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Partial Differential Equations

Capacitary representation of positive solutions of semilinear parabolic equations

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Abstract

We give a global bilateral estimate on the maximal solution $\bar{u}_F$ of $\partial_t u - \Delta u + u^q = 0$ in $\mathbb{R}^N \times (0,\infty)$, $q > 1$, $N \geqslant 1$, which vanishes at $t = 0$ on the complement of a closed subset $F \subset \mathbb{R}^N$. This estimate is expressed by a Wiener test involving the Bessel capacity $C_{2/q,q'}$. We deduce from this estimate that $\bar{u}_F$ is σ-moderate in Dynkin's sense. ***To cite this article: M. Marcus, L. Véron, C. R. Acad. Sci. Paris, Ser. I 342 (2006).***

Résumé

Représentation capacitaire des solutions positives d'équations paraboliques semi-linéaires. Nous donnons une estimation bilatérale précise de la solution maximale $\bar{u}_F$ de $\partial_t u - \Delta u + u^q = 0$ dans $\mathbb{R}^N \times (0,\infty)$, $q > 1$, $N \geqslant 1$, qui s'annulle en $t = 0$ sur le complémentaire d'un sous-ensemble fermé $F \subset \mathbb{R}^N$. Cette estimation s'exprime par un test de Wiener impliquant la capacité de Bessel $C_{2/q,q'}$. Nous déduisons de cette estimation que $\bar{u}_F$ est σ-moderée au sens de Dynkin. ***Pour citer cet article : M. Marcus, L. Véron, C. R. Acad. Sci. Paris, Ser. I 342 (2006).***

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Soit $q > 1$. Si u est une solution positive de

$$\partial_t u - \Delta u + |u|^{q-1}u = 0 \tag{1}$$

dans $\mathbb{R}^N \times (0,\infty)$, nous avons démontré dans [6] qu'elle admet une trace initiale, notée $\mathrm{Tr}(u)$, dans la classe des mesures de Borel positives et régulières, mais pas nécessairement localement bornées. Si F est un sous-ensemble fermé de $\mathbb{R}^N$, nous désignons par $\bar{u}_F$ la solution maximale de (1) dont le support de la trace initiale est inclus dans F. Si $1 < q < q_c := (N+2)/N$, il est montré dans [5] que les inégalités suivantes sont vérifiées

$$t^{-1/(q-1)} f\big(|x-a|/\sqrt{t}\,\big) \leqslant \bar{u}_F(x,t) \leqslant \big((q-1)t\big)^{-1/(q-1)} \quad \forall a \in F,$$

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où f est l'unique fonction positive vérifiant

$$\Delta f + \frac{1}{2} y \cdot Df + \frac{1}{q-1} f - |f|^{q-1} f = 0 \quad \text{dans } \mathbb{R}^N \quad \text{et} \quad \lim_{|y|\to\infty} |y|^{2/(q-1)} f(y) = 0.$$

Ces inégalités jouent un rôle fondamental dans la démonstration du caractère biunivoque de la correspondance, par l'opérateur de trace initiale, entre l'ensemble des solutions positives de (1) et l'ensemble des mesures de Borel positives régulières. Quand $q \geqslant q_c$ la fonction f est identiquement nulle car les singularités isolées de (1) sont éliminables [4].

Définition. Soit $N \geqslant 1$, $q \geqslant q_c$ et F un sous-ensemble fermé de $\mathbb{R}^N$. On définit le potentiel $(2/q, q')$-capacitaire W_F de F par

$$W_F(x,t) = t^{-1/(q-1)} \sum_{n=0}^{\infty} (n+1)^{N/2-1/(q-1)} \mathrm{e}^{-n/4} C_{2/q,q'}\left(\frac{F_n}{\sqrt{(n+1)t}}\right), \tag{2}$$

$\forall (x,t) \in \mathbb{R}^N \times [0,\infty)$, où $F_n = F_n(x,t) = \{y \in F\colon \sqrt{nt} \leqslant |x-y| < \sqrt{(n+1)t}\}$.

