



Group Theory/Algebra

Irreducibility of the Lawrence–Krammer representation of the BMW algebra of type A_{n-1} [☆]

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Abstract

The Lawrence–Krammer representation introduced by Lawrence and Krammer in order to show the linearity of the braid group is generically irreducible. We show this fact and show further that for some values of its two parameters, when these are specialized to complex numbers, the representation becomes reducible. We describe what these values are and give a complete description of the dimensions of the invariant subspaces when the representation is reducible. **To cite this article:** *C. Levailant, C. R. Acad. Sci. Paris, Ser. I 347 (2009).*

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Résumé

Irréductibilité de la représentation de Lawrence–Krammer de l’algèbre BMW de type A_{n-1} . La représentation de Lawrence–Krammer, introduite par Lawrence et Krammer pour montrer la linéarité du groupe de tresses, est génériquement irréductible. On montre ce fait et on montre également que lorsque les deux paramètres de la représentation prennent certaines valeurs complexes, la représentation devient réductible. On donne ici toutes les valeurs des paramètres pour lesquelles la représentation est réductible, ainsi que les dimensions des sous-espaces stables. **Pour citer cet article :** *C. Levailant, C. R. Acad. Sci. Paris, Ser. I 347 (2009).*

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On considère, suivant la définition de [3], l’algèbre de Birman–Murakami–Wenzl \mathcal{B} de type A_{n-1} avec paramètres complexes non nuls l et m sur le corps $\mathbb{Q}(l, r)$, où les paramètres r et m sont reliés par $m = \frac{1}{r} - r$. Cette algèbre a pour générateurs g_1, \dots, g_{n-1} , qui satisfont aux relations de tresses et elle contient aussi d’autres éléments e_1, \dots, e_{n-1} , qui sont définis par les relations :

$$e_i = \frac{l}{m}(g_i^2 + mg_i - 1), \quad i = 1, \dots, n - 1.$$

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On travaille sur une représentation de degré $\frac{n(n-1)}{2}$ de cette algèbre dans l'espace de Lawrence–Krammer. Comme représentation du groupe de tresses à n brins, cette représentation est équivalente, à rééchelonnage près des générateurs, à la représentation du groupe d'Artin de type A_{n-1} basée sur les paramètres r et t , décrite dans [4]. Cette dernière représentation est elle-même équivalente à la représentation de Lawrence–Krammer basée sur les paramètres q et t et utilisée par Krammer et indépendamment par Bigelow pour montrer la linéarité du groupe de tresses à n brins (voir [6] et [1]). Le r de cette note est le $\frac{1}{r}$ de [3] et les deux paramètres r et l de [3] sont reliés aux paramètres t et r de [4] par $lt = \frac{1}{r^3}$. Les paramètres t et r de [4] sont eux-même reliés aux paramètres t et q de [6] par $q = r^2$ (cf. [4], Exemple 3.8). L'irréductibilité de la représentation de Lawrence–Krammer pour des valeurs génériques de q et t est contenue dans les travaux de Zinno (voir [12]), de Cohen–Gijssbers–Wales (voir [3]) et de Marin (voir [10]). D'autres travaux étudient aussi cette représentation et traitent de sa réductibilité. En particulier, certaines conditions suffisantes d'irréductibilité sont implicitement dans la thèse de Marin (voir [9]) et dans [2], Bigelow étudie le cas de réductibilité $t = \frac{1}{q}$. Le théorème suivant donne toutes les valeurs complexes des paramètres pour lesquelles la représentation est réductible et décrit les dimensions des sous-espaces stables irréductibles :

Théorème 0.1. *On se donne trois paramètres complexes non nuls l , m et r , où $m = \frac{1}{r} - r$ et un entier $n \geq 3$ et on suppose que l'algèbre de Iwahori–Hecke du groupe symétrique \mathcal{S}_n avec paramètre r^2 sur le corps $\mathbb{Q}(l, r)$ est semisimple.*

