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C. R. Acad. Sci. Paris, Ser. I 340 (2005) 725–730
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Partial Differential Equations

$\mathcal{D}$-modules associated to the projective space of $n \times n$ matrices

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Received 8 March 2005; accepted after revision 30 March 2005

Available online 4 May 2005

Presented by Jean-Michel Bony

Abstract

Let us consider X the complex vector space of square matrices and $\mathbb{P}(X)$ the associated projective space. Denote $\mathcal{A}$ the quotient algebra of all $\mathrm{SL}_n(\mathbb{C}) \times \mathrm{SL}_n(\mathbb{C})$-invariant differential operators modulo those vanishing on $\mathrm{SL}_n(\mathbb{C}) \times \mathrm{SL}_n(\mathbb{C})$-invariant functions. We show that the inverse image functor π^+, where $\pi : X\backslash\{0\} \to \mathbb{P}(X)$ is the canonical projection, establishes an equivalence of categories between the category of regular holonomic $\mathcal{D}$-modules on the projective space $\mathbb{P}(X)$ and the quotient category of graded $\mathcal{A}$-modules of finite type modulo those supported by $\{0\}$. Then we deduce a combinatorial classification of regular holonomic $\mathcal{D}_{\mathbb{P}(X)}$-modules. ***To cite this article: P. Nang, C. R. Acad. Sci. Paris, Ser. I 340 (2005).***
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Résumé

$\mathcal{D}$-modules associés au projectif des matrices $n \times n$. Considérons X l'espace vectoriel complexe des matrices carrées et $\mathbb{P}(X)$ l'espace projectif associé. Notons $\mathcal{A}$ l'algèbre quotient de tous les opérateurs différentiels $\mathrm{SL}_n(\mathbb{C}) \times \mathrm{SL}_n(\mathbb{C})$-invariants modulo ceux s'annulant sur les fonctions $\mathrm{SL}_n(\mathbb{C}) \times \mathrm{SL}_n(\mathbb{C})$-invariantes. Nous montrons que le foncteur image inverse π^+, où $\pi : X\backslash\{0\} \to \mathbb{P}(X)$ est la projection canonique, établit une équivalence de catégories entre la catégorie des $\mathcal{D}$-modules holonômes réguliers sur l'espace projectif $\mathbb{P}(X)$ et la catégorie quotient des $\mathcal{A}$-modules gradués de type fini modulo ceux portés par $\{0\}$. On en déduit une classification des $\mathcal{D}_{\mathbb{P}(X)}$-modules holonômes réguliers. ***Pour citer cet article : P. Nang, C. R. Acad. Sci. Paris, Ser. I 340 (2005).***
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Version française abrégée

Soient X l'espace vectoriel complexe des matrices carrées d'ordre n, $\widetilde{X}$ l'espace projectif $\mathbb{P}(X) = \mathbb{P}^{n^2-1}$ associé, $\mathcal{D}_{\widetilde{X}}$ (resp. $\mathcal{D}_X$) le faisceau des opérateurs différentiels sur le projectif $\widetilde{X}$ (resp. X). Le groupe $\mathbb{P}(\mathrm{GL}_n(\mathbb{C})) \times$

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[1] Supported by the ICTP Research Fellowship.

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doi:10.1016/j.crma.2005.04.005

$\mathbb{P}(\mathrm{GL}_n(\mathbb{C}))$ opère sur $\widetilde{X}$ par multiplication à gauche et à droite : $((g,h),A) \to gAh^{-1}$. Ce groupe a n orbites $\widetilde{X}_k \subset \widetilde{X}$ les sous ensembles de $\mathbb{P}(X)$ des matrices de rang k. On se propose de classifier les $\mathcal{D}_{\mathbb{P}(X)}$-modules holonômes réguliers $\mathcal{M}$ dont la variété caractéristique car$(\mathcal{M})$ est contenu dans la réunion $\tilde{\Lambda}$ des fibrés conormaux aux orbites $\widetilde{X}_k$:

$$\mathrm{car}(\mathcal{M}) \subset \tilde{\Lambda} := \bigcup_{k=1}^{n} T^*_{\widetilde{X}_k} \widetilde{X}. \tag{1}$$

