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Probability Theory

A new class of interacting Markov chain Monte Carlo methods

Une nouvelle classe d'algorithmes de Monte Carlo par chaînes de Markov en interaction

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ABSTRACT

We present a new class of interacting Markov chain Monte Carlo methods to approximate numerically discrete-time nonlinear measure-valued equations. These stochastic processes belong to the class of self-interacting Markov chains with respect to their occupation measures. We provide several convergence results for these new methods including exponential estimates and a uniform convergence theorem with respect to the time parameter, yielding what seems to be the first results of this kind for this type of self-interacting models. We illustrate these models in the context of Feynman–Kac distribution semigroups arising in physics, biology and in statistics.

RÉSUMÉ

Nous présentons de nouveaux algorithmes Monte Carlo par chaînes de Markov en interaction pour la résolution de processus à valeurs mesures non linéaires à temps discret. Ces modèles appartiennent à la classe des chaînes de Markov en auto interaction avec leurs mesures d'occupations. Nous proposons une variété de résultats de convergence, avec notamment des estimations exponentielles et un théorème de convergence uniforme par rapport au paramètre temporel. Cette analyse semble être la premiere de ce type pour des chaînes de Markov en auto-interaction. Nous illustrons ces modèles dans le cadre de semigroupes de Feynman–Kac couramment utilisés en physique, en biologie, et en statistiques.

Version française abrégée

On notera par la suite $\mathcal{B}(E)$ l'espace des fonctions mesurables et bornées, et $\mathcal{P}(E)$ l'ensemble des mesures de probabilités, sur un espace mesurable $(E,\mathcal{E})$. Nous utiliserons par la suite les notations classiques de la théorie des opérateurs intégraux rappelées au début de la première section de la version anglaise.

On s'intéresse dans cette Note aux flots de mesures de probabilités $(\pi^{(l)})_{l\geqslant 0}$ sur des espaces mesurables $(S^{(l)},\mathcal{S}^{(l)})_{l\geqslant 0}$ solutions d'équations de la forme suivante

$$\forall l\geqslant 1 \quad \pi^{(l)}=\Phi_l\big(\pi^{(l-1)}\big) \quad \text{pour des transformations } \Phi_l:\mathcal{P}\big(S^{(l-1)}\big)\to\mathcal{P}\big(S^{(l)}\big).$$

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La résolution de telles équations est généralement un problème numérique très complexe qui nécessite d'énormes moyens de calculs. Un exemple typique est celui des semigroupes de Feynman–Kac définis en (3). Ces dernières années, ces modèles probabilistes et leurs interprétations particulaires, parfois appelées méthodes de Monte Carlo séquentielles, sont devenus des points de contact très actifs entre les probabilités, les statistiques Bayésiennes, la chimie théorique, la physique quantique et les sciences de l'ingénieur. Les livres [1,5,6] fournissent un panorama complet sur le sujet. Une des difficultés majeures des interprétations de type champ moyen provient du fait qu'il est impossible d'augmenter leur précision lors de leur exécution numérique contrairement aux algorithmes de Monte Carlo par chaîne de Markov. Pour résoudre cette importante question, nous présentons dans cette Note et dans sa version étendue [3] une nouvelle classe d'algorithmes de Monte Carlo par chaîne de Markov en interaction (*en abrégé i-MCMC*). Ces techniques de simulation stochastique complètent de façon naturelle les algorithmes fondés sur des interprétations particulaires de type champ moyen.

Cet algorithme stochastique est défini de la façon suivante : on se donne tout d'abord une transition markovienne $M^{(0)}$ sur l'espace de départ $S^{(0)}$, ainsi qu'une famille de transitions markoviennes $M_\mu^{(l)}$ sur $S^{(l)}$ indexées par le paramètre $l \geqslant 1$ et l'ensemble des mesures $\mu \in \mathcal{P}(S^{(l-1)})$. On se donne enfin une suite de mesures de probabilités ν_l sur les ensembles $S^{(l)}$, avec $l \geqslant 0$. On convient que $\pi^{(0)} = \pi^{(0)} M^{(0)}$ est une mesure invariante de $M^{(0)}$, et les mesures invariantes des opérateurs $M_\mu^{(l)}$ sont données par les mesures $\Phi_l(\mu) = \Phi_l(\mu) M_\mu^{(l)}$. Pour chaque $l \leqslant m$, on notera $\eta^{(l)} \in \mathcal{P}(S^{(l)})$ la mesure image d'une mesure $\eta \in \mathcal{P}(\prod_{0 \leqslant l \leqslant m} S^{(l)})$ sur le l-ième niveau $S^{(l)}$. On note $X^{(0)} := (X_n^{(0)})_{n \geqslant 0}$ une chaîne Markov sur $S^{(0)}$ de loi initiale ν_0 et de transition $M^{(0)}$. Pour chaque indice $k \geqslant 1$, connaissant une réalisation de la chaîne $X^{(k-1)} := (X_n^{(k-1)})_{n \geqslant 0}$ au niveau $(k-1)$, on définit le k-ième processus $X^{(k)} := (X_n^{(k)})_{n \geqslant 0}$ par la donnée d'une chaîne de Markov de loi initiale ν_k et de transitions élémentaires données par la formule suivante

