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Mathematical Problems in Mechanics

Recovery of a displacement field from its linearized strain tensor field in curvilinear coordinates

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Abstract

We establish that, if a symmetric matrix field defined over a simply-connected open set satisfies the Saint Venant equations in curvilinear coordinates, then its coefficients are the linearized strains associated with a displacement field. Our proof provides an explicit algorithm for recovering such a displacement field, which may be viewed as the linear counterpart of the reconstruction of an immersion from a given flat Riemannian metric. ***To cite this article: P.G. Ciarlet et al., C. R. Acad. Sci. Paris, Ser. I 344 (2007).***

Résumé

Reconstruction d'un champ de déplacements à partir de son tenseur des déformations linéarisées en coordonnées curvilignes. Nous montrons que, si un champ de matrices symétriques défini sur un ouvert simplement connexe vérifie les équations de Saint Venant en coordonnées curvilignes, alors c'est le tenseur des déformations linéarisées associé à un champ de déplacements. Notre démonstration fournit un algorithme explicite de reconstruction d'un tel champ de déplacements, qui peut être considéré comme la version linéarisée de la reconstruction d'une immersion à partir d'une métrique riemannienne plate. ***Pour citer cet article : P.G. Ciarlet et al., C. R. Acad. Sci. Paris, Ser. I 344 (2007).***

1. Notations and preliminaries

Latin indices and exponents vary in the set $\{1, 2, 3\}$ and the summation convention with respect to repeated indices and exponents is systematically used in conjunction with this rule.

Let Ω be an open subset of $\mathbb{R}^3$ and let there be given an immersion $\boldsymbol{\Theta} \in \mathcal{C}^3(\overline{\Omega}; \mathbb{R}^3)$. For each $x = (x_i) \in \Omega$, the three vectors $\mathbf{g}_i(x) := \partial_i \boldsymbol{\Theta}(x)$, where $\partial_i := \partial/\partial x_i$, form a basis in the tangent space, identified here with $\mathbb{R}^3$, to the manifold $\boldsymbol{\Theta}(\Omega)$ at the point $\boldsymbol{\Theta}(x)$. The vector fields $\mathbf{g}^j$, defined by $\mathbf{g}_i(x) \cdot \mathbf{g}^j(x) = \delta_i^j$ for all $x \in \Omega$, form the dual basis of the basis formed by the vector fields $\mathbf{g}_i$. The manifold $\boldsymbol{\Theta}(\Omega)$ being naturally endowed with the Euclidean

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metric inherited from the surrounding space $\mathbb{R}^3$, the immersion $\boldsymbol{\Theta}$ induces a Riemannian metric on Ω, defined by its covariant components

$$g_{ij}(x) = \mathbf{g}_i(x) \cdot \mathbf{g}_j(x) \quad \text{for all } x \in \Omega.$$

The contravariant components of this metric are defined by $g^{k\ell}(x) = \mathbf{g}^k(x) \cdot \mathbf{g}^\ell(x)$, or equivalently, by $(g^{k\ell}(x)) = (g_{ij}(x))^{-1}$ for all $x \in \Omega$. This metric induces the Levi-Civita connection in the manifold Ω, defined by the Christoffel symbols

$$\Gamma_{ij}^k := \frac{1}{2} g^{k\ell}(\partial_i g_{j\ell} + \partial_j g_{i\ell} - \partial_\ell g_{ij}) = \Gamma_{ji}^k \quad \text{in } \Omega.$$

Note that the regularity assumption on the immersion $\boldsymbol{\Theta}$ implies that $g_{ij}, g^{k\ell} \in \mathcal{C}^2(\overline{\Omega})$ and that $\Gamma_{ij}^k \in \mathcal{C}^1(\overline{\Omega})$. The covariant derivatives of the covariant components $u_i \in H^1(\Omega)$ of a vector field $u_i \mathbf{g}^i$ are defined by

$$u_{j\|i} := \partial_i u_j - \Gamma_{ij}^k u_k.$$

The covariant derivatives of the covariant components $T_{ij} \in L^2(\Omega)$ of a second-order tensor field are defined by

$$T_{ij\|k} := \partial_k T_{ij} - \Gamma_{ki}^\ell T_{\ell j} - \Gamma_{kj}^\ell T_{i\ell}$$