Notre résultat principal est l'estimation bilatérale :

Théorème 1. *Soit $N \geqslant 1$, $q \geqslant q_c$. Il existe deux constantes $C_1 > C_2 > 0$, ne dépendant que de N et q, telles que pour tout sous-ensemble fermé F de $\mathbb{R}^N$*

$$C_2 W_F(x,t) \leqslant \bar{u}_F(x,t) \leqslant C_1 W_F(x,t) \quad \forall (x,t) \in \mathbb{R}^N \times (0,\infty). \tag{3}$$

Si $\mu \in \mathfrak{M}^b(\mathbb{R}^N)$ (l'espace des mesures de Radon bornées dans $\mathbb{R}^N$) appartient à $W^{-2/q,q}(\mathbb{R}^N)$, il existe une unique solution $u = u_\mu$ de (1) dont la trace initiale est μ [3]. On définit alors

$$\underline{u}_F = \sup\{u_\mu\colon \mu \in \mathfrak{M}^b_+(\mathbb{R}^N) \cap W^{-2/q,q}(\mathbb{R}^N)\colon \mu(F^c) = 0\}. \tag{4}$$

Cette solution est σ-modérée au sens de Dynkin, c'est à dire qu'il existe une suite croissante de mesures positives $\mu_n \in \mathfrak{M}^b_+(\mathbb{R}^N) \cap W^{-2/q,q}(\mathbb{R}^N)$ telles que $u_{\mu_n} \uparrow \underline{u}_F$.

La clef de la démonstration du Théorème 1 est l'estimation inférieure de $\underline{u}_F$.

Théorème 2. *Soit $N \geqslant 1$, $q \geqslant q_c$. Il existe une constante $C = C(N,q) > 0$ telle que pour tout sous-ensemble fermé F de $\mathbb{R}^N$,*

$$\underline{u}_F(x,t) \geqslant C W_F(x,t) \quad \forall (x,t) \in \mathbb{R}^N \times (0,\infty). \tag{5}$$

Une conséquence importante des estimations précédentes est la suivante :

Théorème 3. *Soit $N \geqslant 1$, $q > 1$. Pour tout sous-ensemble fermé F de $\mathbb{R}^N$, $\bar{u}_F = \underline{u}_F$.*

1. Main results

Let u be a nonnegative solution of

$$\partial_t u - \Delta u + |u|^{q-1} u = 0, \quad q > 1 \tag{1}$$

in $\mathbb{R}^N \times (0,\infty)$. It was proved in [6] that u admits an initial trace, denoted by $\mathrm{Tr}(u)$, in the class of outer regular positive Borel measures, not necessarily locally bounded. If F is a closed subset of $\mathbb{R}^N$, we denote by $\bar{u}_F$ the maximal solution of (1) which belongs to $C(F^c \times [0,\infty))$ and vanishes on $F^c \times \{0\}$, where $F^c := \mathbb{R}^N \setminus F$.

If $1 < q < (N+2)/N$, the following inequalities are verified

$$t^{-1/(q-1)} f\left(|x-a|/\sqrt{t}\right) \leqslant \bar{u}_F(x,t) \leqslant \left((q-1)t\right)^{-1/(q-1)} \quad \forall a \in F, \tag{2}$$

where f is the unique positive solution [5] of

$$\Delta f+\frac{1}{2}y\cdot Df+\frac{1}{q-1}f-|f|^{q-1}f=0 \quad \text{in } \mathbb{R}^N \quad \text{s.t.} \quad \lim_{|y|\to\infty}|y|^{2/(q-1)}f(y)=0. \tag{3}$$

These inequalities play a fundamental role in proving that (in the subcritical case) any positive solution is uniquely determined by its initial trace [6].

Definition 1. Let

$$\begin{aligned} &B_{\sqrt{nt}}(x)=\{y\in\mathbb{R}^N:\ |x-y|<\sqrt{nt}\},\\ &T_n(x,t)=\overline{B}_{\sqrt{(n+1)t}}(x)\setminus B_{\sqrt{nt}}(x).\end{aligned} \tag{4}$$

For every $q\geqslant q_c$ and every closed subset $F\subset\mathbb{R}^N$, we define the $(2/q,q')$-capacitary potential W_F of F by

$$W_F(x,t)=t^{-1/(q-1)}\sum_{n=0}^{\infty}(n+1)^{N/2-1/(q-1)}\,\mathrm{e}^{-n/4}C_{2/q,q'}\left(\frac{F\cap T_n(x,t)}{\sqrt{(n+1)t}}\right), \tag{5}$$

$\forall(x,t)\in\mathbb{R}^N\times[0,\infty)$.