$$\forall n \geqslant 0 \quad \mathbb{P}\big(X_{n+1}^{(k)} \in dx \,\big|\, X^{(k-1)}, X_n^{(k)}\big) = M^{(k)}_{\eta_n^{(k-1)}}\big(X_n^{(k)}, dx\big) \quad \text{avec} \quad \eta_n^{(k-1)} := \frac{1}{n+1} \sum_{p=0}^{n} \delta_{X_p^{(k-1)}}$$

L'idée naturelle derrière cet algorithme réside dans le fait que la k-ième chaîne $X_n^{(k)}$ se comporte approximativement comme une chaîne de Markov de transition $M^{(k)}_{\pi^{(k-1)}}$, dès que les mesures d'occupation $\eta_n^{(k-1)}$ de la chaîne au niveau inférieur sont assez proches de la mesure limite $\pi^{(k-1)}$. Par construction, pour chaque indice $m \geqslant 0$ le processus $\overline{X}_n^m := (X_n^{(l)})_{0 \leqslant l \leqslant m} \in E_m := (S^{(0)} \times \cdots \times S^{(m)})$ est une chaîne de Markov en auto-interaction avec sa mesure d'occupation $\overline{\eta}_n^{[m]} = \frac{1}{n+1} \sum_{p=0}^{n} \delta_{\overline{X}_n^m}$, de loi initiale $\bigotimes_{0 \leqslant l \leqslant m} \nu_l$ et de transition

$$\mathbb{P}\big(\overline{X}_{n+1}^m \in dx \,\big|\, \overline{X}_0^m, \ldots, \overline{X}_n^m\big) = K^{(m)}_{\overline{\eta}_n^{[m]}}\big(\overline{X}_n^m, dx\big) := \prod_{0 \leqslant l \leqslant m} M^{(l)}_{\eta_n^{(l-1)}}\big(X_n^{(l)}, dx^l\big)$$

Dans la formule précédente, $dx := dx^0 \times \cdots \times dx^m$ représente un voisinage infinitésimal d'un point générique $x := (x^0, \ldots, x^m)$ dans l'ensemble produit E_m. Sous les conditions décrites en (7) et (8), la mesure produit $\overline{\pi}^{[m]} := \pi^{(0)} \otimes \cdots \otimes \pi^{(m)}$ satisfait l'équation de point fixe $\overline{\pi}^{[m]} = \overline{\pi}^{[m]} K_{\overline{\pi}^{[m]}}$. De plus, nous montrons que pour tout $r \geqslant 1$, $m \geqslant 1$, et toute function $f \in \mathcal{B}(E_m)$ nous avons

$$\sup_{n \geqslant 1} \sqrt{n}\, \mathbb{E}\big(\big|\overline{\eta}_n^{[m]}(f) - \overline{\pi}^{[m]}(f)\big|^r\big) < \infty$$