and they belong to the space $H^{-1}(\Omega)$. The covariant derivatives of the covariant components $T_{ijk} \in H^{-1}(\Omega)$ of a third-order tensor field are defined by

$$T_{ijk\|\ell} := \partial_\ell T_{ijk} - \Gamma_{\ell i}^t T_{tjk} - \Gamma_{\ell j}^t T_{itk} - \Gamma_{\ell k}^t T_{ijt},$$

and they belong to the space $H^{-2}(\Omega)$. If $T_{ij} \in L^2(\Omega)$, the second-order covariant derivatives $T_{ij\|k\ell}$ are defined by the relations

$$T_{ij\|k\ell} := \partial_\ell T_{ij\|k} - \Gamma_{\ell i}^t T_{tj\|k} - \Gamma_{\ell j}^t T_{it\|k} - \Gamma_{\ell k}^t T_{ij\|t} = T_{ij\|\ell k}.$$

A *domain* in $\mathbb{R}^3$ is a bounded and connected open set Ω with a Lipschitz-continuous boundary, the set Ω being locally on the same side of its boundary.

Detailed proofs of the results announced in this Note are given in [4].

2. Poincaré theorem in curvilinear coordinates

Poincaré's Theorem, which is classically proved only for continuously differentiable functions, was generalized by Ciarlet and Ciarlet, Jr. [1] as follows:

Theorem 2.1. *Let Ω be a simply connected domain of $\mathbb{R}^3$. Let $h_k \in H^{-1}(\Omega)$ be distributions that satisfy $\partial_\ell h_k = \partial_k h_\ell$ in $H^{-2}(\Omega)$. Then there exists a function $p \in L^2(\Omega)$, unique up to an additive constant, such that $h_k = \partial_k p$ in $H^{-1}(\Omega)$.*

Clearly, this theorem remains valid if the functions h_k are replaced by matrix fields $\mathbf{H}_k$ with components h_{ijk} in $H^{-1}(\Omega)$, the function p being then replaced by a matrix field $\mathbf{P}$ with components p_{ij} in $L^2(\Omega)$. Using Theorem 2.1, one can then show that a similar 'Poincaré theorem in curvilinear coordinates' holds as well. The mapping $\boldsymbol{\Theta}$ is that introduced in Section 1.

Theorem 2.2. *Let Ω be a simply connected domain of $\mathbb{R}^3$ and let $\boldsymbol{\Theta} \in \mathcal{C}^3(\overline{\Omega}; \mathbb{R}^3)$ be an immersion. Let $\mathbf{H}_k$ be matrix fields with components $h_{ijk} \in H^{-1}(\Omega)$ satisfying*

$$h_{ijk\|\ell} = h_{ij\ell\|k} \quad \text{in } H^{-2}(\Omega).$$

Then there exist a matrix field $\mathbf{P}$ with components $p_{ij} \in L^2(\Omega)$, unique up to additive constants, such that

$$h_{ijk} = p_{ij\|k} \quad \text{in } H^{-1}(\Omega).$$

Note that Theorem 2.2 can also be established as a consequence of Theorem A.4 of [6] establishing the existence of weak solutions to Pfaff systems, of which the equations $p_{ij\|_k} = h_{ijk}$ constitutes a special case.

3. Saint Venant equations in curvilinear coordinates

Let Ω be a bounded open subset of $\mathbb{R}^3$ and let $\boldsymbol{\Theta} \in \mathcal{C}^3(\overline{\Omega}; \mathbb{R}^3)$ be an immersion. The vector fields $\mathbf{g}_i \in \mathcal{C}^2(\overline{\Omega}; \mathbb{R}^3)$ and $\mathbf{g}^i \in \mathcal{C}^2(\overline{\Omega}; \mathbb{R}^3)$ are defined as in Section 1. With every vector field $\boldsymbol{u} \in H^1(\Omega; \mathbb{R}^3)$, we associate the *covariant components of the linearized change of metric tensor*, also known as the *linearized strains in curvilinear coordinates*, defined by

$$\varepsilon_{ij}(\boldsymbol{u}) := \frac{1}{2}(\partial_i \boldsymbol{u} \cdot \mathbf{g}_j + \mathbf{g}_i \cdot \partial_j \boldsymbol{u}).$$

Note that $\varepsilon_{ij}(\boldsymbol{u}) \in L^2(\Omega)$ for all i, j and that $\varepsilon_{ij}(\boldsymbol{u}) = \varepsilon_{ji}(\boldsymbol{u})$.

