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Numerical Analysis

Stabilized explicit coupling for fluid–structure interaction using Nitsche's method

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Abstract

In this Note we propose a stabilized explicit coupling scheme for fluid–structure interaction based on Nitsche's method. The scheme is stable irrespective of the fluid–solid density ratio. Numerical experiments show that optimal time accuracy can be obtained by performing a few defect-correction iterations. ***To cite this article: E. Burman, M.A. Fernández, C. R. Acad. Sci. Paris, Ser. I 345 (2007).***

Résumé

Couplage explicite stabilisé en interaction fluide–structure avec la méthode de Nitsche. Dans cette Note on propose un schéma de couplage explicite stabilisé basé sur la méthode de Nitsche. Le schéma est stable indépendamment du rapport de densités fluide et structure. Des expériences numériques montrent qu'on peut obtenir une précision optimale en temps après quelques itérations d'un algorithme de Résidus Corrigés. ***Pour citer cet article : E. Burman, M.A. Fernández, C. R. Acad. Sci. Paris, Ser. I 345 (2007).***

Version française abrégée

L'objectif de cette Note est de présenter un couplage explicite stable pour la résolution numérique de problèmes d'interaction fluide–structure faisant intervenir un fluide visqueux incompressible avec un effet de masse ajoutée important (c'est-à-dire, lorsque le rapport de densités fluide–structure est proche de 1). Dans de telles situations (les écoulements sanguins par exemple), des instabilités numériques ont été observées (voir [13,16]) pour des schémas explicites, comportant une seule (ou un petit nombre de) résolution fluide et solide par pas de temps. On peut trouver dans [13,5] des explications théoriques de ces instabilités numériques. En général, ces instabilités ont été contournées avec des schémas implicites [14,9,8] ou semi-implicites [7], souvent très coûteux car ils font intervenir la résolution d'un problème couplé (monolithique) à chaque pas de temps. Dans cette Note on propose un schéma de couplage

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explicite stabilisé basé sur la méthode de Nitsche [15,1]. Le schéma de couplage proposé associé à (1), (2) est donné par (4)–(5) avec le choix $\boldsymbol{u}_h^* = \boldsymbol{u}_h^n$, $p_h^* = p_h^n$ et $\gamma_0 > 0$. Grâce aux propriétés dissipatives de la méthode de Nitsche et à un terme de stabilisation sur l'interface, donnant un contrôle des variations des efforts du fluide, on montre (Lemme 1) que les propriétés de stabilité du schéma sont indépendantes du rapport de densités fluide–structure, sous une condition de type CFL. L'inconvénient de la méthode proposée est que l'erreur de troncature du terme de stabilisation est d'ordre $O(\delta t^{1/2})$. Cependant, des expériences numériques (Fig. 1) montrent que la précision optimale des schémas en temps sur chaque sous-domaine peut être récupérée en appliquant un algorithme de Résidus Corrigés (voir [17]).

1. Introduction

In this Note we address the numerical simulation of fluid–structure interaction problems involving a viscous incompressible fluid and an elastic structure. This problem is particularly difficult to treat efficiently when the fluid added-mass, acting on the structure, is strong. In other words, when the fluid and solid densities are close. Indeed, in such situations, *explicit* coupling schemes, i.e. that only involve the solution of the fluid and the structure once (or just a few times) per time step (see e.g. [6] for a review), are known to give rise to numerical instabilities (see e.g. [13,16]). Theoretical explanations of this issue have been reported in [13,5], in particular, it has been pointed out that reducing the time step does not 'cure' the problem. Up to now, these instabilities have been overcome through the use of *implicit* coupling schemes (see e.g. [14,9,8]). Such an approach leads to a monolithic (i.e. fully coupled) problem at each time step, the solution of which often requires a high computational effort. Recent advances suggest the use of *semi-implicit* coupling schemes, involving a reduced monolithic problem [7]. Although significant improvement have been achieved in the last years, to the authors knowledge, none of the existing strategies are able to allow fully explicit coupling without compromising stability.

In this Note we propose a stabilized explicit coupling scheme, based on Nitsche's method [15,1], whose stability properties are independent of the fluid and structure density ratio under a CFL-like condition. This breakthrough is possible thanks to a careful analysis of the dissipative structure of the Nitsche coupling and a stabilization term giving control of the time fluctuations of the interface fluid load. The main disadvantage of the method is that the weak consistency of the stabilization term is of order $O(\delta t^{\frac{1}{2}})$ leading to a scheme that is too dissipative in practice. In order to enhance accuracy, we propose an improved explicit coupling scheme involving a few defect-correction iterations (see e.g. [17]).

