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On the controllability of the N-dimensional Navier–Stokes and Boussinesq systems with $N-1$ scalar controls

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Abstract

In this Note we present several controllability results for nonlinear systems of the Navier–Stokes and Boussinesq kind. We discuss the existence of particular controls with a small number of degrees of freedom. ***To cite this article: E. Fernández-Cara et al., C. R. Acad. Sci. Paris, Ser. I 340 (2005).***

Résumé

Sur la contrôlabilité des systèmes de Navier–Stokes et Boussinesq N-dimensionnels avec $N-1$ contrôles scalaires. Dans cette Note on présente quelques résultats de contrôlabilité pour des systèmes non linéaires du type Navier–Stokes et Boussinesq. On analyse l'existence de contrôles particuliers avec un nombre petit de degrés de liberté. ***Pour citer cet article : E. Fernández-Cara et al., C. R. Acad. Sci. Paris, Ser. I 340 (2005).***

Version française abrégée

Cette Note contient quelques résultats de contrôlabilité pour des systèmes non linéaires du type Navier–Stokes et Boussinesq. On cherche des contrôles particuliers, avec un nombre petit de degrés de liberté.

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On se donne un domaine borné et régulier $\Omega \subset \mathbf{R}^N$ ($N = 2$ ou $N = 3$), un ouvert non vide (et petit) $\mathcal{O} \subset \Omega$ et un nombre $T > 0$. On considère d'abord le système

$$\begin{cases} y_t - \Delta y + (y \cdot \nabla) y + \nabla p = v \mathbb{1}_{\mathcal{O}}, \quad \nabla \cdot y = 0 & \text{dans } Q = \Omega \times (0, T), \\ y = 0 & \text{sur } \Sigma = \partial\Omega \times (0, T), \\ y(0) = y^0 & \text{dans } \Omega, \end{cases} \tag{1}$$

où $\mathbb{1}_{\mathcal{O}}$ est la fonction caractéristique de $\mathcal{O}$. Considérons l'espace de Banach $E = H \cap L^4(\Omega)^N$, où

$$H = \{ w \in L^2(\Omega)^N; \quad \nabla \cdot w = 0 \text{ dans } \Omega, \ w \cdot n = 0 \text{ sur } \partial\Omega \}. \tag{2}$$

On dira que (1) est *localement exactement contrôlable aux trajectoires en temps T* si, pour toute solution suffisamment régulière $(\bar{y}, \bar{p})$ du système non contrôlé

$$\begin{cases} \bar{y}_t - \Delta \bar{y} + (\bar{y} \cdot \nabla) \bar{y} + \nabla \bar{p} = 0, \quad \nabla \cdot \bar{y} = 0 & \text{dans } Q, \\ \bar{y} = 0 & \text{sur } \Sigma, \end{cases}$$

il existe $\delta > 0$ tel que, si $\|y^0 - \bar{y}(0)\|_E \leqslant \delta$, alors on peut trouver des contrôles $v \in L^2(\mathcal{O} \times (0, T))^N$ et des états associés (y, p) solution de (1) satisfaisant $y(T) = \bar{y}(T)$.

On considère maintenant le système de Boussinesq

$$\begin{cases} y_t - \Delta y + (y \cdot \nabla) y + \nabla p = v \mathbb{1}_{\mathcal{O}} + \theta\, e_N, \quad \nabla \cdot y = 0 & \text{dans } Q, \\ \theta_t - \Delta \theta + y \cdot \nabla \theta = h \mathbb{1}_{\mathcal{O}} & \text{dans } Q, \\ y = 0, \quad \theta = 0 & \text{sur } \Sigma, \\ y(0) = y^0, \quad \theta(0) = \theta^0 & \text{dans } \Omega, \end{cases} \tag{3}$$

où les contrôles sont $v \in L^2(\mathcal{O} \times (0, T))^N$ et $h \in L^2(\mathcal{O} \times (0, T))$. On dira que (3) est *localement exactement contrôlable aux trajectoires en temps T* si, pour toute solution suffisamment régulière $(\bar{y}, \bar{p}, \bar{\theta})$ du système