1. Mean field and self-interacting processes

For the convenience of the reader we have collected some of the main notation used in the Note. We denote by $\mathcal{B}(E)$ the Banach space of all bounded and measurable functions f on some measurable space $(E, \mathcal{E})$ equipped with the supremum norm $\|f\| := \sup_{x \in E} |f(x)|$. We also let $\mathcal{B}_1(E) \subset \mathcal{B}(E)$ denote the unit ball of functions f with norm $\|f\| \leqslant 1$, $\mathcal{M}(E)$ the set of all finite signed measures on E, and $\mathcal{P}(E)$ the convex subset of all probability measures. We recall that a bounded integral operator M from a measurable space $(E, \mathcal{E})$ into an auxiliary measurable space $(F, \mathcal{F})$, is an operator $f \mapsto M(f)$ from $\mathcal{B}(F)$ into $\mathcal{B}(E)$ such that the functions $M(f)(x) = \int_F M(x, dy) f(y)$ are $\mathcal{E}$-measurable and bounded, for any $f \in \mathcal{B}(F)$. A bounded integral operator M from $(E, \mathcal{E})$ into an auxiliary measurable space $(F, \mathcal{F})$ also generates a dual operator $\mu \mapsto \mu M$ from $\mathcal{M}(E)$ into $\mathcal{M}(F)$ defined by $(\mu M)(f) := \mu(M(f))$. We slightly abuse the notation and denote by $\|M\| := \sup_{f \in \mathcal{B}_1(F)} \|M(f)\|$ the norm of the operator $f \mapsto M(f)$. Also, we equip the Banach space $\mathcal{M}(E)$ with the corresponding total variation norm $\|\mu\| = \sup_{f \in \mathcal{B}_1(E)} |\mu(f)|$.

Let $(S^{(l)}, \mathcal{S}^{(l)})_{l \geqslant 0}$ be a sequence of measurable spaces. Suppose we have a sequence of probability measures $\pi^{(l)} \in \mathcal{P}(S^{(l)})$ that satisfies a nonlinear equation of the following form

$$\forall l \geqslant 1 \quad \pi^{(l)} = \Phi_l\big(\pi^{(l-1)}\big) \quad \text{for some mappings } \Phi_l : \mathcal{P}\big(S^{(l-1)}\big) \to \mathcal{P}\big(S^{(l)}\big) \tag{1}$$

Except in some particular situations, the numerical solving of these measure-valued equations requires intensive calculations. For instance, the typical example of mappings Φ_l are the Feynman–Kac transformations given by

$$\forall l \geqslant 0 \quad \forall (\mu, f) \in \big(\mathcal{P}(S^{(l)}) \times \mathcal{B}(S^{(l+1)})\big), \qquad \Phi_{l+1}(\mu)(f) := \mu\big(G_l L_{l+1}(f)\big)/\mu(G_l) \tag{2}$$

In the above display, G_l is a positive potential function on $S^{(l)}$, and L_{l+1} stands for a collection of Markov transitions from $S^{(l)}$ into $S^{(l+1)}$. In this situation, the solution of the measure-valued equation (1) is given by the normalized Feynman–Kac distribution flow described by

$$\pi^{(l)}(f) = \gamma^{(l)}(f)/\gamma^{(l)}(1) \quad \text{with} \quad \gamma^{(l)}(f) := \mathbb{E}\left(f(Y_l) \prod_{0 \leqslant k < l} G_k(Y_k) \right) \tag{3}$$

where $(Y_l)_{l \geqslant 0}$ stands for a Markov chain taking values in the state spaces $(S^{(l)})_{l \geqslant 0}$, with initial distribution $\pi^{(0)}$ and Markov transitions $(L_l)_{l \geqslant 1}$. Even if they look simple, these Feynman–Kac distribution flows are very complex mathematical objects.

These probabilistic models and their mean field interacting particle interpretations are one of the most active contact points between probability, Bayesian statistics, theoretical chemistry, quantum physics, and engineering sciences, including rare event analysis and advanced signal processing. These stochastic algorithms are sometimes referred to as sequential Monte Carlo methods, population Monte Carlo methods, or branching and interacting particle systems. It is clearly out of the scope of this article to detail these stochastic models; we recommend the interested reader consult the pair of books [1,5] and the references therein. Contrary to Markov chain Monte Carlo algorithms [7], one drawback of these particle models is that it is impossible to increase iteratively the precision of the algorithm.

To solve this question, we design in this Note and in its extended version [3] a new class of interacting Markov chain Monte Carlo methods (*i-MCMC*). We emphasize that this new i-MCMC methodology can be combined easily with the mean field particle model discussed above. This class of interacting stochastic algorithms can be interpreted as a dynamic and adaptive simulation algorithm which takes advantage of the information carried by the past history to increase the quality of the next series of samples. One critical aspect of i-MCMC as opposed to any other numerical method is that it provides a natural adaptation and reinforced learning strategy of the physical or engineering evolution equation at hand. Another advantage of these probabilistic techniques is that they do not use any regularity assumption on the coefficients of the models. Finally, in contrast to more traditional mean field type particle models and related sequential Monte Carlo techniques, these stochastic algorithms increase iteratively the precision of the numerical approximation.