Ils forment une catégorie abélienne que nous noterons $\mathrm{Mod}^{\mathrm{rh}}_{\tilde{\Lambda}}(\mathcal{D}_{\mathbb{P}(X)})$. Notons que plusieurs auteurs se sont interéssés à la description de certaines catégories de $\mathcal{D}$-modules holonômes réguliers notament [1,3,4,6,12,14–18]. Pour décrire les objets de $\mathrm{Mod}^{\mathrm{rh}}_{\tilde{\Lambda}}(\mathcal{D}_{\mathbb{P}(X)})$, on se sert de la projection canonique $\pi : X\backslash\{0\} \to \mathbb{P}(X)$. On étudie l'image inverse $\pi^+(\mathcal{M})$ d'un $\mathcal{D}_{\mathbb{P}(X)}$-module holonôme régulier $\mathcal{M}$ dans $\mathrm{Mod}^{\mathrm{rh}}_{\tilde{\Lambda}}(\mathcal{D}_{\mathbb{P}(X)})$. Soient $G := \mathrm{SL}_n(\mathbb{C}) \times \mathrm{SL}_n(\mathbb{C})$ et $\bar{A} := \Gamma(X, \mathcal{D}_X)^G$ l'algèbre de Weyl des opérateurs différentiels G-invariants. On note $\mathcal{A}$ le quotient de $\bar{A}$ par l'ideal $\mathcal{J} \subset \bar{A}$ des opérateurs nuls sur les fonctions G-invariantes. Soit θ le champ d'Euler sur X. On introduit la catégorie $\mathcal{C}$ dont les objets sont des $\mathcal{A}$-modules gradués de type fini T tels que $\dim_{\mathbb{C}} \mathbb{C}[\theta]u < \infty \ \forall u \in T$. Notons $\mathcal{C}' \subset \mathcal{C}$ la sous catégorie des $\mathcal{A}$-modules engendrés par les sections homogènes de degré entier (i.e. sections annulées par une puissance de $(\theta - p)$ avec p entier). Soit $\mathcal{C}_0 \subset \mathcal{C}'$ la sous catégorie des modules portés par l'origine $\{0\}$. Posons $\mathcal{C}'' := \mathcal{C}'/\mathcal{C}_0$ la catégorie quotient correspondante. On a le théorème suivant :

Théorème 0.1. *Le foncteur image inverse π^+ établit une équivalence de catégories entre la catégorie $\mathrm{Mod}^{\mathrm{rh}}_{\tilde{\Lambda}}(\mathcal{D}_{\mathbb{P}(X)})$ et la catégorie quotient C''.*

Enfin la catégorie $\mathcal{C}''$ se décrit à l'aide d'espaces vectoriels de dimension finie reliés par des morphismes vérifiant certaines conditions (cf. 3.3).

1. Introduction

Let X be the complex vector space of square matrices of order n, $\widetilde{X}$ its associated projective space $\mathbb{P}(X) = \mathbb{P}^{n^2-1}$. As usual $\mathcal{D}_{\widetilde{X}}$ (resp. $\mathcal{D}_X$) will refer to the sheaf of differential operators on $\widetilde{X}$ (resp. X). The group $\mathbb{P}(\mathrm{GL}_n(\mathbb{C})) \times \mathbb{P}(\mathrm{GL}_n(\mathbb{C}))$ acts on $\widetilde{X}$ by right and left multiplication: $((g,h),A) \to gAh^{-1}$. This group has n orbits $\widetilde{X}_k \subset \widetilde{X}$ the subsets of $\mathbb{P}(X)$ of matrices of rank k. Our purpose is to classify regular holonomic $\mathcal{D}_{\mathbb{P}(X)}$-modules $\mathcal{M}$ whose characteristic variety char$(\mathcal{M})$ is contained in the union of conormal bundles to the orbits $\widetilde{X}_k$:

$$\mathrm{char}(\mathcal{M}) \subset \tilde{\Lambda} := \bigcup_{k=1}^{n} T^*_{\widetilde{X}_k} \widetilde{X}. \tag{2}$$