2. The coupled problem

We consider a low Reynolds regime and assume that the interface undergoes infinitesimal displacements. The fluid is described by the Stokes equations in a fixed domain $\Omega^{\mathrm{f}} \subset \mathbb{R}^d$, $d = 2, 3$. The structure is described by the classical linear elasticity equations in the solid domain $\Omega^{\mathrm{s}} \subset \mathbb{R}^d$. We denote by $\Sigma \stackrel{\text{def}}{=} \partial\Omega^{\mathrm{s}} \cap \partial\Omega^{\mathrm{f}}$ the fluid–structure interface and $\partial\Omega^{\mathrm{f}} = \Gamma^{\mathrm{in}} \cup \Gamma^{\mathrm{out}} \cup \Sigma$, $\partial\Omega^{\mathrm{s}} = \Gamma^{\mathrm{d}} \cup \Gamma^{\mathrm{n}} \cup \Sigma$, are given partitions of the fluid and solid boundaries, respectively. The coupled problem reads as follows: Find the fluid velocity and pressure $(\boldsymbol{u}, p)$ and the structure displacement $\boldsymbol{\eta}$ such that

$$\begin{cases} \rho^{\mathrm{f}}\partial_t \boldsymbol{u} - \nabla\cdot\boldsymbol{\sigma}^{\mathrm{f}}(\boldsymbol{u}, p) = \boldsymbol{0}, & \text{in } \Omega^{\mathrm{f}}, \\ \nabla\cdot\boldsymbol{u} = 0, & \text{in } \Omega^{\mathrm{f}}, \\ \boldsymbol{\sigma}^{\mathrm{f}}(\boldsymbol{u}, p)\boldsymbol{n}^{\mathrm{f}} = \boldsymbol{g}, & \text{on } \Gamma^{\mathrm{in}} \cup \Gamma^{\mathrm{out}}, \end{cases} \quad \text{and} \quad \begin{cases} \rho^s\partial_{tt}^2\boldsymbol{\eta} - \nabla\cdot\boldsymbol{\sigma}^{\mathrm{s}}(\boldsymbol{\eta}) = \boldsymbol{0}, & \text{in } \Omega^{\mathrm{s}}, \\ \boldsymbol{\eta} = \boldsymbol{0}, & \text{on } \Gamma^{\mathrm{d}}, \\ \boldsymbol{\sigma}^{\mathrm{s}}(\boldsymbol{\eta})\boldsymbol{n}^{\mathrm{s}} = \boldsymbol{0}, & \text{on } \Gamma^{\mathrm{n}}, \end{cases} \tag{1}$$

satisfying the interface coupling conditions

$$\begin{cases} \boldsymbol{u} = \partial_t\boldsymbol{\eta}, & \text{on } \Sigma, \\ \boldsymbol{\sigma}^{\mathrm{s}}(\boldsymbol{\eta})\cdot\boldsymbol{n}^{\mathrm{s}} = -\boldsymbol{\sigma}^{\mathrm{f}}(\boldsymbol{u}, p)\cdot\boldsymbol{n}^{\mathrm{f}}, & \text{on } \Sigma, \end{cases} \tag{2}$$

and the initial conditions $\boldsymbol{u}(0) = \boldsymbol{u}_0$, $\boldsymbol{\eta}(0) = \boldsymbol{\eta}_0$ and $\partial_t\boldsymbol{\eta}(0) = \boldsymbol{v}_0$. Here, ρ^{f} and ρ^{s} stand for the fluid and solid densities, $\boldsymbol{\sigma}^{\mathrm{s}}(\boldsymbol{u}, p) \stackrel{\text{def}}{=} -p\boldsymbol{I} + 2\mu\boldsymbol{\epsilon}(\boldsymbol{u})$ and $\boldsymbol{\sigma}^{\mathrm{s}}(\boldsymbol{\eta})$ for the fluid and solid stress tensors, μ for the fluid dynamic viscosity, $\boldsymbol{\epsilon}(\boldsymbol{u})$ for the fluid strain rate tensor and $\boldsymbol{g}$ for a given surface load. Although (1), (2) is a simplified coupled model, it features the main stability issues that appear in complex fluid–structure interaction problems (see e.g. [13,16,5]).