The next theorem shows that the functions $\varepsilon_{ij}(\boldsymbol{u})$ satisfy crucial *compatibility relations*, which constitute the *Saint Venant equations in curvilinear coordinates*, since they generalize the well-known Saint Venant equations in Cartesian coordinates. The proof rests on various computations involving derivatives in the distributional sense.

Theorem 3.1. *The linearized strains in curvilinear coordinates $\varepsilon_{ij}(\boldsymbol{u}) \in L^2(\Omega)$ associated with a vector field $\boldsymbol{u} \in H^1(\Omega; \mathbb{R}^3)$ satisfy the relations*

$$\varepsilon_{ki\|j\ell}(\boldsymbol{u}) + \varepsilon_{\ell j\|ik}(\boldsymbol{u}) - \varepsilon_{kj\|i\ell}(\boldsymbol{u}) - \varepsilon_{\ell i\|jk}(\boldsymbol{u}) = 0 \quad \text{in } H^{-2}(\Omega).$$

4. Recovery of a vector field from the associated linearized change of metric tensor

We now characterize the space of all symmetric matrix fields that satisfy the Saint Venant equations in curvilinear coordinates found in Theorem 3.1.

Theorem 4.1. *Let Ω be a simply-connected domain in $\mathbb{R}^3$ and let $\boldsymbol{\Theta} \in \mathcal{C}^3(\overline{\Omega}; \mathbb{R}^3)$ be an immersion. Let there be given a symmetric matrix field $(e_{ij}) \in L^2(\Omega; \mathbb{S}^3)$ that satisfies the Saint Venant equations in curvilinear coordinates*

$$e_{ki\|j\ell} + e_{\ell j\|ik} - e_{kj\|i\ell} - e_{\ell i\|jk} = 0 \quad \text{in } H^{-2}(\Omega).$$

Then there exists a vector field $\boldsymbol{v} \in H^1(\Omega; \mathbb{R}^3)$ such that

$$e_{ij} = \frac{1}{2}(\partial_i \boldsymbol{v} \cdot \mathbf{g}_j + \mathbf{g}_i \cdot \partial_j \boldsymbol{v}) \quad \text{in } L^2(\Omega).$$

Sketch of proof. Since the Saint Venant equations are satisfied, Theorem 2.2 shows that there exist functions $\tilde{a}_{ij} \in L^2(\Omega)$, unique up to additive constants, such that $\tilde{a}_{ij\|k} = e_{kj\|i} - e_{ki\|j}$ in $H^{-1}(\Omega)$. Since the right-hand side of this equation is antisymmetric in (i, j), it follows that $\tilde{a}_{ij\|k} + \tilde{a}_{ji\|k} = 0$ in $H^{-1}(\Omega)$.

Therefore, again by Theorem 2.2, there exist constants $c_{ij} = c_{ji}$ such that $\tilde{a}_{ij}(x) + \tilde{a}_{ji}(x) = c_{ij}$ for almost all $x \in \Omega$. It then follows that the functions $a_{ij} := \tilde{a}_{ij} + \frac{1}{2}c_{ij}$ are antisymmetric in (i, j), belong to the space $L^2(\Omega)$, and satisfy the equations $a_{ij\|k} = e_{kj\|i} - e_{ki\|j}$.

To prove that there exists a solution $\boldsymbol{v} \in H^1(\Omega; \mathbb{R}3)$ to the system $\partial_i \boldsymbol{v} = (e_{ij} + a_{ij})\mathbf{g}^j$, it is enough to prove that $\partial_k((e_{ij} + a_{ij})\mathbf{g}^j) = \partial_i((e_{kj} + a_{kj})\mathbf{g}^j)$. Since $\Gamma_{ki}^j = \Gamma_{ik}^j$, this relation in fact amounts to proving that $e_{i\ell\|k} + a_{i\ell\|k} = e_{k\ell\|i} + a_{k\ell\|i}$, which is in turn equivalent to proving that $e_{i\ell\|k} + e_{k\ell\|i} - e_{ki\|\ell} = e_{k\ell\|i} + e_{i\ell\|k} - e_{ik\|\ell}$. But this last equation is clearly satisfied, since the matrix field (e_{ij}) is symmetric. The existence of the field $\boldsymbol{v}$ then follows from Theorem 2.1.