$$\begin{cases} \bar{y}_t - \Delta \bar{y} + (\bar{y} \cdot \nabla) \bar{y} + \nabla \bar{p} = \bar{\theta} e_N, \quad \nabla \cdot \bar{y} = 0 & \text{dans } Q, \\ \bar{\theta}_t - \Delta \bar{\theta} + \bar{y} \cdot \nabla \bar{\theta} = 0 & \text{dans } Q, \\ \bar{y} = 0, \quad \bar{\theta} = 0 & \text{sur } \Sigma, \end{cases}$$

il existe $\delta > 0$ tel que, si $\|(y^0, \theta^0) - (\bar{y}(0), \bar{\theta}(0))\|_{E \times L^2} \leqslant \delta$, alors on peut trouver des contrôles $v \in L^2(\mathcal{O} \times (0, T))^N$ et $h \in L^2(\mathcal{O} \times (0, T))$ et des états associés (y, p, θ) satisfaisant $y(T) = \bar{y}(T)$ et $\theta(T) = \bar{\theta}(T)$.

Finalement, lorsque $N = 2$, on considèrera les systèmes avec terme non linéaire *tronqué*

$$\begin{cases} y_t - \Delta y + (y \cdot \nabla) \mathbf{T}_M(y) + \nabla p = v \mathbb{1}_{\mathcal{O}}, \quad \nabla \cdot y = 0 & \text{dans } Q, \\ y \cdot n = 0, \quad \nabla \times y = 0 & \text{sur } \Sigma, \\ y(0) = y^0 & \text{dans } \Omega, \end{cases} \tag{4}$$

où $M > 0$, $\mathbf{T}_M(y) = (T_M(y_1), T_M(y_2))$ et

$$T_M(s) = \begin{cases} -M & \text{si } s \leqslant -M, \\ s & \text{si } -M \leqslant s \leqslant M, \\ M & \text{si } s \geqslant M. \end{cases}$$

On dira que (4) est (globalement) *contrôlable à zéro en temps T* si, pour tout $y^0 \in H$, il existe $v \in L^2(\mathcal{O} \times (0, T))^2$ tel que l'état associé satisfait $y(T) = 0$.

On fait les hypothèses suivantes sur $\mathcal{O}$, $\bar{y}$ et $\bar{\theta}$:

$$\exists x^0 \in \partial\Omega, \exists \varepsilon > 0 \text{ tels que } \overline{\mathcal{O}} \cap \partial\Omega \supset B(x^0; \varepsilon) \cap \partial\Omega, \tag{5}$$

$$\bar{y} \in L^\infty(Q)^N, \quad \bar{y}_t \in L^2\big(0, T; L^\sigma(\Omega)^N\big) \quad \begin{pmatrix} \sigma > 1 & \text{si } N = 2 \\ \sigma > 6/5 & \text{si } N = 3 \end{pmatrix} \tag{6}$$

et

$$\bar{\theta} \in L^\infty(Q), \quad \bar{\theta}_t \in L^2\big(0, T; L^\sigma(\Omega)\big) \quad \begin{pmatrix} \sigma > 1 & \text{si } N = 2 \\ \sigma > 6/5 & \text{si } N = 3 \end{pmatrix}. \tag{7}$$

On a alors les résultats suivants :

Théorème 0.1. *Supposons que $\mathcal{O}$ satisfait* (5). *Alors, pour tout $T > 0$,* (1) *est localement exactement contrôlable en temps T aux trajectoires $(\bar{y}, \bar{p})$ qui satisfont* (6) *avec des contrôles $v \in L^2(\mathcal{O} \times (0,T))^N$ qui possédent une composante nulle.*

Théorème 0.2. *Supposons que $\mathcal{O}$ satisfait* (5) *avec $n_i(x^0) \neq 0$, pour un indice $i < N$. Alors, pour tout $T > 0$,* (3) *est localement exactement contrôlable en temps T aux trajectoires $(\bar{y}, \bar{p}, \bar{\theta})$ qui satisfont* (6), (7) *avec des contrôles v et h tels que $v_i \equiv v_N \equiv 0$. En particulier, si $N = 2$, on a la contrôlabilité exacte locale aux trajectoires avec des contrôles $v \equiv 0$ et $h \in L^2(\mathcal{O} \times (0,T))$.*