The origins of i-MCMC stochastic algorithms can be traced back to the article [2] co-authored by the first author with Laurent Miclo. This article was concerned with abstract biology-inspired self-interacting Markov chains with applications to genetic type algorithms involving a competition between the natural reinforcement mechanisms and the potential attraction of a given exploration landscape. Although our investigations have been partly influenced by this work, the present article focuses on a different class of self-interacting models with a prescribed flow of limiting distributions solving a given measure-valued equation (1). A self-interacting model was presented recently in [4, Section 3.1] by the two authors in collaboration with Ajay Jasra. However, it was not studied theoretically. The models presented here are much more general. Moreover, our main contribution is to develop a refined theoretical analysis based on measure-valued processes and semigroup ideas to analyze their asymptotic behavior as the time parameter tends to infinity. Our analysis includes exponential estimates and a uniform convergence theorem with respect to the level parameter, yielding what seems to be the first results of this kind for this type of self-interacting models.

2. The i-MCMC methodology

We consider a Markov transition $M^{(0)}$ from $S^{(0)}$ into itself, and a collection of Markov transitions $M_\mu^{(l)}$ from $S^{(l)}$ into itself, indexed by the parameter $l \geqslant 1$ and the set of probability measures $\mu \in \mathcal{P}(S^{(l-1)})$. We further assume that the invariant measure of each operator $M_\mu^{(l)}$ is given by $\Phi_l(\mu)$; that is we have that

$$\pi^{(0)} = \pi^{(0)} M^{(0)} \quad \text{and} \quad \forall l \geqslant 1 \quad \forall \mu \in \mathcal{P}\big(S^{(l-1)}\big), \qquad \Phi_l(\mu) = \Phi_l(\mu) M_\mu^{(l)}$$

The choice of the transitions $M^{(0)}$ and $M_\mu^{(l)}$ is clearly not unique, we can choose for instance $M^{(0)}(x, .) = \pi^{(0)}$ and $M_\mu^{(l)}(x, .) = \Phi_l(\mu)$. For every $l \leqslant m$, we denote by $\eta^{(l)} \in \mathcal{P}(S^{(l)})$ the image measure of a measure $\eta \in \mathcal{P}(\prod_{0 \leqslant l \leqslant m} S^{(l)})$ on the l-th level set $S^{(l)}$. We also set a series of probability measures ν_l on $S^{(l)}$, with $l \geqslant 0$. We let $X^{(0)} := (X_n^{(0)})_{n \geqslant 0}$ be a Markov chain on $S^{(0)}$ with initial distribution ν_0 and elementary Markov transition $M^{(0)}$. For every $k \geqslant 1$, given a realization of the chain $X^{(k-1)} := (X_n^{(k-1)})_{n \geqslant 0}$, the k-th level chain $X_n^{(k)}$ is a Markov chain with initial distribution ν_k and with random Markov transitions $M^{(k)}_{\eta_n^{(k-1)}}$ depending on the current occupation measures $\eta_n^{(k-1)}$ of the chain at level $(k-1)$

$$\mathbb{P}\big(X_{n+1}^{(k)} \in dx \,\big|\, X^{(k-1)}, X_n^{(k)}\big) = M^{(k)}_{\eta_n^{(k-1)}}\big(X_n^{(k)}, dx\big) \quad \text{with} \quad \eta_n^{(k-1)} := \frac{1}{n+1} \sum_{p=0}^{n} \delta_{X_p^{(k-1)}} \tag{4}$$

The rationale behind this is that the k-th level chain $X_n^{(k)}$ behaves asymptotically as a Markov chain with time homogeneous elementary transition $M^{(k)}_{\pi^{(k-1)}}$, as long as $\eta_n^{(k-1)}$ is a good approximation of $\pi^{(k-1)}$.

In the special case where $M_\mu^{(k)}(x^k,.)=\Phi_k(\mu)$, the k-th level chain $(X_n^{(k)})_{n\geqslant 1}$ is a collection of independent random variables with distributions $(\Phi_k(\eta_{n-1}^{(k-1)}))_{n\geqslant 1}$. To get an intuitive understanding of the dynamic of these i-MCMC algorithms, we provide a brief description of the i-MCMC model associated with the Feynman–Kac distribution flows (2). In this situation, we first observe that

$$\Phi_k\left(\frac{1}{n+1}\sum_{0\leqslant p\leqslant n}\delta_{X_p^{(k-1)}}\right)=\sum_{0\leqslant p\leqslant n}\frac{G_{k-1}(X_p^{(k-1)})}{\sum_{0\leqslant p'\leqslant n}G_{k-1}(X_{p'}^{(k-1)})}L_k\big(X_p^{(k-1)},.\big)$$