That the vector field $\boldsymbol{v}$ does indeed satisfy the required equations is a consequence of the symmetry of the matrix field (e_{ij}) and of the anti-symmetry of the matrix field (a_{ij}). □

Note that Theorem 3 of [3] shows that, if the open set Ω is *connected*, any other vector field $\tilde{\boldsymbol{v}} \in H^1(\Omega; \mathbb{R}^3)$ that satisfies $e_{ij} = \frac{1}{2}(\partial_i \tilde{\boldsymbol{v}} \cdot \mathbf{g}_j + \mathbf{g}_i \cdot \partial_j \tilde{\boldsymbol{v}})$ in $L^2(\Omega; \mathbb{S}^3)$ is necessarily of the form $\tilde{\boldsymbol{v}}(x) = \boldsymbol{v}(x) + (\mathbf{a} + \mathbf{b} \wedge \boldsymbol{\Theta}(x))$ for almost all $x \in \Omega$, where $\mathbf{a}$ and $\mathbf{b}$ are vectors in $\mathbb{R}^3$.

5. The Riemann curvature tensor and the Saint Venant equations

We now show that the Saint Venant equations in curvilinear coordinates are nothing but an infinitesimal version of the compatibility conditions that a three-dimensional Riemannian space must satisfy in order to be isometrically immersed in the three-dimensional Euclidean space. These compatibility conditions are first recalled in the next theorem, which is a straightforward extension of a well-known result in differential geometry, classically established only for smoother immersions $\boldsymbol{\Theta} \in \mathcal{C}^3(\Omega; \mathbb{R}^3)$. The symbol $\mathbb{S}^3$, resp. $\mathbb{S}^3_>$, designates the set of all symmetric, resp. positive-definite symmetric, real matrices of order three.

Theorem 5.1. *Let Ω be an open subset of $\mathbb{R}^3$ and let $p > 3$. Given any immersion $\boldsymbol{\Theta} \in W^{2,p}_{\mathrm{loc}}(\Omega; \mathbb{R}^3)$, let the matrix field $(g_{ij}) \in W^{1,p}_{\mathrm{loc}}(\Omega; \mathbb{S}^3_>)$ be defined by*

$$g_{ij} = \partial_i \boldsymbol{\Theta} \cdot \partial_j \boldsymbol{\Theta} \quad \text{in } \Omega.$$

Then the Riemann curvature tensor associated with the matrix field (g_{ij}) vanishes in the distributional sense, i.e.,

$$R_{skij} := g_{s\ell}\big(\partial_i \Gamma^\ell_{jk} - \partial_j \Gamma^\ell_{ik} + \Gamma^r_{jk}\Gamma^\ell_{ir} - \Gamma^r_{ik}\Gamma^\ell_{jr}\big) = 0 \quad \text{in } \mathcal{D}'(\Omega).$$

As shown in Theorem 4.4 of S. Mardare [5], the converse of Theorem 5.1 is also true:

Theorem 5.2. *Let Ω be a connected and simply-connected open subset of $\mathbb{R}^3$ and let $(g_{ij}) \in W^{1,\infty}_{\mathrm{loc}}(\Omega; \mathbb{S}^3_>)$ be a field of positive-definite symmetric matrices. If the Riemann curvature tensor associated with the matrix field (g_{ij}) vanishes in the distributional sense, i.e., if*

$$R_{skij} := g_{s\ell}\big(\partial_i \Gamma^\ell_{jk} - \partial_j \Gamma^\ell_{ik} + \Gamma^r_{jk}\Gamma^\ell_{ir} - \Gamma^r_{ik}\Gamma^\ell_{jr}\big) = 0 \quad \text{in } \mathcal{D}'(\Omega),$$

then there exists an immersion $\boldsymbol{\Theta} \in W^{2,\infty}_{\mathrm{loc}}(\Omega; \mathbb{R}^3)$ such that

$$g_{ij} = \partial_i \boldsymbol{\Theta} \cdot \partial_j \boldsymbol{\Theta} \quad \text{in } \Omega.$$

In order to show that Theorems 3.1 and 4.1 are nothing but the 'infinitesimal' versions of Theorems 5.1 and 5.2, respectively, we show that the left-hand side of the Saint Venant equations is in a specific sense the *linear part* of the Riemann curvature tensor.

