fonction $f \in L^2(\mathbb{R}^2)$ telle que $f = 0$ dans $\mathbb{R}^2 \setminus \mathcal{O}$, on a*

$$\left\| f \circ \overline{\boldsymbol{\psi}}(t; t_{k+1}, \cdot) \right\|_{L^2(\mathcal{O})} \leqslant \|f\|_{L^2(\mathcal{O})} \quad \forall t \in [t_k, t_{k+1}]. \tag{1}$$

Les propriétés i) et ii) indiquent que la caractéristique préserve la rigidité du solide. Ce n'est plus le cas si on utilise une fonction caractéristique standard comme cela est fait dans [7]. La propriété iii) permet de contrôler ce qui se passe en dehors du domaine $\mathcal{O}$ bien que $\overline{\mathbf{X}}^k(\mathcal{O}) \nsubseteq \mathcal{O}$. Ces propriétés sont essentielles pour établir le résultat de convergence suivant :

Théorème 0.1. *On suppose que $\mathcal{O}$ est un domaine borné régulier de $\mathbb{R}^2$, que $\mathbf{f}$ et $\mathbf{u}_0$ vérifient les conditions de compatibilité* (24) *et que $(\mathbf{u}, p, \boldsymbol{\zeta}, \omega)$ est une solution de* (3)–(9) *satisfaisant les propriétés de régularité* (25) *et* (26)*. Alors il existe des constantes $C > 0$ et $\tau^* > 0$ indépendantes de Δt telles que pour tout $0 < \Delta t \leqslant \tau^*$, la solution $(\mathbf{u}^k, p^k, \boldsymbol{\zeta}^k)$ du schéma de semi-discrétisation en temps* (16)*,* (19)–(20) *vérifie*

$$\sup_{1 \leqslant k \leqslant N} \left(\left| \boldsymbol{\zeta}(t_k) - \boldsymbol{\zeta}^k \right| + \left\| \mathbf{u}(t_k) - \mathbf{u}^k \right\|_{L^2(\mathcal{O})^2} \right) \leqslant C \Delta t. \tag{2}$$

La preuve complète de ce résultat ainsi qu'une analyse d'erreur pour un schéma complet de discrétisation en temps et en espace est établie dans [5].

1. Introduction

The aim of this Note is to present a modified characteristics method for the semi-discretization in time of a fluid–solid system when the densities of the fluid and the solid are different. We consider a fluid–rigid interaction system for a viscous incompressible fluid in which a rigid body is immersed. The fluid–rigid system occupies a bounded and regular domain $\mathcal{O} \subset \mathbb{R}^2$. The solid is assumed to be a ball of radius 1. At time t, the position of the ball in the fluid is given by $B(t) \subset \mathcal{O}$ and the fluid fills the part $\Omega(t) = \mathcal{O} \setminus B(t)$. We denote by $\boldsymbol{\zeta}(t)$ the center of mass of the rigid ball $B(t) = B(\boldsymbol{\zeta}(t))$ at time t and its angular velocity $\omega(t)$. The unknowns are the velocity field $\mathbf{u}(\mathbf{x}, t)$, the pressure $p(\mathbf{x}, t)$ of the fluid, the center of mass $\boldsymbol{\zeta}(t)$ and the angular velocity $\omega(t)$ of the ball. For $t \in [0, T]$, we have the following Navier–Stokes system coupled with Newton's laws:

$$\rho_f \left(\frac{\partial \mathbf{u}}{\partial t} + (\mathbf{u} \cdot \nabla)\mathbf{u} \right) - \mu \Delta \mathbf{u} + \nabla p = \rho_f \mathbf{f}, \quad \mathbf{x} \in \Omega(t),\ t \in [0, T], \tag{3}$$

$$\operatorname{div} \mathbf{u} = 0, \quad \mathbf{x} \in \Omega(t),\ t \in [0, T], \tag{4}$$

$$\mathbf{u} = 0, \quad \mathbf{x} \in \partial\mathcal{O},\ t \in [0, T], \tag{5}$$

$$\mathbf{u} = \boldsymbol{\zeta}'(t) + \omega(t)\big(\mathbf{x} - \boldsymbol{\zeta}(t)\big)^{\perp}, \quad \mathbf{x} \in \partial B(t),\ t \in [0, T], \tag{6}$$