Pour notre troisième résultat, on introduit l'espace

$$W = \big\{ \nabla \times z = (\partial_2 z, -\partial_1 z);\ z \in L^2\big(0,T; H^1(\Omega)\big) \big\}.$$

Alors on a :

Théorème 0.3. *Soit $N = 2$. Alors, pour tout $T > 0$ et tout $M > 0$, le système* (4) *est contrôlable à zéro en temps T avec des contrôles $v\mathbb{1}_{\mathcal{O}}$, où $v \in W$.*

Les démonstrations détaillées de ces résultats seront données dans un travail à paraître.

1. Introduction and main results

Let $\Omega \subset \mathbf{R}^N$ be a bounded and regular domain ($N = 2$ or $N = 3$), let $\mathcal{O} \subset \Omega$ be a nonempty (small) open subset and let $T > 0$ be given. In this Note, we present several controllability properties for some nonlinear systems of the Navier–Stokes and Boussinesq kind.

We will first be concerned with the Navier–Stokes system

$$\begin{cases} y_t - \Delta y + (y \cdot \nabla) y + \nabla p = v\mathbb{1}_{\mathcal{O}}, \quad \nabla \cdot y = 0 & \text{in } Q = \Omega \times (0,T), \\ y = 0 & \text{on } \Sigma = \partial\Omega \times (0,T), \\ y(0) = y^0 & \text{in } \Omega, \end{cases} \tag{8}$$

where $\mathbb{1}_{\mathcal{O}}$ is the characteristic function of $\mathcal{O}$. Let us introduce the Banach space $E = H \cap L^4(\Omega)^N$, where

$$H = \big\{ w \in L^2(\Omega)^N;\ \nabla \cdot w = 0 \text{ in } \Omega,\ w \cdot n = 0 \text{ on } \partial\Omega \big\}. \tag{9}$$

It will be said that (8) is *locally exactly controllable to the trajectories at time T* if, for each sufficiently regular solution $(\bar{y}, \bar{p})$ to the uncontrolled system

$$\begin{cases} \bar{y}_t - \Delta \bar{y} + (\bar{y} \cdot \nabla) \bar{y} + \nabla \bar{p} = 0, \quad \nabla \cdot \bar{y} = 0 & \text{in } Q, \\ \bar{y} = 0 & \text{on } \Sigma, \end{cases}$$

there exists $\delta > 0$ such that, whenever $\|y^0 - \bar{y}(0)\|_E \leqslant \delta$, we can find controls $v \in L^2(\mathcal{O} \times (0,T))^N$ and associated states (y, p) satisfying $y(T) = \bar{y}(T)$.

We will also consider the Boussinesq system

$$\begin{cases} y_t - \Delta y + (y \cdot \nabla) y + \nabla p = v\mathbb{1}_{\mathcal{O}} + \theta\, e_N, \quad \nabla \cdot y = 0 & \text{in } Q, \\ \theta_t - \Delta \theta + y \cdot \nabla \theta = h\mathbb{1}_{\mathcal{O}} & \text{in } Q, \\ y = 0, \quad \theta = 0 & \text{on } \Sigma, \\ y(0) = y^0, \quad \theta(0) = \theta^0 & \text{in } \Omega, \end{cases} \tag{10}$$

where the controls are now $v \in L^2(\mathcal{O} \times (0,T))^N$ and $h \in L^2(\mathcal{O} \times (0,T))$. It will be said that (10) is *locally exactly controllable to the trajectories at time* T if, for any sufficiently regular solution $(\bar{y}, \bar{p}, \bar{\theta})$ of the system

$$\begin{cases} \bar{y}_t - \Delta\bar{y} + (\bar{y}\cdot\nabla)\bar{y} + \nabla\bar{p} = \bar{\theta}e_N, \quad \nabla\cdot\bar{y} = 0 & \text{in } Q, \\ \bar{\theta}_t - \Delta\bar{\theta} + \bar{y}\cdot\nabla\bar{\theta} = 0 & \text{in } Q, \\ \bar{y} = 0, \quad \bar{\theta} = 0 & \text{on } \Sigma, \end{cases}$$