Our main result is the following bilateral estimate:

Theorem 1. *Let $N\geqslant 1$, $q\geqslant q_c$. There exist two positive constants C_1 and C_2, depending on N and q such that for any closed subset F of $\mathbb{R}^N$,*

$$C_2W_F(x,t)\leqslant \bar{u}_F(x,t)\leqslant C_1W_F(x,t)\quad \forall(x,t)\in\mathbb{R}^N\times(0,\infty). \tag{6}$$

Remark. It is important to notice that although W_F is not a solution of (1), it is invariant with respect to the similarity transformation associated with this equation in the sense that:

$$k^{1/(q-1)}W_F(\sqrt{k}x,kt)=W_{F/\sqrt{k}}(x,t)\quad \forall(x,t)\in\mathbb{R}^N\times(0,\infty),\ \forall k>0. \tag{7}$$

Clearly $\bar{u}_F$ is also similarly invariant with respect to the above transformation.

If μ is a bounded Borel measure such that $|\mu|\in W^{-2/q,q}(\mathbb{R}^N)$ then, there exists a unique solution $u=u_\mu$ of (1) with initial trace μ (see [3]). We define

$$\underline{u}_F=\sup\{u_\mu:\ \mu\in W_+^{-2/q,q}(\mathbb{R}^N),\ \mu(F^c)=0\}.$$

This solution is σ-moderate in the sense of Dynkin, which means that there exists an increasing sequence of positive measures $\mu_n\in W_+^{-2/q,q}(\mathbb{R}^N)$ such that $u_{\mu_n}\uparrow\underline{u}_F$.

Clearly $\underline{u}_F\leqslant\bar{u}_F$. Therefore the next result implies the lower estimate in Theorem 1.

Theorem 2. *Let $N\geqslant 1$, $q\geqslant q_c$. There exists a positive $C=C(N,q)$, such that for any closed subset F of $\mathbb{R}^N$,*

$$\underline{u}_F(x,t)\geqslant CW_F(x,t)\quad \forall(x,t)\in\mathbb{R}^N\times(0,\infty). \tag{8}$$

As a consequence of Theorems 1 and 2 we find that $\bar{u}_F\leqslant c\underline{u}_F$. Using this fact we obtain the following result (already known [6] in the case $1<q<q_c$).

Theorem 3. *Let $N\geqslant 1$, $q>1$. For any closed subset F of $\mathbb{R}^N$ one has $\bar{u}_F=\underline{u}_F$. In particular $\bar{u}_F$ is σ-moderate.*

2. Proof of the upper estimate in Theorem 1

In the following we denote by c a positive constant which depends only on N and q; its value may change from one occurrence to another. Without loss of generality, we assume that F is compact. Denote $B_r(x)=\{y:\ |x-y|<r\}$ and $B_r=B_r(0)$.

Let $r>0$ be a positive number such that $F\subset B_r$. We start by deriving an upper estimate depending on r. Let $\rho>0$ be a positive number, to be later determined as a function of r,t. Let $\eta\in C_0^\infty(B_{r+\rho})$ be such that $\eta=1$ on F and $0\leqslant\eta\leqslant 1$. Put $\eta^*=1-\eta$ and choose $\zeta:=(\mathrm{e}^{t\Delta}[\eta^*])^{2q'}$ as a test function. Then

$$\int_0^1\int_{\mathbb{R}^N} u(\partial_t-\Delta)\zeta\,\mathrm{d}x\,\mathrm{d}t+\int_0^1\int_{\mathbb{R}^N} u^q\zeta\,\mathrm{d}x\,\mathrm{d}t=-\int_{\mathbb{R}^N} u(x,1)\,\mathrm{d}x. \tag{9}$$

