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Non-mutants with equal colored Jones polynomial

Alexander Stoimenow

BK21 Project, Department of Mathematical Sciences, KAIST, Daejeon 307-701, Republic of Korea

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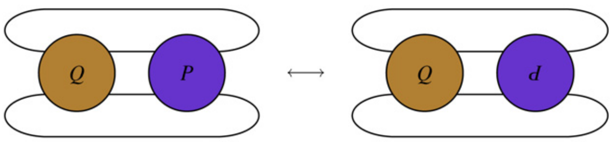
Abstract

We construct arbitrarily large (finite) families of hyperbolic non-mutant knots with equal colored Jones polynomial. ***To cite this article: A. Stoimenow, C. R. Acad. Sci. Paris, Ser. I 347 (2009).***

Résumé

Nœuds non-mutants avec le même polynôme de Jones colorié. On construit des familles (finies) de taille quelconque de nœuds hyperboliques non-mutants avec le même polynôme de Jones colorié. ***Pour citer cet article : A. Stoimenow, C. R. Acad. Sci. Paris, Ser. I 347 (2009).***

Mutation was introduced by Conway [1]. It is a procedure of rotating by π a 2-string tangle in a tangle decomposition of a link diagram D, along the axis perpendicular to the projection plane, or horizontal or vertical in the projection plane. For example:



(1)

(For oriented diagrams D one may need to reverse the strands of one of P or Q to get in- and outputs compatible.)

Knots are *mutants* if they are transformable into each other by a sequence of mutations. Mutants have been known as difficult to distinguish, and some attention is given in the literature to the question what invariants can separate mutants [3,13,14], and more particularly the application of link polynomials [6,4,10,7] on which satellites can do so [2].

Some time ago gradually the project emerged to find out exactly which low crossing knots in the tables of [5] are mutants. This work began implicitly in [16], and the first subtle examples it turned up were given in [15]. A more

E-mail address: stoimeno@mathsci.kaist.ac.kr.
URL: http://mathsci.kaist.ac.kr/~stoimeno.

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detailed study of such examples led to collaboration with Toshifumi Tanaka, and later Daniel Matei. The forthgoing work in [17] was motivated by a question in [8], also related to the Volume conjecture:

Question 1. (See [8, problem 1.91(2)].) Let K be a simple unoriented knot. Are there any knots K' other than mutants of K, which cannot be distinguished from K by the Jones polynomial and all its satellites? (Mutants and their satellites have equal Jones polynomial by [12].)

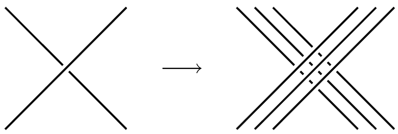
The *skein (HOMFLY) polynomial* P is given here by being 1 on the unknot and the *skein relation*

$$l^{-1}P\big(\times\big) + lP\big(\times\big) = -mP\big(\,)\,(\,\big). \qquad (2)$$

This convention uses the variables of [10], but differs from theirs by the interchange of l and l^{-1}. The *Jones polynomial* V can be obtained from P by the substitution

$$V(t) = P\big(-\sqrt{-1}t,\ \sqrt{-1}\big(t^{-1/2} - t^{1/2}\big)\big).$$

Let L be a link embedded in the solid torus $T = S^1 \times D^2$. If we embed T in S^3 so that its core $S^1 \times \{0\}$ represents a knot K, then we call the resulting embedding K' of L the *satellite of (companion) K with pattern L*. Satellite links involve a number of *parallel* strands



(3)

The satellite is defined up to the choice of framing of T, which is fixed by performing (3) in a diagram of writhe 0. The *colored Jones polynomial (CJP) J_L* is the set of all invariants obtained by applying the Jones polynomial to all satellites of L.

Two pairs of knots from [5] found in [17] answered Question 1, establishing that in general K' may exist for some K. The first found pair is 14_{41721} and 14_{42125}. To confirm equality of the CJP, we used a fusion formula due to Masbaum and Vogel [11]. To rule out mutation, we used the result of [9] and distinguished the knots by the Whitehead double HOMFLY polynomial. Later we obtained infinitely many pairs.

In this short Note we announce the following result, which uses an extension of the preceding construction. For space reasons the proof cannot be included here and will be given subsequently.

Theorem 2. *For any number n, there exists a family of n distinct hyperbolic knots with equal CJP, which are not mutants.*

We noted in [17] that by Thurston's work such an *infinite* family would contradict the Volume conjecture. Thus, regarding size at least, our statement is probably the maximal possible.

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