Theorem 5.3. *Let Ω be a bounded open subset in $\mathbb{R}^3$ and let there be given a matrix field $(g_{ij}) \in \mathcal{C}^2(\overline{\Omega}; \mathbb{S}^3_>)$ whose associated Riemann curvature tensor field vanishes in Ω. Then, for all 'increments' symmetric matrix fields $(e_{ij}) \in W^{1,p}(\Omega; \mathbb{S}^3)$, $p > 3$, the linear part with respect to (e_{ij}) of the covariant components of the Riemann curvature tensor associated with the metric $(g_{ij} + e_{ij})$ are given by*

$$R^{\mathrm{lin}}_{skij}(e_{ij}) = e_{ki\|js} + e_{sj\|ik} - e_{kj\|is} - e_{si\|jk}, \tag{1}$$

where $e_{ki\|js}$ denote the second-order covariant derivatives of e_{ki} (cf. Section 1).

Sketch of proof. For all $\varepsilon > 0$, define the matrix field

$$\big(g_{ij}(\varepsilon)\big) := (g_{ij}) + \varepsilon(e_{ij}) \in W^{1,p}\big(\Omega; \mathbb{S}^3\big).$$

Since $W^{1,p}(\Omega) \subset \mathcal{C}^0(\overline{\Omega})$ by the Sobolev embedding theorem, there exists a number $\varepsilon_0 > 0$ such that, for all $0 < \varepsilon < \varepsilon_0$, the matrix field $(g_{ij}(\varepsilon))$ is positive definite in $\overline{\Omega}$. This implies that $g^{k\ell}(\varepsilon) \in W^{1,p}(\Omega)$, where $(g^{k\ell}(\varepsilon)) = (g_{ij}(\varepsilon))^{-1}$ is the inverse of the matrix field $(g_{ij}(\varepsilon))$. Hence the Christoffel symbols

$$\Gamma_{rjk}(\varepsilon) := \frac{1}{2}\big\{\partial_j g_{rk}(\varepsilon) + \partial_k g_{jr}(\varepsilon) - \partial_r g_{jk}(\varepsilon)\big\} \quad \text{and} \quad \Gamma^\ell_{jk}(\varepsilon) := g^{\ell r}(\varepsilon)\Gamma_{rjk}(\varepsilon)$$

belong to the space $L^p(\Omega)$. Consequently, the Riemann curvature tensor associated with the metric $(g_{ij}(\varepsilon))$ is well defined in the sense of distributions by its mixed components

$$R^\ell_{\cdot kij}(\varepsilon) := \partial_i \Gamma^\ell_{jk}(\varepsilon) - \partial_j \Gamma^\ell_{ik}(\varepsilon) + \Gamma^r_{jk}(\varepsilon)\Gamma^\ell_{ir}(\varepsilon) - \Gamma^r_{ik}(\varepsilon)\Gamma^\ell_{jr}(\varepsilon),$$

or by its covariant components

$$R_{skij}(\varepsilon) = g_{s\ell}(\varepsilon) R^{\ell}_{\cdot kij}(\varepsilon).$$

The linear part with respect to (e_{ij}) of each covariant component of the Riemann curvature tensor associated with the metric $(g_{ij} + e_{ij})$ is then defined by the limit

$$R^{\text{lin}}_{skij} := \lim_{\substack{\varepsilon \to 0 \\ \varepsilon \neq 0}} \frac{R_{skij}(\varepsilon)}{\varepsilon},$$

since the Riemann curvature tensor of the metric (g_{ij}) vanishes in Ω by assumption.