- *La représentation de Lawrence–Krammer de l'algèbre BMW $\mathcal{B}(A_{n-1})$ avec paramètres l et m sur le corps $\mathbb{Q}(l, r)$ est irréductible, sauf quand :*
 - $l = r$ et $n \geq 4$: il existe alors un unique sous-espace stable irréductible de dimension $\frac{n(n-3)}{2}$ dans l'espace de Lawrence–Krammer.
 - $l = -r^3$: il existe alors un sous-espace stable irréductible de dimension $\frac{(n-1)(n-2)}{2}$ dans l'espace de Lawrence–Krammer. De plus, celui-ci est unique sauf pour $n = 3$ et $r^6 = -1$.
 - $l = \frac{1}{r^{2n-3}}$: il existe alors un sous-espace stable unidimensionnel dans l'espace de Lawrence–Krammer. De plus, celui-ci est unique, sauf pour $n = 3$ et $r^6 = -1$.
 - $l = \frac{1}{r^{n-3}}$ ou $l = -\frac{1}{r^{n-3}}$: il existe alors un unique sous-espace stable irréductible de dimension $(n-1)$ dans l'espace de Lawrence–Krammer.
- *Quand $n = 3$ et $l = -r^3 = \frac{1}{r^3}$, il existe exactement deux sous-espaces stables unidimensionnels.*
- *Les sous-espaces stables irréductibles mentionnés sont les seuls sous-espaces stables irréductibles pouvant apparaître dans l'espace de Lawrence–Krammer.*
- *Pour ces valeurs des paramètres, la représentation est réductible et indécomposable, donc l'algèbre BMW n'est pas semisimple.*

On se donne deux paramètres complexes non nuls t et q , où q n'est pas une racine k -ième de l'unité pour tout entier k tel que $1 \leq k \leq n$. La représentation de Lawrence–Krammer du groupe de tresses B_n à n brins, basée sur les paramètres q et t est réductible si et seulement si

$$t \in \left\{ \frac{1}{q}, -1, \frac{1}{q^n}, \frac{1}{\sqrt{q^n}}, -\frac{1}{\sqrt{q^n}} \right\} \quad \text{quand } n \geq 4,$$

$$t \in \left\{ -1, \frac{1}{q^3}, \frac{1}{\sqrt{q^3}}, -\frac{1}{\sqrt{q^3}} \right\} \quad \text{dans le cas de } B_3.$$

1. The representation

Let l and m be two nonzero complex parameters and let r and $-\frac{1}{r}$ be the two complex roots of the quadratics $X^2 + mX - 1$. Let n be an integer with $n \geq 3$. Throughout the note, we will assume that the Iwahori–Hecke algebra \mathcal{H}_n of the Symmetric group \mathcal{S}_n with parameter r^2 over the field $\mathbb{Q}(l, r)$ is semisimple. This assumption is met exactly when $r^{2k} \neq 1$ for every integer k with $1 \leq k \leq n$ (see [8]). We consider the BMW algebra \mathcal{B} of type A_{n-1} with parameters l and m over the field $\mathbb{Q}(l, r)$, as defined in [3]. This algebra has two sets of $n-1$ elements: the g_i s,

$i = 1, \dots, n - 1$, that satisfy the braid relations and generate \mathcal{B} and the e_i s, $i = 1, \dots, n - 1$, that are related to the g_i s by

$$e_i = \frac{l}{m}(g_i^2 + mg_i - 1).$$

We define \mathcal{H} to be the Iwahori–Hecke algebra of the symmetric group \mathcal{S}_{n-2} over the field $\mathbb{Q}(l, r)$ with generators g_3, \dots, g_{n-1} that satisfy the braid relations and the relation $g_i^2 + mg_i = 1$ for all i . The base field $\mathbb{Q}(l, r)$ is an \mathcal{H} -module for the action $g_i \cdot 1 = r$, where $i = 3, \dots, n - 1$. We define \mathcal{B}_1 to be the \mathcal{B} -module- \mathcal{H} :

$$\mathcal{B}e_1 / (\mathcal{B}e_i e_1 | i = 3, \dots, n - 1).$$

When $n = 3$, \mathcal{B}_1 is simply $\mathcal{B}e_1$. We now obtain a \mathcal{B} -module of dimension $\frac{n(n-1)}{2}$ over $\mathbb{Q}(l, r)$ by considering the tensor product:

$$\mathcal{V}^{(n)} = \mathcal{B}_1 \otimes_{\mathcal{H}} \mathbb{Q}(l, r).$$

This \mathcal{B} -module is precisely the generically irreducible representation that we study throughout this note. As a matter of fact, each product of the algebra $e_i e_j$, for non-adjacent nodes i and j , acts trivially on $\mathcal{V}^{(n)}$. Then, by [3], this left representation of \mathcal{B} must be equivalent to the Lawrence–Krammer representation of the BMW algebra of type A_{n-1} . As a representation of the braid group on n strands, it is equivalent up to some rescaling of the generators to the representation based on the two parameters q and t , introduced by Krammer in [6] to show that the braid group on n strands is linear. The link between the parameters l and r of this paper and the parameters q and t of Krammer’s representation is given by $q = \frac{1}{r^2}$ and $lt = r^3$.

2. Reducibility of the representation

Our main theorem is the following; details of the proof appear in [7]:

Theorem 2.1. *Let l, m and r be three nonzero complex parameters, where m and r are related by $m = \frac{1}{r} - r$. Assume that the Iwahori–Hecke algebra \mathcal{H}_n of the symmetric group \mathcal{S}_n with parameter r^2 over the field $\mathbb{Q}(l, r)$ is semisimple, that is assume that $r^{2k} \neq 1$ for every integer $k \in \{1, \dots, n\}$.*

When $n \geq 4$, the Lawrence–Krammer representation of the BMW algebra $\mathcal{B}(A_{n-1})$ with parameters l and m over the field $\mathbb{Q}(l, r)$ is irreducible, except when $l \in \{r, -r^3, \frac{1}{r^{2n-3}}, \frac{1}{r^{n-3}}, -\frac{1}{r^{n-3}}\}$ when it is reducible.

When $n = 3$, the Lawrence–Krammer representation of the BMW algebra $\mathcal{B}(A_2)$ with parameters l and m over the field $\mathbb{Q}(l, r)$ is irreducible except when $l \in \{-r^3, \frac{1}{r^3}, 1, -1\}$ when it is reducible.

Proof. (Sketch) We show that if there exists a proper invariant subspace \mathcal{W} of $\mathcal{V}^{(n)}$, then all the e_i s and g_k conjugates of the e_i s, $i = 1, \dots, n - 1$, must annihilate \mathcal{W} . In particular, the action of \mathcal{B} on \mathcal{W} is a Iwahori–Hecke algebra action. For the small values $n \in \{3, 4, 5, 6\}$, we choose a basis of $\mathcal{V}^{(n)}$ and compute the matrix $M(n)$ of the left action of the BMW algebra element

$$\sum_{1 \leq i \leq n-1} e_i + \sum_{1 \leq i < j-1 < n} g_{j-1}^{-1} \cdots g_{i+1}^{-1} e_i g_{i+1} \cdots g_{j-1}$$

in this basis. If \mathcal{W} is nontrivial, the determinant of this matrix must be zero. By using Maple, it yields the values for l and r that are described in the theorem. Conversely, for the values of l and r that annihilate this determinant, we find some nonzero vectors in the kernel $K(n)$ of the matrix $M(n)$. We show that $K(n)$ is a proper submodule of $\mathcal{V}^{(n)}$. It then gives the equivalence of Theorem 2.1 for these small values of n . When n is large enough, that is when $n \geq 7$, the irreducible representations of \mathcal{H}_n have degrees $1, n - 1, \frac{n(n-3)}{2}, \frac{(n-1)(n-2)}{2}$ or degrees greater than $\frac{(n-1)(n-2)}{2}$, except in the case $n = 8$, when they have degrees $1, 7, 14, 20$ or 21 (see [5] and [8]). We show that there exists a one-dimensional invariant subspace of $\mathcal{V}^{(n)}$ if and only if $l = \frac{1}{r^{2n-3}}$ and that there exists an irreducible $(n - 1)$ -dimensional invariant subspace of $\mathcal{V}^{(n)}$ if and only if $l \in \{\frac{1}{r^{n-3}}, -\frac{1}{r^{n-3}}\}$. We then proceed by induction on n . First, let $n = 7$ or $n \geq 9$ and suppose Theorem 2.1 holds for representations of $\mathcal{B}(A_k)$ where $k \in \{n - 3, n - 2\}$. Suppose that there exists a