3. The Nitsche formulation

Nitsche's method [15] is a classical method for imposing essential boundary conditions weakly. Unlike the penalty method, it is consistent with the original differential equation. Herein lies the advantage of Nitsche's method. Indeed, optimal convergence is retained without perturbing the conditioning of the matrix. Recently the Nitsche's method was proposed in a domain decomposition framework in [1,3], and with special focus on iterative solving in [4].

In the context of fluid–structure interaction, using implicit coupling, some results are given for vibration problems (acoustics) in [10] and for fluid–structure interaction problems with moving fluid domains in [11].

Let $V_h \times Q_h$ denote an inf-sup stable fluid finite element pair and X_h a suitable space for the solid displacement. We may write the space semi-discretized Nitsche's formulation of (1)–(2) as: For all $t > 0$, find $(\boldsymbol{u}_h, p_h, \boldsymbol{\eta}_h, \dot{\boldsymbol{\eta}}_h) \in V_h \times Q_h \times X_h \times X_h$ such that

$$\begin{aligned}
&\rho^{\mathrm{f}} \int_{\Omega^{\mathrm{f}}} \partial_t \boldsymbol{u}_h \cdot \boldsymbol{v}_h \,\mathrm{d}\boldsymbol{x} + \int_{\Omega^{\mathrm{f}}} \boldsymbol{\sigma}^{\mathrm{f}}(\boldsymbol{u}_h, p_h) : \boldsymbol{\epsilon}(\boldsymbol{v}_h) \,\mathrm{d}\boldsymbol{x} + \int_{\Omega^{\mathrm{f}}} q_h \nabla \cdot \boldsymbol{u}_h \,\mathrm{d}\boldsymbol{x} + \rho^{\mathrm{s}} \int_{\Omega^{\mathrm{s}}} \partial_t \dot{\boldsymbol{\eta}}_h \cdot \dot{\boldsymbol{w}}_h \,\mathrm{d}\boldsymbol{x} \\
&\quad + a^{\mathrm{s}}(\boldsymbol{\eta}_h, \dot{\boldsymbol{w}}_h) - \int_{\Sigma} \sigma^{\mathrm{f}}(\boldsymbol{u}_h, p_h)\boldsymbol{n}^{\mathrm{f}} \cdot (\boldsymbol{v}_h - \dot{\boldsymbol{w}}_h) \,\mathrm{d}\boldsymbol{s} - \int_{\Sigma} (\boldsymbol{u}_h - \partial_t \boldsymbol{\eta}_h) \cdot \boldsymbol{\sigma}^{\mathrm{f}}(\boldsymbol{v}_h, -q_h)\boldsymbol{n}^{\mathrm{f}} \,\mathrm{d}\boldsymbol{s} \\
&\quad + \gamma \frac{\mu}{h} \int_{\Sigma} (\boldsymbol{u}_h - \partial_t \boldsymbol{\eta}_h) \cdot (\boldsymbol{v}_h - \dot{\boldsymbol{w}}_h) \,\mathrm{d}\boldsymbol{s} + \int_{\Omega^{\mathrm{s}}} (\dot{\boldsymbol{\eta}}_h - \partial_t \boldsymbol{\eta}_h) \cdot \boldsymbol{w}_h \,\mathrm{d}\boldsymbol{x} = \int_{\Gamma^{\mathrm{in}} \cup \Gamma^{\mathrm{out}}} \boldsymbol{g} \cdot \boldsymbol{v}_h \,\mathrm{d}\boldsymbol{s}, \qquad (3)
\end{aligned}$$

for all $(\boldsymbol{v}_h, q_h, \dot{\boldsymbol{w}}_h, \boldsymbol{w}_h) \in V_h \times Q_h \times X_h \times X_h$. Here, a^{s} stands for a general solid bi-linear form associated to its internal energy and $\gamma > 0$ is the Nitsche's penalty parameter. Note that $\boldsymbol{\sigma}^{\mathrm{f}}(\boldsymbol{u}_h, p_h)_{|\Sigma}$ is evaluated in a standard fashion, facewise, on all the element faces in Σ.