In order to compute this linear part, we then expand all the above functions as power series in ε. After some lengthy computations (all justified in the sense of distributions), we find in this fashion that

$$R^{\ell}_{\cdot kij}(\varepsilon) = \varepsilon g^{\ell r}(e_{rj\|ki} - e_{jk\|ri} - e_{ri\|kj} + e_{ik\|rj}) + \mathrm{O}(\varepsilon^2) \quad \text{in } H^{-1}(\Omega),$$

so that

$$\begin{aligned} R_{\ell kij}(\varepsilon) &= \varepsilon g_{\ell r} g^{rs}(e_{sj\|ki} - e_{jk\|si} - e_{si\|kj} + e_{ik\|sj}) + \mathrm{O}(\varepsilon^2) \\ &= e_{\ell j\|ki} - e_{jk\|\ell i} - e_{\ell i\|kj} + e_{ik\|\ell j} + \mathrm{O}(\varepsilon^2) \quad \text{in } H^{-1}(\Omega). \qquad \square \end{aligned}$$

Note that the matrix field (e_{ij}) is assumed in Theorem 5.3 to be in the space $W^{1,p}(\Omega; \mathbb{S}^3)$ for $p > 3$, and not only in $L^2(\Omega; \mathbb{S}^3)$, in order to have $(g_{ij}(\varepsilon)) \in W^{1,p}(\Omega; \mathbb{S}^3)$, which is the minimal regularity assumption under which the components $R_{\ell kij}(\varepsilon)$ of the Riemannian curvature tensor are well defined in the sense of distributions. By contrast, the functions R^{lin}_{skij} can be extended by continuity to matrix fields (e_{ij}) that belong only to the space $L^2(\Omega; \mathbb{S}^3)$.

Let $\widehat{\Omega}$ be an open subset of $\mathbb{R}^3$. The Cartesian coordinates of a point $\hat{x} \in \widehat{\Omega}$ are denoted $\hat{x}_i$ and the partial derivative operators of the first and second order of functions defined over $\widehat{\Omega}$ are denoted $\hat{\partial}_i := \partial/\partial\hat{x}_i$ and $\hat{\partial}_{ij} := \partial^2/\partial\hat{x}_i\partial\hat{x}_j$. With these notations, the following theorem was proved by Ciarlet and Ciarlet, Jr. [1].

Theorem 5.4. *Let $\widehat{\Omega}$ be a simply-connected domain of $\mathbb{R}^3$ and let $(\hat{e}_{ij}) \in L^2(\widehat{\Omega}; \mathbb{S}^3)$ be a symmetric matrix field that satisfies the following compatibility conditions*

$$\hat{\partial}_{\ell j}\hat{e}_{ik} + \hat{\partial}_{ki}\hat{e}_{j\ell} - \hat{\partial}_{\ell i}\hat{e}_{jk} - \hat{\partial}_{kj}\hat{e}_{i\ell} = 0 \quad \text{in } H^{-2}(\widehat{\Omega}).$$

Then there exists a vector field $\hat{\boldsymbol{v}} = (\hat{v}_i) \in H^1(\widehat{\Omega}; \mathbb{R}^3)$ such that

$$\hat{e}_{ij} = \frac{1}{2}(\hat{\partial}_j\hat{v}_i + \hat{\partial}_i\hat{v}_j).$$

The compatibility relations in Theorem 5.4 are the *Saint Venant equations in Cartesian coordinates*. Note that the *Saint Venant equations in curvilinear coordinates* (Section 3) corresponds to the particular case where $\boldsymbol{\Theta} = \mathbf{id}_{\Omega}$, which thus justifies their name. Therefore, Theorem 4.1 implies Theorem 5.4. Remarkably, the converse is also true; see [4] for details.

Finally, note that these equations have been likewise extended to 'Saint Venant equations on a surface'; cf. [2].

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References

[1] P.G. Ciarlet, P. Ciarlet Jr., Another approach to linearized elasticity and a new proof of Korn's inequality, Math. Models Methods Appl. Sci. 15 (2005) 259–271.

[2] P.G. Ciarlet, L. Gratie, C. Mardare, M. Shen, Saint Venant compatibility equations on a surface – application to intrinsic shell theory, preprint, 2006, in press.

[3] P.G. Ciarlet, C. Mardare, On rigid and infinitesimal rigid displacements in three-dimensional elasticity, Math. Models Methods Appl. Sci. 13 (2003) 1589–1598.
[4] P.G. Ciarlet, C. Mardare, M. Shen, Saint Venant compatibility equations in curvilinear coordinates, Anal. Appl., in press.
[5] S. Mardare, On isometric immersions of a Riemannian space with little regularity, Anal. Appl. 2 (2004) 193–226.
[6] S. Mardare, Sur quelques problèmes de géométrie différentielle liés à la théorie de l'élasticité, Doctoral Disertation, Université Paris 6, 2003.