A straightforward computation yields

$$\int_{Q_{r+\rho}} u^q\,\mathrm{d}x\,\mathrm{d}t+\int_{\mathbb{R}^N} u(x,1)\,\mathrm{d}x\leqslant\int_0^1\int_{\mathbb{R}^N}\big(R(\eta)\big)^{q'}\,\mathrm{d}x\,\mathrm{d}t,$$

$$R(\eta):=\left|D\,\mathrm{e}^{t\Delta}[\eta]\right|^2+\left|\partial_t\,\mathrm{e}^{t\Delta}[\eta]+\Delta\,\mathrm{e}^{t\Delta}[\eta]\right|,\qquad Q_r:=\left\{(x,t):\ t>0,\ |x|^2+t\geqslant r^2\right\}.$$

Using interpolation inequalities [8] one obtains,

$$\int_0^1\int_{\mathbb{R}^N}\big(R(\eta)\big)^{q'}\,\mathrm{d}x\,\mathrm{d}t\leqslant C_1\|\eta^{q'}_{W^{2/q,q'}}. \tag{10}$$

This implies

$$\int_{\mathbb{R}^N} u(x,1)\,\mathrm{d}x+\int_{Q_{r+\rho}} u^q\,\mathrm{d}x\,\mathrm{d}t\leqslant cC^{B_{r+\rho}}_{2/q,q'}(F).$$

Further it can be shown that, for $0<s<1$,

$$\int_{\mathbb{R}^N} u(x,1)\,\mathrm{d}x+\int_s^1\int_{\mathbb{R}^N} u^q\,\mathrm{d}x\,\mathrm{d}t=\int_{\mathbb{R}^N} u(x,s)\,\mathrm{d}x. \tag{11}$$

Clearly $u(x,t+s)<w_s(x,t):=\mathrm{e}^{t\Delta}[u(\cdot,s)]$ for $t>0$. Therefore, by (10) and (11),

$$u\big(x,(r+2\rho)^2\big)\leqslant\frac{c}{(\rho^2+r\rho)^{N/2}}C^{B_{r+\rho}}_{2/q,q'}(F). \tag{12}$$

Let v be the solution of the initial-boundary value problem

$$\partial_t v-\Delta v=0\quad\text{in } Q^*_{r,\rho}:=\big(\overline{B}_{r+\rho}\big)^c\times\big(0,(r+2\rho)^2\big),$$

$$v(x,0)=0\quad\forall x\in B^c_{r+\rho},\qquad v=u\quad\forall(x,t)\in\partial B_{r+\rho}\times\big(0,(r+2\rho)^2\big).$$

Using the fact that $u<v$ in $Q^*_{r,\rho}$ and (12) we obtain,

Lemma 4. *If $F\subset B_r$, there exists $c>0$ such that*

$$\bar u_F(x,t)\leqslant c\left(1+\frac{r}{\rho}\right)^{N/2}\frac{\mathrm{e}^{-(|x|-r-3\rho)^2/4t}}{t^{N/2}}\,C^{B_{r+\rho}}_{2/q,q'}(F), \tag{13}$$

for any $(x,t)\in\mathbb{R}^N\times[(r+3\rho)^2,\infty)$.

The upper estimate in (6) is obtained by slicing F, relative to a given point $(x,t)\in\mathbb{R}^N\times(0,\infty)$, in such a way that each slice satisfies the assumption of Lemma 4 for an appropriate value of r depending on the point. Put

$$T_n(x,t)=B_{\sqrt{(n+1)t}}(x)\setminus B_{\sqrt{nt}}(x),\quad F_n(x,t)=F\cap T_n(x,t)\quad\forall n\in\mathbb{N}.$$

Since a sum of positive solutions of (1) is a super-solution,

$$\bar u_F\leqslant\sum_{n=0}^{\infty}\bar u_{F_n(x,t)}.$$

Using this fact and Lemma 4 we show that

$$\bar{u}_F(x,t) \leqslant c W_F(x,t) \tag{14}$$

for every $(x,t) \in \mathbb{R}^N \times (0,\infty)$. In view of the fact that both sides of this inequality are invariant with respect to the similarity transformation (7), it is sufficient to prove it in the case $(x,t) = (0,1)$. We denote $F_n = F_n(0,1)$ and $T_n = T_n(0,1)$.