proper invariant subspace \mathcal{W} of $\mathcal{V}^{(n)}$ of dimension greater than or equal to $\frac{n(n-3)}{2}$. Then the intersections $\mathcal{W} \cap \mathcal{V}^{(n-1)}$ and $\mathcal{W} \cap \mathcal{V}^{(n-2)}$ must be nontrivial. By induction, l must then belong to

$$\left\{ r, -r^3, \frac{1}{r^{2n-5}}, \frac{1}{r^{n-4}}, -\frac{1}{r^{n-4}} \right\} \cap \left\{ r, -r^3, \frac{1}{r^{2n-7}}, \frac{1}{r^{n-5}}, -\frac{1}{r^{n-5}} \right\}.$$

We assumed that $r^{2k} \neq 1$ for every $k = 1, \dots, n$. Then, by inspection, we see that the only possibility is to have $l \in \{r, -r^3\}$. Let us deal with the case $n = 8$. The case $n = 8$ is in fact not different. Indeed, if \mathcal{W} has dimension greater than or equal to 14, then again, the spaces $\mathcal{W} \cap \mathcal{V}^{(7)}$ and $\mathcal{W} \cap \mathcal{V}^{(6)}$ are nontrivial. Thus, we have proven that if the representation is reducible, it forces the values of the theorem for l and r . It remains to show that when $l = r$ or $l = -r^3$, the representation is reducible. In each case, we verify that the nonzero vector belonging to $K(5) \cap \mathcal{V}^{(4)}$ that we found with Maple also belongs to all the $K(n)$ s for $n \geq 6$. This shows the reducibility of the representation in both cases. \square

Remark 1. In [7], we also give a complete proof without using Maple for the small cases $n \in \{3, 4, 5, 6\}$.

Remark 2. In [11], Hans Wenzl states that $\mathcal{B}(A_{n-1})$ is semisimple, except possibly if r is a root of unity or l is some power of r , where he also considers complex parameters. Here Theorem 2.1 and the method that we use imply that for these specific values of l and r , the algebra is not semisimple as the representation is then reducible and indecomposable.

3. Dimensions of the invariant subspaces when the representation is reducible

In this section we give a series of theorems on the dimensions of the invariant subspaces when the representation is reducible. We still assume that \mathcal{H}_n is semisimple. We have the following results:

Theorem 3.1. *Let n be an integer with $n \geq 4$. There exists a one-dimensional invariant subspace of $\mathcal{V}^{(n)}$ if and only if $l = \frac{1}{r^{2n-3}}$. If so, it is unique.*

(Case $n = 3$) *There exists a one-dimensional invariant subspace of $\mathcal{V}^{(3)}$ if and only if $l \in \{-r^3, \frac{1}{r^3}\}$. If $r^6 \neq -1$, it is unique. If $r^6 = -1$, there exists exactly two one-dimensional invariant subspaces of $\mathcal{V}^{(3)}$.*

Theorem 3.2. *Let n be an integer with $n \geq 3$ and $n \neq 4$. There exists an irreducible $(n-1)$ -dimensional invariant subspace of $\mathcal{V}^{(n)}$ if and only if $l \in \{\frac{1}{r^{n-3}}, -\frac{1}{r^{n-3}}\}$. If so, it is unique.*

(Case $n = 4$) *There exists a 3-dimensional invariant subspace of $\mathcal{V}^{(4)}$ if and only if $l \in \{\frac{1}{r}, -\frac{1}{r}, -r^3\}$. If so, it is unique.*

Theorem 3.3. *Let n be an integer with $n \geq 4$. There exists an irreducible $\frac{n(n-3)}{2}$ -dimensional invariant subspace of $\mathcal{V}^{(n)}$ if and only if $l = r$. If so, it is unique.*

Theorem 3.4. *Let n be an integer with $n \geq 5$. There exists an irreducible $\frac{(n-1)(n-2)}{2}$ -dimensional invariant subspace of $\mathcal{V}^{(n)}$ if and only if $l = -r^3$. If so, it is unique.*

Theorem 3.5. *These are the only irreducible invariant subspaces that may appear in $\mathcal{V}^{(n)}$.*