$$m\boldsymbol{\zeta}''(t) = -\int_{\partial B(t)} \boldsymbol{\sigma}\mathbf{n}\, d\Gamma + \rho_s \int_{B(t)} \mathbf{f}(\mathbf{x}, t)\, d\mathbf{x}, \quad t \in [0, T], \tag{7}$$

$$J\omega'(t) = -\int_{\partial B(t)} \big(\mathbf{x} - \boldsymbol{\zeta}(t)\big)^{\perp} \cdot \boldsymbol{\sigma}\mathbf{n}\, d\Gamma + \rho_s \int_{B(t)} \big(\mathbf{x} - \boldsymbol{\zeta}(t)\big)^{\perp} \cdot \mathbf{f}(\mathbf{x}, t)\, d\mathbf{x}, \quad t \in [0, T], \tag{8}$$

$$\mathbf{u}(\mathbf{x}, 0) = \mathbf{u}_0(\mathbf{x}) \quad \text{for } \mathbf{x} \in \Omega(0), \qquad \boldsymbol{\zeta}(0) = \boldsymbol{\zeta}_0 \in \mathbb{R}^2, \qquad \boldsymbol{\zeta}'(0) = \boldsymbol{\zeta}_1 \in \mathbb{R}^2, \qquad \omega(0) = \omega_0 \in \mathbb{R}. \tag{9}$$

In the above system, $\boldsymbol{\sigma} = -p\mathbf{Id} + 2\mu \mathrm{D}(\mathbf{u})$ denotes the Cauchy stress tensor with $\mathrm{D}(\mathbf{u}) = (\nabla\mathbf{u} + \nabla\mathbf{u}^T)/2$. The positive constant μ is the dynamic viscosity of the fluid and the constants m and J are the mass and the moment of inertia of the ball. We suppose that the density ρ_f of the fluid and the density ρ_s of the solid are constant, but not equal, that is

$$\rho_f \neq \rho_s. \tag{10}$$

In the case $\rho_f = \rho_s$, a convergence analysis has been already done (see [6,7]) for a numerical scheme based on a standard characteristic function (i.e. classical formulation of the material derivative in the Navier–Stokes equations with a characteristic function). The analysis performed in [7] cannot be easily extended to the case $\rho_f \neq \rho_s$ without modifying the characteristic function. The numerical scheme we present in this Note for the semi-discretization in time is based on a global/monolithic weak formulation of (3)–(9). In such approach, velocities of the fluid and the rigid body are treated synchronously. In this framework, we also mention another approach based on fictitious domain method with Lagrange multiplier to take into account the rigid motion constraint of the solid [3] or also penalization methods [1,2].

Let us introduce some notation. For $\boldsymbol{\zeta} \in \mathcal{O}$, we introduce the space of rigid functions in $B(\boldsymbol{\zeta}) = \{\mathbf{x} \in \mathbb{R}^2 : \ |\mathbf{x} - \boldsymbol{\zeta}| \leqslant 1\}$,

$$\mathcal{K}(\boldsymbol{\zeta}) = \big\{\mathbf{u} \in H_0^1(\mathcal{O})^2 \ \big| \ \mathrm{D}(\mathbf{u}) = 0 \text{ in } B(\boldsymbol{\zeta})\big\}, \tag{11}$$

and the space of the pressure

$$M(\boldsymbol{\zeta}) = \big\{p \in L_0^2(\mathcal{O}) \ \big| \ p = 0 \text{ in } B(\boldsymbol{\zeta})\big\} \quad \text{with } L_0^2(\mathcal{O}) = \left\{f \in L^2(\mathcal{O}) \ \Big| \ \int_{\Omega} f \, \mathrm{d}\mathbf{x} = 0\right\}. \tag{12}$$

We also introduce the bilinear forms

$$a(\mathbf{u}, \mathbf{v}) = 2\mu \int_{\mathcal{O}} \mathrm{D}(\mathbf{u}) : \mathrm{D}(\mathbf{v}) \, \mathrm{d}\mathbf{x} \quad \forall \mathbf{u}, \mathbf{v} \in H^1(\mathcal{O})^2,$$

$$b(\mathbf{u}, p) = -\int_{\mathcal{O}} \mathrm{div}(\mathbf{u}) p \, \mathrm{d}\mathbf{x} \quad \forall \mathbf{u} \in H^1(\mathcal{O})^2, \ \forall p \in L_0^2(\mathcal{O}).$$