there exists $\delta > 0$ such that, if $\|(y^0, \theta^0) - (\bar{y}(0), \bar{\theta}(0))\|_{E\times L^2} \leqslant \delta$, then we can find controls $v \in L^2(\mathcal{O} \times (0,T))^N$ and $h \in L^2(\mathcal{O} \times (0,T))$ and associated states (y, p, θ) satisfying $y(T) = \bar{y}(T)$ and $\theta(T) = \bar{\theta}(T)$.

Finally, for $N = 2$ we will also consider the systems with *truncated* nonlinearity

$$\begin{cases} y_t - \Delta y + (y\cdot\nabla)\mathbf{T}_M(y) + \nabla p = v\mathbb{1}_{\mathcal{O}}, \quad \nabla\cdot y = 0 & \text{in } Q, \\ y\cdot n = 0, \quad \nabla\times y = 0 & \text{on } \Sigma, \\ y(0) = y^0 & \text{in } \Omega, \end{cases} \tag{11}$$

where the boundary conditions are of the Navier kind. Here, $M > 0$, $\mathbf{T}_M(y) = (T_M(y_1), T_M(y_2))$ and

$$T_M(s) = \begin{cases} -M & \text{if } s \leqslant -M, \\ s & \text{if } -M \leqslant s \leqslant M, \\ M & \text{if } s \geqslant M. \end{cases}$$

It will be said that (11) is (globally) *null controllable at time* T if, for each $y^0 \in H$, there exists $v \in L^2(\mathcal{O} \times (0,T))^2$ such that the associated state satisfies $y(T) = 0$.

In the previous systems, $v = v(x,t)$ and $h = h(x,t)$ are control functions. The goal of this Note is to prove the existence of controls with a reduced number of degrees of freedom such that the previous controllability properties hold.

Some hypotheses will be imposed on the control domain and the trajectories. More precisely, we will frequently assume that

$$\exists x^0 \in \partial\Omega,\ \exists\varepsilon > 0 \text{ such that } \overline{\mathcal{O}} \cap \partial\Omega \supset B(x^0;\varepsilon) \cap \partial\Omega, \tag{12}$$

($B(x^0;\varepsilon)$ is the ball centered at x^0 of radius ε),

$$\bar{y} \in L^\infty(Q)^N, \quad \bar{y}_t \in L^2\big(0,T; L^\sigma(\Omega)^N\big) \quad \begin{pmatrix} \sigma > 1 & \text{if } N = 2 \\ \sigma > 6/5 & \text{if } N = 3 \end{pmatrix} \tag{13}$$

and

$$\bar{\theta} \in L^\infty(Q), \quad \bar{\theta}_t \in L^2\big(0,T; L^\sigma(\Omega)\big) \quad \begin{pmatrix} \sigma > 1 & \text{if } N = 2 \\ \sigma > 6/5 & \text{if } N = 3 \end{pmatrix}. \tag{14}$$

Our first two results are the following:

Theorem 1.1. *Assume that* $\mathcal{O}$ *satisfies* (12). *Then, for any* $T > 0$, (8) *is locally exactly controllable at time* T *to the trajectories* $(\bar{y}, \bar{p})$ *satisfying* (13) *with controls* $v \in L^2(\mathcal{O} \times (0,T))^N$ *such that* $v_i \equiv 0$ *for some* i.

Theorem 1.2. *Assume that* $\mathcal{O}$ *satisfies* (12) *with* $n_k(x^0) \neq 0$ *for some* $k < N$. *Then, for each* $T > 0$, (10) *is locally exactly controllable at time* T *to the trajectories* $(\bar{y}, \bar{p}, \bar{\theta})$ *satisfying* (13), (14) *with controls* v *and* h *such that* $v_k \equiv v_N \equiv 0$. *In particular, if* $N = 2$, *we have local exact controllability to the trajectories with controls* $v \equiv 0$ *and* $h \in L^2(\mathcal{O} \times (0,T))$.