Proof of Theorem 3.1. (Sketch) Theorem 3.1 is easily proven by hand. Indeed, if v is a spanning vector of a one-dimensional invariant subspace of $\mathcal{V}^{(n)}$, and if $\lambda_1, \dots, \lambda_{n-1}$ are scalars such that $g_i \cdot v = \lambda_i v$, then since the e_i s all annihilate v , we get $\lambda_i^2 + m\lambda_i - 1 = 0$ for each i , so that the λ_i s must take the values r or $-\frac{1}{r}$. Moreover, by the braid relations $g_i g_{i+1} g_i = g_{i+1} g_i g_{i+1}$ for every node $i \in \{1, \dots, n-2\}$, and the fact that $(r^2)^2 \neq 1$, we see that all the λ_i s must take the same value r or $-\frac{1}{r}$. Except when $n = 3$, where both values are possible, each of which respectively forcing $l = \frac{1}{r^3}$ and $l = -r^3$, in the case when $n \geq 4$, the λ_i s must all equal r . This forces the value $\frac{1}{r^{2n-3}}$ for l as well as a unique (up to multiplication by a scalar) spanning vector. \square

Proof of Theorem 3.2. (Sketch) When $n \geq 4$ and $n \neq 6$ (resp. $n = 6$) there are exactly two (resp. four) inequivalent irreducible representations of \mathcal{H}_n of degree $(n - 1)$. When $n \geq 5$ and $n \neq 6$ (resp. $n = 6$), we show that only one of these two (resp. four) representations may occur inside $\mathcal{V}^{(n)}$. Moreover, this representation occurs when $l \in \{-\frac{1}{r^{n-3}}, -\frac{1}{r^{n-3}}\}$ and leads in each case to a unique irreducible invariant subspace of $\mathcal{V}^{(n)}$ of dimension $(n - 1)$. When $n = 4$, both may occur, the first one leading to the values $l \in \{\frac{1}{r}, -\frac{1}{r}\}$ and the conjugate one forcing the value $l = -r^3$. For each of the three (distinct) values of l , we can show the uniqueness of the 3-dimensional irreducible invariant subspace of $\mathcal{V}^{(4)}$. When $n = 3$, the irreducible representation of \mathcal{H}_3 of degree 2 is self-conjugate and occurs in $\mathcal{V}^{(3)}$ for $l \in \{1, -1\}$. Again, for each of these values, there is a unique irreducible 2-dimensional invariant subspace in $\mathcal{V}^{(3)}$. \square

Proof of Theorem 3.3. (Sketch) We deal with the case $n = 4$ directly by hand. When $n \geq 5$, we show that when $l = r$, the rank of the matrix $M(n)$ is greater than or equal to n . This is simply achieved by exhibiting an invertible submatrix of $M(n)$ of size n . If $k(n)$ denotes the dimension of the kernel $K(n)$, this then yields the inequality on the dimension $k(n) \leq \frac{n(n-3)}{2}$. Also, we show that the \mathcal{B} -module $K(n)$ is irreducible. Let us recall from above that any irreducible proper invariant subspace of $\mathcal{V}^{(n)}$ is an irreducible \mathcal{H}_n -module. When $n \geq 5$ and $n \neq 8$, it thus has dimension 1, $(n - 1)$, $\frac{n(n-3)}{2}$, $\frac{(n-1)(n-2)}{2}$ or dimension greater than $\frac{(n-1)(n-2)}{2}$ by [5] and [8]. This fact on the dimensions and Theorems 3.1 and 3.2 imply that the dimension $k(n)$ of $K(n)$ is in fact greater than or equal to $\frac{n(n-3)}{2}$. The latter inequality also holds for $n = 8$, but this case needs to be dealt with separately. Details can be found in [7], §9.2. Thus, when $l = r$, the \mathcal{B} -module $K(n)$ is an irreducible invariant subspace of $\mathcal{V}^{(n)}$ of dimension $\frac{n(n-3)}{2}$. Since from our discussion at the beginning of part 2, any proper invariant subspace of $\mathcal{V}^{(n)}$ must be contained in $K(n)$, the uniqueness is established.