Finally, we define the density function ρ into the whole domain $\mathcal{O}$, as the following discontinuous and piecewise constant function

$$\rho(\mathbf{x}) = \begin{cases} \rho_s & \text{if } \mathbf{x} \in B(\boldsymbol{\zeta}), \\ \rho_f & \text{if } \mathbf{x} \in \mathcal{O} \setminus B(\boldsymbol{\zeta}). \end{cases}$$

The space (11) is specific to this problem. In fact, if the solution $\mathbf{u}$ of (3)–(9) is extended by

$$\mathbf{u}(\mathbf{x}, t) = \boldsymbol{\zeta}'(t) + \omega(t)\big(\mathbf{x} - \boldsymbol{\zeta}(t)\big)^{\perp} \quad \forall \mathbf{x} \in B\big(\boldsymbol{\zeta}(t)\big), \tag{13}$$

then, we have that $\mathbf{u}(t) \in \mathcal{K}(\boldsymbol{\zeta}(t))$. In the reminder of this Note, the solution $\mathbf{u}$ of (3)–(9) will be extended as above.

An important ingredient of the numerical method we use is given by the characteristic functions whose level lines are the integral curves of the velocity field. More precisely (see, for instance, [4,8]), considering the characteristic function $\tilde{\boldsymbol{\psi}} : [0, T]^2 \times \mathcal{O} \to \mathcal{O}$ defined as the solution of the initial value problem

$$\begin{cases} \dfrac{\mathrm{d}}{\mathrm{d}t} \tilde{\boldsymbol{\psi}}(t; s, \mathbf{x}) = \mathbf{u}\big(\tilde{\boldsymbol{\psi}}(t; s, \mathbf{x}), t\big), \\ \tilde{\boldsymbol{\psi}}(s; s, \mathbf{x}) = \mathbf{x}, \end{cases} \tag{14}$$

the material derivative $D_t\mathbf{u} = \partial\mathbf{u}/\partial t + (\mathbf{u} \cdot \nabla)\mathbf{u}$ of the velocity $\mathbf{u}$ at time t_0 satisfies:

$$D_t\mathbf{u}(\mathbf{x}, t_0) = \frac{\mathrm{d}}{\mathrm{d}t}\big[\mathbf{u}\big(\tilde{\boldsymbol{\psi}}(t; t_0, \mathbf{x}), t\big)\big]\big|_{t=t_0}. \tag{15}$$

Using (13) and (14), we can obtain a global weak formulation of the system (3)–(9) posed in the whole domain $\mathcal{O}$. We refer to [7] for more details.

2. The time discretization scheme and main result

System (3)–(9) is discretized in time by using a characteristic function and a global weak formulation which involves the whole domain $\mathcal{O}$. For $N \in \mathbb{N}^*$ we denote $\Delta t = T/N$ and $t_k = k\Delta t$ for $k = 0, \ldots, N$. We denote by $(\mathbf{u}^k, p^k, \boldsymbol{\zeta}^k) \in \mathcal{K}(\boldsymbol{\zeta}^k) \times M(\boldsymbol{\zeta}^k) \times \mathcal{O}$ the approximation of the solution of (3)–(9) at time $t = t_k$. We describe the numerical scheme for computing the approximate solution $(\mathbf{u}^{k+1}, p^{k+1}, \boldsymbol{\zeta}^{k+1})$ at the discrete time $t = t_{k+1}$.

First, we update the position of the rigid ball at instant t_{k+1} by computing

$$\boldsymbol{\zeta}^{k+1} = \boldsymbol{\zeta}^k + \mathbf{u}^k\big(\boldsymbol{\zeta}^k\big)\Delta t. \tag{16}$$

We then define the characteristic function $\overline{\boldsymbol{\psi}}$ associated with the semi-discretized velocity field as the solution of

$$\begin{cases} \dfrac{\mathrm{d}}{\mathrm{d}t} \overline{\boldsymbol{\psi}}(t; t_{k+1}, \mathbf{x}) = \mathbf{u}^k\big(\overline{\boldsymbol{\psi}}(t; t_{k+1}, \mathbf{x})\big) - \mathbf{u}^k\big(\boldsymbol{\zeta}^k\big), \\ \overline{\boldsymbol{\psi}}(t_{k+1}; t_{k+1}, \mathbf{x}) = \mathbf{x} - \mathbf{u}^k\big(\boldsymbol{\zeta}^k\big)\Delta t \end{cases} \tag{17}$$

and we denote

$$\overline{\mathbf{X}}^k(\mathbf{x}) = \overline{\boldsymbol{\psi}}(t_k; t_{k+1}, \mathbf{x}) \quad \forall \mathbf{x} \in \mathcal{O}. \tag{18}$$