If $N = 1$, each of the sets F_n satisfies the condition of Lemma 4 with $r = \sqrt{n}$. But this is not the case when $N \geqslant 2$. Therefore, if $N \geqslant 2$, a secondary slicing is needed.

For every $n \in \mathbb{N}$ there exists a set of points $\Theta_n = \{a_{j,n}\}_{j=1}^{J_n}$ on the sphere $|y| = (\sqrt{n+1} + \sqrt{n})/2$ such that

$$|a_{n,j} - a_{n,k}| \geqslant 1/\sqrt{2(n+1)} \quad \text{for } j \neq k, \qquad T_n \subset \bigcup_{1 \leqslant j \leqslant J_n} B_{\sqrt{1/(n+1)}}(a_{n,j}).$$

Clearly $J_n \leqslant (\sqrt{2}(n+1))^{N-1} < (4n)^{N-1}$. If $F_{n,j} := F_n \cap B_{\sqrt{1/(n+1)}}(a_{n,j})$,

$$\bar{u}_F(0,1) \leqslant \sum_{n=0}^{\infty} \sum_{1 \leqslant j \leqslant J_n} \bar{u}_{F_{n,j}}(0,1). \tag{15}$$

It is not difficult to verify that

$$C_{2/q,q'}^{B_{2/\sqrt{n+1}}(a_{n,j})}(F_{n,j}) \approx (n+1)^{N/2-1/(q-1)} C_{2/q,q'}(\sqrt{n+1}\, F_{n,j}),$$

where the capacity on the right-hand side (resp. left-hand side) is the Bessel capacity relative to $\mathbb{R}^N$ (resp. relative to $B_{2/\sqrt{n+1}}(a_{n,j})$). The symbol $\approx$ stands for two-sided inequalities with constants $c = c(N,q)$. Further, the quasi-additivity of Bessel capacities [2] implies

$$\sum_{1 \leqslant j \leqslant J_n} C_{2/q,q'}(\sqrt{n+1}\, F_{n,j}) \leqslant c(N,q)\, C_{2/q,q'}(\sqrt{n+1}\, F_n). \tag{16}$$

Each set $F_{n,j}$ satisfies the condition of Lemma 4 with $r = \sqrt{n}$. Therefore, estimating $\bar{u}_{F_{n,j}}(0,1)$ as in (13) and using (15) and (16) we obtain (14) for $(x,t) = (0,1)$.

3. Proof of Theorem 2

As in the elliptic case [7], for each $(x,t) \in \mathbb{R}^N \times (0,\infty)$, we construct a measure $\mu = \mu_{x,t} \in W_+^{-2/q,q}(\mathbb{R}^N)$, concentrated on F, such that

$$u_\mu(x,t) \geqslant c W_F(x,t). \tag{17}$$

Since $\underline{u}_F > u_\mu$, (17) implies (8).

For every measure $\mu \in W_+^{-2/q,q}(\mathbb{R}^N)$, $0 \leqslant u_\mu \leqslant \mathrm{e}^{t\Delta}[\mu]$. Therefore

$$u_\mu \geqslant \mathrm{e}^{t\Delta}[\mu] - \int_0^t \mathrm{e}^{(t-s)\Delta}\big[\big(\mathrm{e}^{s\Delta}[\mu]\big)^q\big]\,\mathrm{d}s. \tag{18}$$

Let ν_n be the capacitary measure of $F_n(x,t)/\sqrt{t(n+1)}$ (see [1]) and define the measure μ_n by

$$\mu_n(A) = \big(t(n+1)\big)^{N/2-1/(q-1)} \nu_n\big(A/\sqrt{t(n+1)}\big),$$

for every Borel set A. Thus μ_n is concentrated on F_n. Finally put $\mu := \mu_{x,t} = \sum_n \mu_n$. With this choice of μ it is not difficult to show that

$$\mathrm{e}^{t\Delta}[\mu](x,t) \geqslant \frac{1}{(4\pi t)^{N/2}} \sum_{n=0}^{\infty} \big(\sqrt{(n+1)t}\big)^{N-2/(q-1)} \mathrm{e}^{-(n+1)/4} C_{2/q,q'}\bigg(\frac{F_n(x,t)}{\sqrt{(n+1)t}}\bigg). \tag{19}$$