4. An explicit coupling scheme

In what follows, we will use the following general notation for the first order backward difference $\partial_{\delta t} X^{n+1} \stackrel{\mathrm{def}}{=} \delta t^{-1}(X^{n+1} - X^n)$. The semi-discrete problem (3) can be discretized in time using a first order backward difference in the fluid and a Newmark scheme for the structure as follows:

– **Solid substep**: Given $(\boldsymbol{u}_h^*, p_h^*) \in V_h \times Q_h$, find $(\boldsymbol{\eta}_h^{n+1}, \dot{\boldsymbol{\eta}}_h^{n+1}) \in X_h \times X_h$ such that

$$\begin{aligned}
&\rho^{\mathrm{s}} \int_{\Omega^{\mathrm{s}}} \partial_{\delta t} \dot{\boldsymbol{\eta}}_h^{n+1} \cdot \dot{\boldsymbol{w}}_h \,\mathrm{d}\boldsymbol{x} + \frac{1}{2} a^{\mathrm{s}}(\boldsymbol{\eta}_h^{n+1} + \boldsymbol{\eta}_h^n, \dot{\boldsymbol{w}}_h) + \int_{\Sigma} \boldsymbol{\sigma}^{\mathrm{f}}\big(\boldsymbol{u}_h^*, p_h^*\big)\boldsymbol{n}^{\mathrm{f}} \cdot \dot{\boldsymbol{w}}_h \,\mathrm{d}\boldsymbol{s} \\
&\quad - \gamma \frac{\mu}{h} \int_{\Sigma} \big(\boldsymbol{u}_h^* - \partial_{\delta t} \boldsymbol{\eta}_h^{n+1}\big) \cdot \dot{\boldsymbol{w}}_h \,\mathrm{d}\boldsymbol{s} + \int_{\Omega^{\mathrm{s}}} \left(\frac{\dot{\boldsymbol{\eta}}_h^{n+1} + \dot{\boldsymbol{\eta}}_h^n}{2} - \partial_{\delta t} \boldsymbol{\eta}_h^{n+1} \right) \cdot \boldsymbol{w}_h \,\mathrm{d}\boldsymbol{x} = 0, \qquad (4)
\end{aligned}$$

for all $(\dot{\boldsymbol{w}}_h, \boldsymbol{w}_h) \in X_h \times X_h$.

– **Fluid substep**: Given $\partial_{\delta t} \boldsymbol{\eta}_h^{n+1} \in X_h$, find $(\boldsymbol{u}_h^{n+1}, p_h^{n+1}) \in V_h \times Q_h$ such that

$$\begin{aligned}
&\rho^{\mathrm{f}} \int_{\Omega^{\mathrm{f}}} \partial_{\delta t} \boldsymbol{u}_h^{n+1} \cdot \boldsymbol{v}_h \,\mathrm{d}\boldsymbol{x} + \int_{\Omega^{\mathrm{f}}} \boldsymbol{\sigma}^{\mathrm{f},n+1} : \boldsymbol{\epsilon}(\boldsymbol{v}_h) \,\mathrm{d}\boldsymbol{x} + \int_{\Omega^{\mathrm{f}}} q_h \nabla \cdot \boldsymbol{u}_h^{n+1} \,\mathrm{d}\boldsymbol{x} \\
&\quad - \int_{\Sigma} \boldsymbol{\sigma}^{\mathrm{f},n+1} \boldsymbol{n}^{\mathrm{f}} \cdot \boldsymbol{v}_h \,\mathrm{d}\boldsymbol{s} - \int_{\Sigma} \big(\boldsymbol{u}_h^{n+1} - \partial_{\delta t} \boldsymbol{\eta}_h^{n+1}\big) \cdot \boldsymbol{\sigma}^{\mathrm{f}}(\boldsymbol{v}_h, -q_h)\boldsymbol{n}^{\mathrm{f}} \,\mathrm{d}\boldsymbol{s} \\
&\quad + \gamma \frac{\mu}{h} \int_{\Sigma} \big(\boldsymbol{u}_h^{n+1} - \partial_{\delta t} \boldsymbol{\eta}_h^{n+1}\big) \cdot \boldsymbol{v}_h \,\mathrm{d}\boldsymbol{s} + \gamma_0 \frac{\delta t}{\mu} \int_{\Sigma} \partial_{\delta t} \boldsymbol{\sigma}^{\mathrm{f},n+1} \boldsymbol{n}^{\mathrm{f}} \cdot \boldsymbol{\sigma}^{\mathrm{f}}(\boldsymbol{v}_h, q_h)\boldsymbol{n}^{\mathrm{f}} = \int_{\Gamma^{\mathrm{in}} \cup \Gamma^{\mathrm{out}}} \boldsymbol{g} \cdot \boldsymbol{v}_h \,\mathrm{d}\boldsymbol{s}, \qquad (5)
\end{aligned}$$

for all $(\boldsymbol{v}, q) \in V_h \times Q_h$ and with the notation $\boldsymbol{\sigma}^{\mathrm{f},n+1} \stackrel{\mathrm{def}}{=} \boldsymbol{\sigma}^{\mathrm{f}}(\boldsymbol{u}_h^{n+1}, p_h^{n+1})$.