From this observation, we see that each random state $X_n^{(k)}$ is sampled according to two separate genetic type mechanisms. First, we randomly select one state $X_p^{(k-1)}$ at level $(k-1)$, with a probability proportional to its potential value $G_{k-1}(X_p^{(k-1)})$. Second, we randomly evolve from this state according to the exploration transition L_k. This biology-inspired i-MCMC model can be interpreted as a spatial branching and interacting process. In this interpretation, the k-th chain tends to duplicate individuals with large potential functions. The selected offspring randomly evolve from the state space $S^{(k-1)}$ to the state space $S^{(k)}$ at the next level. The same description for path space models coincides with the evolution of genealogical tree based i-MCMC models.

We end this section with a self-interacting Markov chain interpretation of the stochastic algorithm discussed above. We consider the product space $E_m:=(S^{(0)}\times\cdots\times S^{(m)})$ and we let $(K_\eta^{(m)})_{\eta\in\mathcal{P}(E_m)}$ be the collection of Markov transitions from E_m into itself given by

$$\forall x:=\big(x^0,\ldots,x^m\big)\in E_m,\qquad K_\eta^{[m]}(x,dy)=\prod_{0\leqslant l\leqslant m}M_{\eta^{(l-1)}}^{(l)}\big(x^l,dy^l\big)\tag{5}$$

Here $dy:=dy^0\times\cdots\times dy^m$ stands for an infinitesimal neighborhood of a generic point $y:=(y^0,\ldots,y^m)\in E_m$, and $\eta^{(l)}\in\mathcal{P}(S^{(l)})$ stands for the image measure of a measure $\eta\in\mathcal{P}(E_m)$ on the l-th level set $S^{(l)}$, where $m\geqslant l$. In this notation, it is readily checked that $\overline{X}_n^m:=(X_n^{(l)})_{0\leqslant l\leqslant m}$ is an E_m-valued self-interacting Markov chain with elementary transitions defined by

$$\forall n\geqslant 0\quad \mathbb{P}\big(\overline{X}_{n+1}^m\in dy\mid \overline{X}_0^m,\ldots,\overline{X}_n^m\big)=K_{\overline{\eta}_n^{[m]}}^{[m]}\big(\overline{X}_n^m,dy\big)\quad\text{with}\quad \overline{\eta}_n^{[m]}=\frac{1}{n+1}\sum_{p=0}^{n}\delta_{\overline{X}_n^m}\tag{6}$$

3. Asymptotic analysis

We assume that there exists a collection of bounded integral operators Γ_l from $\mathcal{B}(S^{(l)})$ into $\mathcal{B}(S^{(l-1)})$ such that the mappings $\Phi_l:\mathcal{P}(S^{(l-1)})\to\mathcal{P}(S^{(l)})$ satisfy the following regularity conditions

$$\big|\big[\Phi_l(\mu)-\Phi_l(\nu)\big](f)\big|\leqslant\int\big|[\mu-\nu](g)\big|\Gamma_l(f,dg)\quad\text{with}\quad\int\Gamma_l(f,dg)\|g\|\leqslant\Lambda_l\|f\|\tag{7}$$

for some finite constant $\Lambda_l<\infty$. We also suppose that there exist some integer $n_l\geqslant 0$ and some constant c_l such that for any pair of measures $(\eta,\mu)\in\mathcal{P}(S^{(l-1)})^2$ we have

$$\big\|M_\eta^{(l)}-M_\mu^{(l)}\big\|\leqslant c_l\|\eta-\mu\|\quad\text{and}\quad b_l(n_l):=\sup_{\eta\in\mathcal{P}(S^{(l-1)})}\beta\big(\big(M_\eta^{(l)}\big)^{n_l}\big)<1\tag{8}$$

In the special case where $M_\mu^{(l)}(x^l,.)=\Phi_l(\mu)$, the second condition is trivially met for $n_l=1$ with $b_l(n_l)=0$. The first condition is related to the Lipschitz property of the mapping Φ_l. In this particular situation, it takes the following form $\|\Phi_l(\eta)-\Phi_l(\mu)\|\leqslant c_l\|\eta-\mu\|$. For the Feynman–Kac transformations (2), we prove in [1] that this condition is satisfied with $c_l=\beta(L_l)/\epsilon_{l-1}(G)$ as long as the potential functions G_l are chosen so that $G_l(x)\geqslant\epsilon_l(G)G_l(y)$ for any $(x,y)\in(S^{(l)})^2$, for some sequence $(\epsilon_l(G))_{l\geqslant 0}\in(0,1)^{\mathbb{N}}$.