Let us observe that due to the initial condition of (17), if we consider $\mathbf{x} \in \mathcal{O}$, then $\overline{\boldsymbol{\psi}}(t_{k+1}; t_{k+1}, \mathbf{x})$ does not necessarily belong to $\mathcal{O}$. For this reason, the velocity field $\mathbf{u}^k$ is extended by zero outside of domain $\mathcal{O}$.

We next define $(\mathbf{u}^{k+1}, p^{k+1}) \in \mathcal{K}(\boldsymbol{\zeta}^{k+1}) \times M(\boldsymbol{\zeta}^{k+1})$ as the solution of the following Stokes type system

$$\left(\rho^{k+1}\frac{\mathbf{u}^{k+1} - \mathbf{u}^k \circ \overline{\mathbf{X}}^k}{\Delta t}, \boldsymbol{\varphi}\right)_{L^2(\mathcal{O})^2} + a\big(\mathbf{u}^{k+1}, \boldsymbol{\varphi}\big) + b\big(\boldsymbol{\varphi}, p^{k+1}\big) = \big(\rho^{k+1}\mathbf{f}^{k+1}, \boldsymbol{\varphi}\big)_{L^2(\mathcal{O})^2} \quad \forall \boldsymbol{\varphi} \in \mathcal{K}\big(\boldsymbol{\zeta}^{k+1}\big), \tag{19}$$

$$b\big(\mathbf{u}^{k+1}, q\big) = 0 \quad \forall q \in M\big(\boldsymbol{\zeta}^{k+1}\big), \tag{20}$$

where $\mathbf{f}^{k+1} = \mathbf{f}(t_{k+1})$ and ρ^{k+1} is the discontinuous density function defined by

$$\rho^{k+1}(\mathbf{x}) = \begin{cases} \rho_s & \text{if } \mathbf{x} \in B(\boldsymbol{\zeta}^{k+1}), \\ \rho_f & \text{if } \mathbf{x} \in \mathcal{O} \setminus B(\boldsymbol{\zeta}^{k+1}). \end{cases} \tag{21}$$

The mixed formulation (19)–(20) is a well-posed problem since it can be proved that the spaces $\mathcal{K}(\boldsymbol{\zeta})$, $M(\boldsymbol{\zeta})$ and the bilinear form b satisfy an *inf-sup* condition.

Let us note that our characteristic function $\overline{\mathbf{X}}^k$ defined in (17)–(18) does not map $\mathcal{O}$ onto $\mathcal{O}$, that is $\overline{\mathbf{X}}^k(\mathcal{O}) \not\subseteq \mathcal{O}$, nevertheless the following essential properties hold:

Lemma 1. *For any $k \in \{0, \dots, N\}$, the characteristic function $\overline{\mathbf{X}}^k$ defined in* (17)–(18) *satisfies the following properties*:

i) $\overline{\mathbf{X}}^k(B(\boldsymbol{\zeta}^{k+1})) = B(\boldsymbol{\zeta}^k)$.
ii) *If we extend by ρ_f the density field ρ^k outside of $\mathcal{O}$, we have $\rho^{k+1} = \rho^k \circ \overline{\mathbf{X}}^k$.*
iii) *For any $f \in L^2(\mathbb{R}^2)$ such that $f = 0$ in $\mathbb{R}^2 \setminus \mathcal{O}$, we have*

$$\big\| f \circ \overline{\boldsymbol{\psi}}(t; t_{k+1}, \cdot)\big\|_{L^2(\mathcal{O})} \leqslant \|f\|_{L^2(\mathcal{O})} \quad \forall t \in [t_k, t_{k+1}]. \tag{22}$$