If $\gamma_0 = 0$, $\boldsymbol{u}_h^* = \boldsymbol{u}_h^{n+1}$ and $p_h^* = p_h^{n+1}$ for $n \geqslant 0$, the scheme (4), (5) is implicit. Therefore, the resulting system has to be solved monolithically or by 'sub-iterating' between the fluid and the solid solvers. On the other hand, if $\boldsymbol{u}_h^* = \boldsymbol{u}_h^n$ and $p_h^* = p_h^n$ for $n \geqslant 0$, the scheme is fully explicit. For $\gamma_0 = 0$ the explicit scheme is known to be unstable. However, for γ_0 sufficiently large it is stable under moderate conditions on the discretization parameters, as will be discussed in the next paragraph. The explicit scheme requires an initial pressure p_h^0 which can be obtained, for instance, by initiating the time-stepping procedure with a fully implicit step.

4.1. Stability analysis

For $n \geqslant 0$, we define the total discrete energy of the fluid–structure system, at the time level n, as

$$E^n \stackrel{\text{def}}{=} \frac{\rho^{\mathrm{f}}}{2}\|\boldsymbol{u}_h^n\|_{0,\Omega^{\mathrm{f}}}^2 + \frac{\rho^{\mathrm{s}}}{2}\|\partial_{\delta t}\boldsymbol{\eta}_h^n\|_{0,\Omega^{\mathrm{s}}}^2 + \frac{1}{2}a^{\mathrm{s}}(\boldsymbol{\eta}_h^n, \boldsymbol{\eta}_h^n) + \mu\left(1 - \frac{16C_T}{\gamma}\right)\sum_{k=0}^{n-1}\delta t\left\|\boldsymbol{\epsilon}(\boldsymbol{u}_h^{k+1})\right\|_{0,\Omega^{\mathrm{f}}}^2.$$

We then have the following stability lemma, whose proof can be found in the forthcoming paper [2]:

Lemma 1. *Assume that the fluid–structure system is isolated, i.e.* $\boldsymbol{g} = \boldsymbol{0}$. *For the above discrete coupling schemes* (4)–(5) *we have the following results*:

- *Implicit coupling* ($\boldsymbol{u}_h^* = \boldsymbol{u}_h^{n+1}$ *and* $p_h^* = p_h^{n+1}$): *If* $\gamma > 16C_T$ *there holds*
$$E^n \leqslant E^0, \quad \forall n \geqslant 0.$$
- *Explicit coupling* ($\boldsymbol{u}_h^* = \boldsymbol{u}_h^n$ *and* $p_h^* = p_h^n$): *If* $\gamma > 16C_T$, $\gamma_0 > (h + C_{\Omega^{\mathrm{f}}})/C_T$ *and* $\delta t < C_\Sigma h(\gamma\mu)^{-1}$ *there holds*
$$\frac{1}{4}E^n \leqslant E^0 + C_\Sigma\|\boldsymbol{u}_h^0\|_{0,\Sigma}^2 + \gamma_0\frac{\delta t}{\mu}\left\|\sigma^{\mathrm{f}}(\boldsymbol{u}_h^0, p_h^0)\boldsymbol{n}^{\mathrm{f}}\right\|_{0,\Sigma}^2, \quad \forall n \geqslant 0.$$

With C_T *denoting the constant of the element trace inequality,* $C_{\Omega^{\mathrm{f}}}$ *a constant depending only on the geometry of the fluid domain and* C_Σ *the CFL-like condition constant of the explicit interface coupling.*

Some remarks are in order: (i) The above lemma shows that the explicit coupling scheme is conditionally stable in the energy norm, irrespective of the fluid–solid density ratio; (ii) A standard consistency analysis shows that the explicit scheme is expected to be of order $\mathrm{O}(\delta t^{\frac{1}{2}})$ in time. The reduced order is caused by the term stabilizing the time fluctuations of the fluid load on the interface.