For our last result, let us introduce the space

$$W = \big\{\nabla\times z = (\partial_2 z, -\partial_1 z);\ z \in L^2\big(0,T; H^1(\Omega)\big)\big\}.$$

We have the following:

Theorem 1.3. *Let $N = 2$. Then, for any $T > 0$ and any $M > 0$, (11) is null controllable at time T with controls of the form $v\mathbb{1}_{\mathcal{O}}$, where $v \in W$.*

In the following sections, we will indicate the main ideas used in the proofs of the previous results (for simplicity, we will only refer to Theorems 1.1 and 1.3). The detailed proofs will be given in a forthcoming paper.

2. The null controllability of some similar linear systems

Following well known arguments, we will first deduce null controllability results for linearized versions of (8) and (11), namely:

$$\begin{cases} y_t - \Delta y + (\bar{y} \cdot \nabla) y + (y \cdot \nabla)\bar{y} + \nabla p = f + v\mathbb{1}_{\mathcal{O}}, \quad \nabla \cdot y = 0 & \text{in } Q, \\ y = 0 & \text{on } \Sigma, \\ y(0) = y^0 & \text{in } \Omega \end{cases} \tag{15}$$

(where $\bar{y}$ satisfies (13) and $f = f(x,t)$ satisfies appropriate decay assumptions near $t = T$) and

$$\begin{cases} y_t - \Delta y + (y \cdot \nabla)\bar{y} + \nabla p = v\mathbb{1}_{\mathcal{O}}, \quad \nabla \cdot y = 0 & \text{in } Q, \\ y \cdot n = 0, \quad \nabla \times y = 0 & \text{on } \Sigma, \\ y(0) = y^0 & \text{in } \Omega \end{cases} \tag{16}$$

(where we assume that $N = 2$ and $\bar{y} \in L^\infty(Q)^2$).

For the null controllability of (15) with $v_k \equiv 0$, the main tool is a *global Carleman estimate* for the solutions of the associated adjoint system

$$\begin{cases} -\varphi_t - \Delta\varphi - (D\varphi)\bar{y} + \nabla\pi = g, \quad \nabla \cdot \varphi = 0 & \text{in } Q, \\ \varphi = 0 & \text{on } \Sigma, \\ \varphi(T) = \varphi^0 & \text{in } \Omega, \end{cases} \tag{17}$$

where $D\varphi = \nabla\varphi + {}^t\nabla\varphi$ and $g \in L^2(Q)^N$. Indeed, assume that (for instance) $N = 3$ and $n_1(x^0) \neq 0$. Then the task is to prove that the solutions of (17) satisfy

$$\iint_Q \rho_1^2 |\varphi|^2 \,\mathrm{d}x\,\mathrm{d}t \leqslant C(\Omega, \mathcal{O}, T, \bar{y}) \left(\iint_Q \rho_2^2 |g|^2 \,\mathrm{d}x\,\mathrm{d}t + \iint_{\mathcal{O}\times(0,T)} \rho_3^2 \left(|\varphi_2|^2 + |\varphi_3|^2\right) \mathrm{d}x\,\mathrm{d}t \right) \tag{18}$$

for some appropriate weights $\rho_i = \rho_i(x,t)$ and some $C(\Omega, \mathcal{O}, T, \bar{y}) > 0$. This can be proved using first a global Carleman inequality established in [1]. At this point $|\varphi_1|^2$ appears in the second term of the right hand side of (18). Using hypothesis (12) and the facts that $\nabla \cdot \varphi = 0$ in Q and $\varphi_1 = 0$ on Σ we can get rid of this local term in $|\varphi_1|^2$.