We have to derive a corresponding upper estimate for the nonlinear term in (18). To this end we employ a partitioning $\{\mathcal{T}_n\colon n\in\mathbb{Z}\}$ of $\mathbb{R}^N\times(0,t)$ defined by,

$$\mathcal{T}_n=\begin{cases}\bigl\{(y,s)\colon tn\leqslant|x-y|^2+t-s\leqslant t(n+1),\ 0<s<t\bigr\}, & \text{if } n\in\mathbb{N}_*,\\ \bigl\{(y,s)\colon t\alpha^{-n}\leqslant|x-y|^2+t-s\leqslant t\alpha^{-n-1},\ 0<s<t\bigr\}, & \text{if } n\leqslant 0,\end{cases}$$

where $\alpha\in(0,1)$ must be appropriately chosen. If $G(\xi,\tau):=(4\pi|\tau|)^{-N/2}\exp(-|\xi|^2/4\tau))$ then

$$\int_0^t \mathrm{e}^{-(t-s)\Delta}\bigl(\mathrm{e}^{s\Delta}[\mu]\bigr)^q\,\mathrm{d}s=C\sum_{p\in\mathbb{Z}}\iint_{\mathcal{T}_p}G(y-x,s-t)\left(\sum_{n=0}^{\infty}\int_{\mathbb{R}^N}G(z-y,s)\,\mathrm{d}\mu_n(z)\right)^q\mathrm{d}y\,\mathrm{d}s.$$

We denote

$$J_1=\sum_{p\in\mathbb{Z}}\iint_{\mathcal{T}_p}G(y-x,s-t)\left(\sum_{n=0}^{p+2}\int_{\mathbb{R}^N}G(z-y,s)s^{N/2}\,\mathrm{d}\mu_n(z)\right)^q\mathrm{d}y\,\mathrm{d}s,$$

$$J_2=\sum_{p\in\mathbb{Z}}\iint_{\mathcal{T}_p}G(x-y,s-t)\left(\sum_{n=p+3}^{\infty}\int_{\mathbb{R}^N}G(z-y,s)s^{N/2}\,\mathrm{d}\mu_n(z)\right)^q\mathrm{d}y\,\mathrm{d}s.$$

J_1 is estimated using Hölder's inequality and the following inequalities [8]

$$\frac{1}{C}\|\lambda\|_{W^{-2/q,q}}\leqslant\bigl\|\mathrm{e}^{t\Delta}[\lambda]\bigr\|_{L^q(\mathbb{R}^N\times(0,1))}\leqslant C\|\lambda\|_{W^{-2/q,q}},\tag{20}$$

valid for any $\lambda\in W^{-2/q,q}$. The estimate of J_2, in the case $N>1$, requires a more delicate argument using the secondary slicing introduced in the previous section and the quasi-additivity of Bessel capacities. For a suitable choice of α one obtains

$$J_1+J_2\leqslant\frac{C}{t^{N/2}}\sum_{n=0}^{\infty}\bigl(\sqrt{(n+1)t}\bigr)^{N-2/(q-1)}\mathrm{e}^{-(n+1)/4}C_{2/q,q'}\left(\frac{F_n}{\sqrt{(n+1)t}}\right).\tag{21}$$

The inequalities (18), (19) and (21) imply (17), with μ replaced by $\epsilon\mu$, provided that $\epsilon>0$ is sufficiently small, depending on q and the constants in (19) and (21). This in turn implies (8).

It follows that

$$\underline{u}_F(x,t)\leqslant\bar{u}_F(x,t)\leqslant c\underline{u}_F(x,t).$$

By the uniqueness argument used in [6] we conclude that $\bar{u}_F(x,t)=\underline{u}_F(x,t)$.

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