5. Numerical experiment

Due to the sub-optimality in time of the explicit scheme (4), (5), we propose to improve accuracy by a defect-correction procedure [17]. This consists in performing a few (explicit) correction iterations in the explicit scheme in order to recover the accuracy of the underlying implicit scheme (we refer to [2] for the details). In order to illustrate the performance of the method we consider a simplified version of the fluid–structure benchmark proposed in [16], by coupling the 2D Stokes equations (using the Laplacian operator) with the 2D wave equation. The fluid domain is given by $\Omega^{\mathrm{f}} \stackrel{\text{def}}{=} [0, 5] \times [0, 0.5]$ and the solid domain by $\Omega^{\mathrm{s}} \stackrel{\text{def}}{=} [0, 5] \times [0.5, 0.6]$, all the space units are in cm. At $x = 0$ we impose a sinusoidal pressure of maximal amplitude $P = 10^4$ dyne/cm^2 during 5×10^{-3} s corresponding to half a period. Zero pressure is enforced at $x = 5$ and a symmetry condition is applied on the lower wall $y = 0$. The structure is clamped on $x = 0$ and $x = 5$ and zero traction is applied on $y = 0.6$. The fluid physical parameters are given by $\rho^{\mathrm{f}} = 1.0$ g/cm^3, $\mu = 0.035$ poise. For the solid we have $\rho^{\mathrm{s}} = 1.2$ g/cm^3 and the elastic modulus is $\lambda = 3 \times 10^7$ dyne/cm^2. For the fluid we used the Taylor–Hood finite element and for the structure a standard $\mathbb{P}_1$-continuous discretization with mesh sizes of $h = 0.1$ cm. The time step was fixed to $\delta t = 10^{-4}$ s. We fixed $\gamma = 100$ and $\gamma_0 = 7 \times 10^{-4}$ for the explicit scheme without correction, and $\gamma_0 = 1.2 \times 10^{-4}$ with correction. All the computations have been performed with FreeFem++ [12].

In Fig. 1 we report a comparison of the proposed method with the fully implicit coupling scheme. As expected the explicit coupling scheme without correction provides a stable approximation, however it has a very poor accuracy.

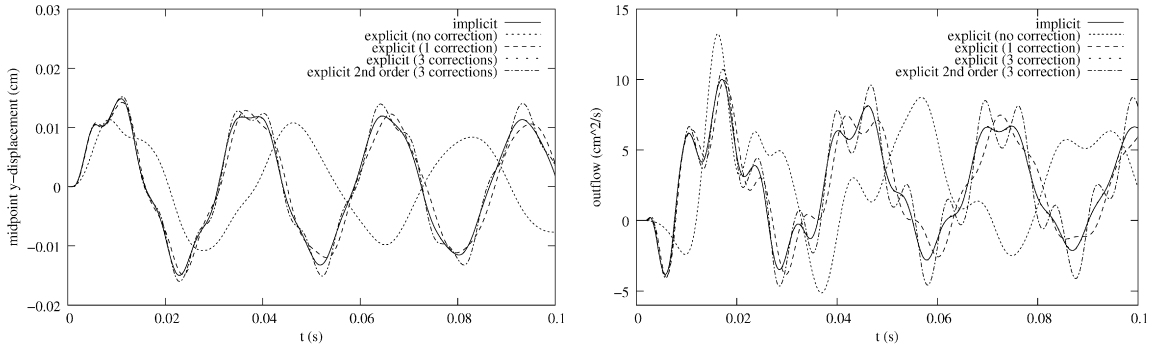

Fig. 1. Mid-point y-displacement (left) and outflow rate (right) vs. time.

Fig. 1. Déplacement vertical du point milieu (gauche) et flux a la sortie (droite).

After one correction step, the explicit scheme recovers first order accuracy $O(\delta t)$. After three correction steps it provides a solution undistinguishable from the implicit scheme solution. This procedure can also be combined with high order discretization. Indeed, although the use of high order time discretization in the stabilized explicit scheme is not relevant (due to the truncation error of the stabilization), after a few correction iterations we recover high order accuracy. For example, a second order scheme can be obtained by replacing the first order backward difference in the fluid by a second order backward difference. The enhanced accuracy is clearly visible (Fig. 1), in particular, in the outflow rate.

6. Conclusion

We have proposed a stabilized explicit coupling scheme for the efficient solution of fluid–structure interaction problems. The stability of the method is obtained by the addition of a weakly consistent penalization term of the (time) fluctuations of the fluid load at the interface. We show that the explicit coupling is stable irrespective of the fluid–solid density ratio. Optimal accuracy is recovered using a defect-correction approach. The method is flexible with respect to the choice of time stepping schemes for the fluid and the structure and allows for independent meshing of both domains. A numerical example shows that one correction step suffices to recover first order accuracy and if three correction steps are applied the explicit method is as precise as the implicit one. A second order scheme is also tested and the second order accuracy is obtained after three correction steps.

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