On the other hand, we can also deduce a null controllability result for (16) whenever $\bar{y} \in L^\infty(Q)^2$. To this end, let us introduce the streamline-vorticity formulation of (16), namely

$$\begin{cases} \omega_t - \Delta\omega + \nabla \times \left((\nabla \times \psi) \cdot \nabla\right)\bar{y} = \nabla \times (v\mathbb{1}_{\mathcal{O}}), \quad \Delta\psi = \omega & \text{in } Q, \\ \psi = 0, \quad \omega = 0 & \text{on } \Sigma, \\ \omega(0) = \nabla \times y^0 & \text{in } \Omega \end{cases} \tag{19}$$

and the associated adjoint system

$$\begin{cases} -\rho_t - \Delta\rho - \nabla \times \left((\bar{y} \cdot \nabla\times)\nabla\theta\right) = 0, \quad \Delta\theta = \rho & \text{in } Q, \\ \theta = 0, \quad \rho = 0 & \text{on } \Sigma, \\ \rho(T) = \rho^0 & \text{in } \Omega. \end{cases} \tag{20}$$

Then the task amounts to prove the observability inequality

$$\left\| \nabla\theta(0) \right\|_{L^2}^2 \leqslant C \iint_{\mathcal{O}\times(0,T)} |\nabla\theta|^2 \,\mathrm{d}x\,\mathrm{d}t \tag{21}$$

for the solutions of (20). The proof of (21) relies on some global Carleman inequalities established in [3] and [4].

3. The local exact controllability of the Navier–Stokes systems (8) and (11)

Theorem 1.1 is proved by applying an inverse mapping theorem. In the framework of the Navier–Stokes equations, this strategy was introduced by O.Yu. Imanuvilov in [2] and has been used recently in [1], relaxing considerably the regularity requirements on the trajectories $(\bar{y}, \bar{p})$.

On the other hand, in order to prove Theorem 1.3 we can use a fixed point argument. This is possible because the unique assumption on $\bar{y}$ needed for the null controllability of (16) is $\bar{y} \in L^\infty(Q)^2$. At this level, the fact that $N = 2$ and the particular form of the boundary conditions in (11) are essential. Thus, we can introduce a set-valued mapping $z \mapsto \Lambda(z)$ where, for each $z \in L^2(Q)^2$, $\Lambda(z)$ is the family of functions y which solve (together with some p) the linear system (16) with $\bar{y} = \mathbf{T}_M(z)$ and satisfy $y(T) = 0$ (and suitable estimates). It can be seen that an appropriate version of Kakutani's fixed point theorem can be applied to Λ.

Remark 1. Assume that $N = 2$. The arguments in [1] implicitly show that, under hypotheses (13), we can find controls $v\mathbb{1}_{\mathcal{O}}$ with $v \in W$ such that the associated solutions to (8) satisfy $y(T) = \bar{y}(T)$. Observe that the assumption (12) on the control domain is not necessary here.

Remark 2. Assume that $N = 3$. It is natural to ask whether a result similar to Theorem 1.1 holds with controls having two zero components. In general, the answer is no.

In fact, it seems difficult to identify the open sets Ω and $\mathcal{O}$ such that one has null controllability for all $T > 0$, even for the linear problems, with a reduced number of controls. This is unknown even for the classical Stokes equations for which, up to now, the unique known results concern *approximate controllability*; see [5].

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References

[1] E. Fernández-Cara, S. Guerrero, O.Yu. Imanuvilov, J.P. Puel, Local exact controllability of the Navier–Stokes system, J. Math. Pures Appl., in press.

[2] O.Yu. Imanuvilov, Remarks on exact controllability for the Navier–Stokes equations, ESAIM Control Optim. Calc. Var. 6 (2001) 39–72.

[3] O.Yu. Imanuvilov, J.P. Puel, Global Carleman estimates for weak elliptic non homogeneous Dirichlet problem, Int. Math. Res. Notices 16 (2003) 883–913.

[4] O.Yu. Imanuvilov, M. Yamamoto, Carleman Estimate for a Parabolic Equation in a Sobolev Space of Negative Order and its Applications, Lecture Notes in Pure and Appl. Math., vol. 218, Dekker, New York, 2001.

[5] J.-L. Lions, E. Zuazua, A generic uniqueness result for the Stokes system and its control theoretical consequences, in: P. Marcellini, G. Talenti, E. Visentini (Eds.), Partial Differential Equations and Applications, in: Lecture Notes in Pure and Appl. Math., vol. 177, Dekker, New York, 1996, pp. 221–235.