They form an Abelian category we shall denote by $\mathrm{Mod}^{\mathrm{rh}}_{\tilde{\Lambda}}(\mathcal{D}_{\mathbb{P}(X)})$. Note that several authors were interested in the description of certain categories of regular holonomic $\mathcal{D}$-modules such as [1,3,4,6,12,14–18]. To describe the objects in $\mathrm{Mod}^{\mathrm{rh}}_{\tilde{\Lambda}}(\mathcal{D}_{\mathbb{P}(X)})$, we use the canonical projection $\pi : X\backslash\{0\} \to \mathbb{P}(X)$. We study the inverse image $\pi^+(\mathcal{M})$ of a regular holonomic $\mathcal{D}_{\mathbb{P}(X)}$-module $\mathcal{M}$ in $\mathrm{Mod}^{\mathrm{rh}}_{\tilde{\Lambda}}(\mathcal{D}_{\mathbb{P}(X)})$. Denote $G := \mathrm{SL}_n(\mathbb{C}) \times \mathrm{SL}_n(\mathbb{C})$ and θ the Euler vector field on X. We show that $\pi^+(\mathcal{M})$ is generated over $\mathcal{D}_{X\backslash\{0\}}$by a finite number of global 'homogeneous sections of integral degree' (i.e. sections annihilated by a power of $(\theta - p)$ with p an integer). Moreover, these sections are G-invariant. In the sequel $\bar{A} := \Gamma(X, \mathcal{D}_X)^G$ will denote the Weyl algebra on X of G-invariant differential operators. We denote $\mathcal{A}$ the quotient of $\bar{A}$ by $\mathcal{J} \subset \bar{A}$ the ideal of operators vanishing on G-invariant functions. Let us introduce the category $\mathcal{C}$ consisting of graded $\mathcal{A}$-modules of finite type T such that $\dim_{\mathbb{C}} \mathbb{C}[\theta]u < \infty \ \forall u \in T$. Denote $\mathcal{C}' \subset \mathcal{C}$ the subcategory of graded $\mathcal{A}$-modules generated by 'homogeneous sections of integral degree'.

Next, denote $\mathcal{C}_0 \subset \mathcal{C}^{'}$ the subcategory consisting of modules with support at the origin $\{0\}$. Now, put $\mathcal{C}^{''} := \mathcal{C}^{'}/\mathcal{C}_0$ the associated quotient category. We have the following theorem:

Theorem 1.1. *The inverse image functor π^+ establishes an equivalence of categories between the category $\mathrm{Mod}^{\mathrm{rh}}_{\tilde{\Lambda}}(\mathcal{D}_{\mathbb{P}(X)})$ and the quotient category $\mathcal{C}''$.*

Finally, the quotient category $\mathcal{C}'' := \mathcal{C}'/\mathcal{C}_0$ can be encoded by means of finite dimension complex vector spaces related by morphisms satisfying certain conditions (see 3.3).

2. Review

First, we refer the reader to [2,7–11] for notions on analytic $\mathcal{D}$-modules. Recall $X = M_n(\mathbb{C})$, $\widetilde{X} = \mathbb{P}(X)$. We denote by X_k (resp. $\widetilde{X}_k$) the subset of matrices in X (resp. $\widetilde{X}$) of rank k. Let $\Lambda := \bigcup_{k=0}^{n} T^*_{X_k} X$ (resp. $\tilde{\Lambda} := \bigcup_{k=1}^{n} T^*_{\widetilde{X}_k} \widetilde{X}$) be the union of conormal bundles to these strata. Denote by $\mathrm{Mod}^{\mathrm{rh}}_{\Lambda}(\mathcal{D}_X)$ (resp. $\mathrm{Mod}^{\mathrm{rh}}_{\tilde{\Lambda}}(\mathcal{D}_{\mathbb{P}(X)})$) the category whose objects are regular holonomic $\mathcal{D}_X$ (resp. $\mathcal{D}_{\widetilde{X}}$)-modules with characteristic variety contained in Λ (resp. $\tilde{\Lambda}$).