Property i) ensures that $\overline{\mathbf{X}}^k$ maps the rigid solid at the discrete time level t_{k+1} into the rigid solid at time t_k. Hence the rigidity of the solid is preserved by $\overline{\mathbf{X}}^k$ at discrete time level. Let us point out that if instead of (17), we choose the more "standard" characteristic function by considering the problem

$$\begin{cases} \dfrac{\mathrm{d}}{\mathrm{d}t}\overline{\boldsymbol{\psi}}(t; t_{k+1}, \mathbf{x}) = \mathbf{u}^k\big(\overline{\boldsymbol{\psi}}(t; t_{k+1}, \mathbf{x})\big), \\ \overline{\boldsymbol{\psi}}(t_{k+1}; t_{k+1}, \mathbf{x}) = \mathbf{x}, \end{cases} \tag{23}$$

as it is done in [7], for the case of equal densities $\rho_s = \rho_f$, then the properties i) and ii) are not satisfied anymore. In that case, the estimate of error $\|\mathbf{u}(t_k) - \mathbf{u}^k\|_{L^2(\mathcal{O})^2}$ shall involve terms such as $\|\rho^{k+1} - \rho^k \circ \overline{\mathbf{X}}^k\|_{L^2(\mathcal{O})}$ which do not vanish when considering (23). With the characteristic function (23), the term $\|\rho^{k+1} - \rho^k \circ \overline{\mathbf{X}}^k\|_{L^2(\mathcal{O})}$ is actually hard to handle since the density function is discontinuous. Finally, the property iii) allows us to overcome the difficulty produced by the fact that $\overline{\mathbf{X}}^k(\mathcal{O}) \not\subseteq \mathcal{O}$.

The above lemma is one of the key ingredients used in the proof of our main result which asserts the convergence of the numerical scheme (16), (19)–(20):

Theorem 2. *Suppose that $\mathcal{O}$ is an open smooth bounded domain in $\mathbb{R}^2$ and that $\mathbf{f}$ and $\mathbf{u}_0$ satisfy*

$$\begin{aligned} &\mathbf{f} \in C\big([0, T]; H^1(\mathcal{O})^2\big), \qquad \mathbf{u}_0 \in H^2(\Omega)^2, \qquad \operatorname{div}(\mathbf{u}_0) = 0 \quad \text{in } \Omega, \\ &\mathbf{u}_0 = 0 \quad \text{on } \partial\mathcal{O}, \qquad \mathbf{u}_0(\mathbf{y}) = \boldsymbol{\zeta}_1 + \omega_0(\mathbf{y} - \boldsymbol{\zeta}_0)^\perp \quad \text{on } \partial B, \end{aligned} \tag{24}$$

where $\boldsymbol{\zeta}_0, \boldsymbol{\zeta}_1 \in \mathbb{R}^2$ and $\omega_0 \in \mathbb{R}$. Let also assume that $(\mathbf{u}, p, \boldsymbol{\zeta}, \omega)$ is a solution of (3)–(9) *satisfying*

$$\begin{cases} \mathbf{u} \in C\big([0, T]; H^2\big(\Omega(t)\big)^2\big) \cap H^1\big(0, T; L^2\big(\Omega(t)\big)^2\big), \\ D_t^2\mathbf{u} \in L^2\big(0, T; L^2\big(\Omega(t)\big)^2\big), \quad \mathbf{u} \in C\big([0, T]; C^{0,1}(\overline{\mathcal{O}})^2\big), \\ p \in C\big([0, T]; H^1\big(\Omega(t)\big)\big), \quad \boldsymbol{\zeta} \in H^3(0, T)^2, \quad \omega \in H^2(0, T) \end{cases} \tag{25}$$

and

$$\operatorname{dist}\big(B(t), \partial\mathcal{O}\big) > 0 \quad \forall t \in [0, T]. \tag{26}$$

Then there exist two positive constants C *and* τ^* *not depending on* Δt *such that for all* $0 < \Delta t \leqslant \tau^*$ *the solution* $(\mathbf{u}^k, p^k, \boldsymbol{\zeta}^k)$ *of the semi-discretization problem* (16), (19)–(20) *satisfies*

$$\sup_{1\leqslant k\leqslant N} \left(\left|\boldsymbol{\zeta}(t_k) - \boldsymbol{\zeta}^k\right| + \left\|\mathbf{u}(t_k) - \mathbf{u}^k\right\|_{L^2(\mathcal{O})^2}\right) \leqslant C\,\Delta t. \tag{27}$$

The complete proof of this theorem, as well as an error analysis for a fully space–time discretization scheme of Lagrange–Galerkin type are given in [5].

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