Under the conditions (8), the mapping

$$\omega_{K_\eta^{[m]}}:\eta\in\mathcal{P}(E_m)\to\omega_{K_\eta^{(m)}}(\eta):=\pi^{(0)}\otimes\Phi_1\big(\eta^{(0)}\big)\otimes\cdots\otimes\Phi_m\big(\eta^{(m-1)}\big)\in\mathcal{P}(E_m)$$

which associates to a measure $\eta\in\mathcal{P}(E_m)$ the invariant measure $\omega_{K_\eta^{[m]}}(\eta)$ of $K_\eta^{[m]}$ is a well-defined mapping with a fixed point $\overline{\pi}^{[m]}:=\pi^{(0)}\otimes\cdots\otimes\pi^{(m)}$. Our main results are basically stated as follows:

Theorem 3.1. *For any $r\geqslant 1$, $m\geqslant 1$, and any function $f\in\mathcal{B}(E_m)$ we have*

$$\sup_{n\geqslant 1}\sqrt{n}\,\mathbb{E}\big(\big|\overline{\eta}_n^{[m]}(f)-\overline{\pi}^{[m]}(f)\big|^r\big)<\infty$$

When the integral operators Γ_k have a uniformly finite support $\sup_{f\in\mathcal{B}(S^{(k)})} Card(Supp(\Gamma_k(f,.))) < \infty$*, we have the exponential inequality*

$$\forall t>0 \quad \limsup_{n\to\infty} \frac{1}{n}\log \mathbb{P}\left(\left|\left[\overline{\eta}_n^{[m]} - \overline{\pi}^{[m]}\right](f)\right| > t\right) < -\frac{t^2}{2\overline{\sigma}_m^2} \quad \text{for some } \overline{\sigma}_m < \infty$$

Theorem 3.2. *We suppose that the mappings Φ_l are given by the Feynman–Kac transformations* (2)*, with a pair of time homogeneous potential/transitions $(G_l, L_l) := (G, L)$ on some measurable state space $(S^{(l)}, \mathcal{S}^{(l)}) = (S, \mathcal{S})$. We further assume that there exist some integer $m \geqslant 1$ and a pair of constants $\epsilon(G), \epsilon(L) > 0$ such that*

$$\forall A\in\mathcal{S} \quad L^m(x,A) := \int L^{m-1}(x,dy)L(y,A) \geqslant \epsilon(L)L^m(y,A)$$

and $G(x) \geqslant \epsilon(G)G(y)$, for any $(x,y)\in S^2$. In this situation, we have the following uniform convergence estimate

$$\sup_{k\geqslant 0}\sup_{n\geqslant 1} n^{\alpha/2}\mathbb{E}\left(\left|\eta_n^{(k)}(f) - \pi^{(k)}(f)\right|^r\right) < \infty$$

for some parameter $\alpha\in(0,1]$ and for any bounded measurable function $f\in\mathcal{B}(S)$.

References

[1] P. Del Moral, Feynman–Kac Formulae. Genealogical and Interacting Particle Systems, with Applications, Probability and Its Applications, Springer-Verlag, New York, 2004.

[2] P. Del Moral, L. Miclo, On convergence of chains with time empirical self-interactions, Proc. Royal Soc. Lond. A 460 (2003) 325–346.

[3] P. Del Moral, A. Doucet, Interacting Markov chain Monte Carlo methods for solving nonlinear measure-valued equations, INRIA Research Report, January 2008, Ann. Appl. Probab. (2010), http://www.imstat.org/aap/future_papers.html, in press.

[4] P. Del Moral, A. Doucet, A. Jasra, Sequential Monte Carlo for Bayesian computation (with discussion), in: Bayesian Statistics, vol. 8, Oxford University Press, 2007, pp. 1–34.

[5] A. Doucet, N. de Freitas, N. Gordon (Eds.), Sequential Monte Carlo Methods in Practice, Statistics for Engineering and Information Science, Springer-Verlag, New York, 2001.

[6] J.S. Liu, Monte Carlo Methods for Scientific Computing, Statistics, Springer-Verlag, New York, 2001.

[7] C.P. Robert, G. Casella, Monte Carlo Statistical Methods, 2nd edition, Springer-Verlag, New York, 2004.