Let $\mathcal{W}$ be the Weyl algebra on X. Denote by $G := \mathrm{SL}_n(\mathbb{C}) \times \mathrm{SL}_n(\mathbb{C})$ and $\bar{\mathcal{A}} := \Gamma(X, \mathcal{D}_X)^G \subset \mathcal{W}$ the subalgebra of G-invariant differential operators. Let $x = (x_{ij})$, $d =^t (\partial/\partial x_{ij})$ be matrices with entries in $\mathcal{D}_X$. The group G acts on these matrices by: $(g,h)\cdot(x,d) = (gxh^{-1}, hdg^{-1})\ \forall (g,h) \in G$. Denote by Tr the trace map. We set $\delta = \det(x)$, $\Delta = \det(d)$, $\theta = \mathrm{Tr}\, xd$. Consider $\mathcal{J} := \{P \in \bar{\mathcal{A}},\ Pf = 0\ \forall f = f(\delta)\}$ the two sided ideal in $\bar{\mathcal{A}}$ of operators vanishing on G-invariant functions. Put $\mathcal{A} := \bar{\mathcal{A}}/\mathcal{J}$. We recall (see [16, Corollary 4, p. 75]) the following proposition:

Proposition 2.1. *The quotient algebra $\mathcal{A}$ is generated over $\mathbb{C}$ by δ, Δ, θ such that*

$$[\theta,\delta] = n\delta, \quad [\theta,\Delta] = -n\Delta, \quad \delta\Delta = \prod_{l=0}^{n-1}\left(\frac{\theta}{n}+l\right), \quad \Delta\delta = \prod_{l=1}^{n}\left(\frac{\theta}{n}+l\right).$$

Now, let us denote by $\mathcal{C}$ the category consisting of graded $\mathcal{A}$-module of finite type T such that $\dim_{\mathbb{C}} \mathbb{C}[\theta]u < \infty$ $\forall u \in T$. We recall [16, Theorem 9, p. 77] the following result which will be effectively used in the next section.

Theorem 2.2. *The categories $\mathrm{Mod}^{\mathrm{rh}}_{\Lambda}(\mathcal{D}_X)$ and $\mathcal{C}$ are equivalent.*

3. Study of the inverse image

Definition 3.1. Let $\mathcal{N}$ be a $\mathcal{D}_X$-module. A section s in $\mathcal{N}$ is said to be homogeneous of integral degree $p \in \mathbb{Z}$, if there exists $j \in \mathbb{N}$ such that $(\theta - p)^j s = 0$.

As in the introduction, denote by $\mathcal{C}^{'} \subset \mathcal{C}$ the subcategory consisting of graded $\mathcal{A}$-modules of finite type generated by homogeneous sections of integral degree. Denote by $\mathcal{C}_0 \subset \mathcal{C}^{'}$ the subcategory consisting of modules supported by the origin $\{0\}$. We consider $\mathcal{C}'' := \mathcal{C}^{'}/\mathcal{C}_0$ the corresponding quotient category. Let $\mathcal{M}$ be an object in $\mathrm{Mod}^{\mathrm{rh}}_{\tilde{\Lambda}}(\mathcal{D}_{\mathbb{P}(X)})$. In this section, we study the inverse image of $\mathcal{M}$ by the canonical projection $\pi : X\backslash\{0\} \to \mathbb{P}(X)$. It is a regular holonomic $\mathcal{D}_{X\backslash\{0\}}$-module (see [11, Corollary 5.4.8]). We show that $\pi^+(\mathcal{M})$ is an object in the quotient category $\mathcal{C}''$. To do so let us first recall that $\mathcal{M}$ has a good filtration $\mathcal{M} = \bigcup_{k\in\mathbb{Z}} \mathcal{M}_k$ (see [11, Corollary 5.1.11]). By using the Cartan Theorem A (see [5], [19, Lemme 7]) we can see that for a large enough integer n, the module

$\mathcal{M}_k \otimes_{\mathcal{O}_{\mathbb{P}(X)}} \mathcal{O}(n)$ is generated over $\mathcal{O}_{\mathbb{P}(X)}$ by its global sections and $H^j(\mathbb{P}(X), \mathcal{M}_k \otimes \mathcal{O}(n)) = 0$ for $j > 0$ (see Cartan Theorem B in [5], [19, Lemme 8]). Next $\mathcal{M} \otimes \mathcal{O}(n)$ is a $\mathcal{D}(n)$-module (with $\mathcal{D}(n) := \mathcal{O}(n) \otimes \mathcal{D}_{\mathbb{P}(X)} \otimes \mathcal{O}(-n)$). Then the sections in $\pi^*(\mathcal{M} \otimes \mathcal{O}(n))$ give the homogeneous sections of integral degree n in $\pi^+(\mathcal{M})$. Thus the regular holonomic $\mathcal{D}_{X\setminus\{0\}}$-module $\pi^+(\mathcal{M})$ is generated by homogeneous sections of integral degree $p \in \mathbb{Z}$. Moreover these homogeneous generators are invariant under the action of G. We have the following theorem:

Theorem 3.2. *$\pi^+(\mathcal{M})$ is generated over $\mathcal{D}_{X\setminus\{0\}}$ by a finite number of global homogeneous sections of "integral degree" and G-invariant.*

3.1. Characterisation of $\pi^+(\mathcal{M})$

In this subsection, we deal first with the extension of the inverse image $\pi^+(\mathcal{M})$. In other word, we see that there exists a regular holonomic $\mathcal{D}_X$-module $\mathcal{N}$ whose restriction on $X\setminus\{0\}$ is isomorphic to $\pi^+(\mathcal{M})$ that is there is a surjective morphism $\mathcal{N} \to \pi^+(\mathcal{M})$. Note $i : X\setminus\{0\} \hookrightarrow X$ the open embedding. We consider $\mathcal{N} := i_+(\pi^+(\mathcal{M}))$ the direct image of $\pi^+(\mathcal{M})$ by the inclusion i. It is a regular holonomic $\mathcal{D}_X$-module (see [11, Theorem 6.2.1]) in $\mathrm{Mod}^{\mathrm{rh}}_{\Lambda}(\mathcal{D}_X)$ which extends $\pi^+(\mathcal{M})$ (see [13, Proposition 2.3]). Next, since the category $\mathrm{Mod}^{\mathrm{rh}}_{\Lambda}(\mathcal{D}_X)$ is equivalent to the category $\mathcal{C}$ (see Theorem 2.2), we can see that $i_+(\pi^+(\mathcal{M}))$ corresponds to an object in the category $\mathcal{C}$. Therefore by using Theorem 3.2, we can see that $\pi^+(\mathcal{M})$ is an object in the quotient category $\mathcal{C}''$. We have the following proposition:

Proposition 3.3. *For any object $\mathcal{M}$ in $\mathrm{Mod}^{\mathrm{rh}}_{\tilde{\Lambda}}(\mathcal{D}_{\mathbb{P}(X)})$, the inverse image $\pi^+(\mathcal{M})$ is an object in the quotient category $\mathcal{C}''$.*

3.2. Equivalence of categories

Now, let $\widetilde{\mathcal{N}}$ be an object in the quotient category $\mathcal{C}''$, we associate to it the module

$$\mathcal{N}_0 := i^+\big(\mathcal{D}_X \otimes_{\mathcal{A}} \widetilde{\mathcal{N}}\big) \tag{3}$$

with $i : X\setminus\{0\} \hookrightarrow X$ the inclusion. This last is a regular holonomic module over $\mathcal{D}_{X\setminus\{0\}}$. Then we can see that its direct image $\pi_+(\mathcal{N}_0)$ by the projection π is an object in the category $\mathrm{Mod}^{\mathrm{rh}}_{\tilde{\Lambda}}(\mathcal{D}_{\mathbb{P}(X)})$. Since for any object $\mathcal{M}$ in $\mathrm{Mod}^{\mathrm{rh}}_{\tilde{\Lambda}}(\mathcal{D}_{\mathbb{P}(X)})$, we can associate to it an object $\pi^+(\mathcal{M})$ in the quotient category $\mathcal{C}''$ (see Proposition 3.3), we have then constructed two functors

$$\begin{cases} \pi^+ : \mathrm{Mod}^{\mathrm{rh}}_{\tilde{\Lambda}}(\mathcal{D}_{\mathbb{P}(X)}) \to \mathcal{C}'', \\ \pi_+ : \mathcal{C}'' \to \mathrm{Mod}^{\mathrm{rh}}_{\tilde{\Lambda}}(\mathcal{D}_{\mathbb{P}(X)}). \end{cases} \tag{4}$$

We get the following theorem:

Theorem 3.4. *The inverse image functor π^+ establishes an equivalence of categories between the category $\mathrm{Mod}^{\mathrm{rh}}_{\tilde{\Lambda}}(\mathcal{D}_{\mathbb{P}(X)})$ and the quotient category $\mathcal{C}''$.*