Conversely, we show that the existence of an irreducible $\frac{n(n-3)}{2}$ -dimensional invariant subspace of $\mathcal{V}^{(n)}$ implies that $l = r$ and we take the following path. First, we examine the case $n = 5$ by hand. We then proceed by induction on n . Except when $n = 7$, there are exactly two inequivalent irreducible representations of \mathcal{H}_n of degree $\frac{n(n-3)}{2}$. By inspection for $n = 5$, only one of them may occur inside $\mathcal{V}^{(5)}$. This is part of our induction hypothesis. The proof is then completed by using the branching rule. The case $n = 7$ is in fact not an exception and also follows from the branching rule, after showing that the Specht modules $S^{(3,3)}$ and $S^{(2,2,2)}$ cannot occur in $\mathcal{V}^{(6)}$. For details, see [7], §8.3. \square

Proof of Theorem 3.4. (Sketch) First, if there exists an irreducible $\frac{(n-1)(n-2)}{2}$ -dimensional invariant subspace \mathcal{W} of $\mathcal{V}^{(n)}$, for $n \geq 6$, its dimension is large enough to make the intersections $\mathcal{W} \cap \mathcal{V}^{(n-1)}$ and $\mathcal{W} \cap \mathcal{V}^{(n-2)}$ nontrivial and to thus force $l \in \{r, -r^3\}$. Since when $l = r$, we know that $k(n) = \frac{n(n-3)}{2}$, which implies in particular that any proper invariant subspace of \mathcal{V} must have dimension less than or equal to $\frac{n(n-3)}{2} = \frac{(n-1)(n-2)}{2} - 1$, we conclude that it is impossible to have $l = r$. Thus, $l = -r^3$. The case $n = 5$ needs to be dealt with separately. Details appear in [7], §9.2.

Conversely, suppose $l = -r^3$ and let $n \geq 5$. There are two cases:

- If $r^{2n} \neq -1$, then we can show that $K(n)$ is irreducible (this fact also holds for $n = 4$). Moreover, we can show that the rank of the matrix $M(n)$ is greater than or equal to $(n - 1)$, by exhibiting an invertible submatrix of $M(n)$ of size $(n - 1)$. Then $k(n) \leq \frac{(n-1)(n-2)}{2}$. By the previous studies it forces $k(n) = \frac{(n-1)(n-2)}{2}$, the case $n = 8$ being dealt with separately (see [7], §9.3). Hence there exists an irreducible $\frac{(n-1)(n-2)}{2}$ invariant subspace of $\mathcal{V}^{(n)}$.
- If $r^{2n} = -1$, then $K(n)$ is no longer irreducible, but $r^{2(n-1)} \neq -1$, so that $K(n - 1)$ is irreducible. Still by the previous case when $n \geq 6$ and by Theorem 3.2 when $n = 5$, we have $k(n - 1) = \frac{(n-2)(n-3)}{2}$. By the discussion at the end of part 2, we know that $K(n) \cap \mathcal{V}^{(n-1)}$ is nontrivial. It follows that

$$K(n) \cap \mathcal{V}^{(n-1)} = K(n - 1),$$

by irreducibility of $K(n - 1)$. This yields on the dimensions: $k(n) \leq k(n - 1) + (n - 1)$. Also, we have $K(n - 1) \subset K(n)$. Further, we can show that: $k(n) \geq k(n - 1) + (n - 2)$. With $k(n - 1) = \frac{(n-2)(n-3)}{2}$, both inequalities now yield $k(n) \in \{\frac{(n-1)(n-2)}{2}, \frac{(n-1)(n-2)}{2} + 1\}$. We then get $k(n) = 1 + \frac{(n-1)(n-2)}{2}$, as otherwise, the existence of a one-dimensional invariant subspace would force the existence of an irreducible $\frac{n(n-3)}{2}$ -dimensional invariant subspace by semisimplicity of \mathcal{H}_n . By Theorem 3.3, l would then be r , which contradicts our assumption $l = -r^3$. From there, it appears that there exists an irreducible $\frac{(n-1)(n-2)}{2}$ -dimensional invariant subspace of $\mathcal{V}^{(n)}$. We can show that it is unique. \square

Remark 3. The joint proof of Theorems 3.3 and 3.4 is given in [7], where we do not use the branching rule.

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