3.3. Diagrams associated to graded $\mathcal{A}$-modules

Let us mention that the objects in the quotient category $\mathcal{C}'' := \mathcal{C}'/\mathcal{C}_0$ can be understood by means of finite diagrams of linear maps. This section consists in the classification of such diagrams. To put it more precisely, a graded $\mathcal{A}$-module T in $\mathcal{C}'$ defines an infinite diagram consisting of finite dimensional complex vector spaces T_p

(with $(\theta - p)$ being nilpotent on each $T_p,\ p \in \mathbb{Z}$) and linear maps between them deduced from the action of θ, δ, Δ:

$$\cdots \rightleftarrows T_p \underset{\Delta}{\overset{\delta}{\rightleftarrows}} T_{p+n} \rightleftarrows \cdots \tag{5}$$

satisfying the relations of Proposition 2.1 and the following $(\theta - p)T_p \subset T_p$,

$$\delta\Delta = \frac{\theta}{n}\left(\frac{\theta}{n}+1\right)\cdots\left(\frac{\theta}{n}+n-1\right), \quad \Delta\delta = \left(\frac{\theta}{n}+1\right)\cdots\left(\frac{\theta}{n}+n\right) \quad \text{on } T_p. \tag{6}$$

First, for any module in $\mathcal{C}_0$ supported by $\{0\}$, let us describe the corresponding diagram.

Remark 1. The module $\mathcal{B}_{\{0\}|X}$ is generated by an element e_{-n^2} such that $\theta e_{-n^2} = -n^2 e_{-n^2}$ and $\delta e_{-n^2} = 0$. Then its associated graded $\mathcal{A}$-module T has a basis (e_m) where $m = -n^2 - nk$ $(k \in \mathbb{N})$ such that $\delta e_{-n^2} = 0$ and satisfying the following system:

$$S = \begin{cases} \theta e_m = m e_m \quad (m = -n^2 - nk,\ k \in \mathbb{N}), \\ \Delta e_m = e_{m-n}, \\ \delta e_m = \left(\dfrac{m+n}{n}\right)\left(\dfrac{m+2n}{n}\right)\cdots\left(\dfrac{m+n^2}{n}\right) e_{m+n}. \end{cases} \tag{7}$$

Since $\delta e_{-n^2} = 0$ (i.e. $\delta T_{-n^2} = 0$), the arrows at the right of T_{-n^2} in the diagram vanish.

Now, any object in the quotient category $\mathcal{C}''$ is a diagram $\widetilde{T} = T$ **modulo** $\mathcal{C}_0$ $(T \in \mathcal{C}')$

$$\cdots \rightleftarrows T_p \underset{\Delta}{\overset{\delta}{\rightleftarrows}} T_{p+n} \rightleftarrows \cdots \textbf{modulo}\ \mathcal{C}_0, \quad p \in \mathbb{Z}, \tag{8}$$

satisfying the previous relations (6). Such a diagram is completely determined by a finite subset of objects and arrows. Indeed, we distinguish two cases:

(a) If $p \equiv 0 \bmod n\mathbb{Z}$, then $\widetilde{T}$ is completely determined by a diagram with $(n+1)$ elements

$$T_{-n^2} \underset{\Delta}{\overset{\delta}{\rightleftarrows}} T_{-n(n-1)} \underset{\Delta}{\overset{\delta}{\rightleftarrows}} T_{-n(n-2)} \cdots T_{-n} \underset{\Delta}{\overset{\delta}{\rightleftarrows}} T_0 \ \textbf{modulo}\ \mathcal{C}_0. \tag{9}$$

In the other degrees δ or Δ are bijective. Indeed, we have $T_0 \simeq \delta^k T_0 \simeq T_{nk}$ and $T_{-n^2} \simeq \Delta^k T_{-n^2} \simeq T_{-n^2-nk}$ $(k \in \mathbb{N})$ thanks to the relations (6). The operator $\delta\Delta$ (resp. $\Delta\delta$) on T_p has only one eigenvalue $\frac{p}{n}(\frac{p}{n}+1)(\frac{p}{n}+2)\cdots(\frac{p}{n}+n-1)$ (resp. $(\frac{p}{n}+1)(\frac{p}{n}+2)\cdots(\frac{p}{n}+n)$). Then the equation $\delta\Delta = \prod_{l=0}^{n-1}(\frac{\theta}{n}+l)$ (resp. $\Delta\delta = \prod_{l=1}^{n}(\frac{\theta}{n}+l)$) has a unique solution θ of eigenvalue p if p is not a critical value. Here $p = 0, -n, -2n, \ldots, -n^2$ thus it is always the case.

(b) If $p \neq 0 \bmod n\mathbb{Z}$ (p integer), then δ and Δ are bijective. Thus $\widetilde{T}$ is completely determined up to a isomorphism by one element T_p **modulo** $\mathcal{C}_0$ equipped with the nilpotent action of $(\theta - p)$.

Acknowledgements

We would like to express our deep gratitude to Professor Louis Boutet de Monvel for extremely useful conversations and helpful insights.

References

[1] L. Boutet de Monvel, D-modules holonômes réguliers en une variable, in: Mathématiques et Physique, Séminaire de L'ENS 1979–1982, in: Progr. Math., vol. 37, 1983, pp. 313–321.
[2] L. Boutet de Monvel, Revue sur le théorie des D-modules et modèles d'opérateurs pseudodifférentiels, in: Math. Phys. Stud., vol. 12, Kluwer Academic, 1991, pp. 1–31.
[3] T. Braden, Perverse sheaves on Grassmannians, Canad. J. Math. 54 (3) (2002) 493–532.
[4] T. Braden, M. Grinberg, Perverse sheaves on rank stratifications, Duke Math. J. 96 (2) (1999) 317–362.
[5] H. Cartan, Variétés analytiques réelles et variétés analytiques complexes, Bull. Soc. Math. France 85 (1957) 77–99.
[6] S.I. Gelfand, S.M. Khoroshkin, Algebraic description of certain categories of D-modules, Funktsional. Anal. i Prilozhen. 19 (3) (1985) 56–57.
[7] M. Kashiwara, On the maximal overdetermined systems of linear partial differential equations I, Publ. Res. Inst. Math. Sci. 10 (1974/1975) 563–579.
[8] M. Kashiwara, On holonomic systems of linear partial differential equations II, Invent. Math. 49 (1978) 121–135.
[9] M. Kashiwara, Algebraic study of systems of partial differential equations, Mém. Soc. Math. France 63 (123 (4)) (1995).
[10] M. Kashiwara, D-modules and microlocal calculus, in: Iwanami Series in Modern Mathematics, in: Transl. Math. Monographs, vol. 217, American Mathematical Society, 2003.
[11] M. Kashiwara, M. Kawai, On holonomic systems of microdifferential equations III: systems with regular singularities, Publ. Res. Inst. Math. Sci. 17 (1981) 813–979.
[12] R. Macpherson, K. Vilonen, Perverse sheaves with regular singularities along the curve $x^n = y^m$, Comment. Math. Helv. 63 (1988) 89–102.
[13] B. Malgrange, Connexions méromorphes, in: London Math. Soc. Lecture Note Ser., vol. 201, 1994, pp. 251–261.
[14] P. Nang, D-modules associated to the group of similitudes, Publ. Res. Inst. Math. Sci. 35 (2) (1999) 223–247.
[15] P. Nang, D-modules associated to 3×3 matrices, C. R. Acad. Sci. Paris, Ser. I 338 (2004) 139–144.
[16] P. Nang, D-modules associated to the determinantal singularities, Proc. Japan. Acad. Ser. A 80 (5) (2004) 74–78.
[17] L. Narvaez Macarro, Faisceaux pervers dont le support singulier est le germe d'une courble plane irréductible, Thèse de 3$^{\text{ème}}$ Cycle, Université Paris VII, 1984.
[18] L. Narvaez Macarro, Cycles évanescents et faisceaux pervers. II. Cas des courbes planes réductibles, in: London Math. Soc. Lecture Note Ser., vol. 201, 1994, pp. 285–323.
[19] J.P. Serre, Géométrie algébrique et géométrie analytique, Ann. Inst. Fourier (Grenoble) 6 (1955–1956) 1–42.