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Algebra

Cup products in Hopf-cyclic cohomology

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Abstract

We construct cup products of two different kinds for Hopf-cyclic cohomology. When the Hopf algebra reduces to the ground field our first cup product reduces to Connes' cup product in ordinary cyclic cohomology. The second cup product generalizes Connes–Moscovici's characteristic map for actions of Hopf algebras on algebras. ***To cite this article: M. Khalkhali, B. Rangipour, C. R. Acad. Sci. Paris, Ser. I 340 (2005).***

Résumé

Cup-produits dans la cohomologie Hopf-cyclique. Nous construisons deux types de cup-produits pour la cohomologie Hopf-cyclique. Lorsque l'algèbre de Hopf se réduit au corps de base, notre premier cup-produit se réduit au cup-produit de Connes en cohomologie cyclique ordinaire. Le deuxième cup-produit généralise l'application caractéristique de Connes–Moscovici pour l'action des algèbres de Hopf sur les algèbres. ***Pour citer cet article : M. Khalkhali, B. Rangipour, C. R. Acad. Sci. Paris, Ser. I 340 (2005).***

Version française abrégée

Soient H une algèbre de Hopf, A une H-module algèbre, B une H-comodule algèbre et C une H-module cogèbre. Soit M un module anti-Yetter–Drinfeld stable (SAYD). On dénote les cohomologies Hopf-cycliques de A, B, et C avec coefficients dans M par $HC^n_H(A, M)$, $HC^{n,H}(B, M)$, et $HC^n_H(C, M)$ respectivement. Notre premier *cup-produit* est une transformation naturelle $HC^p_H(A, M) \otimes HC^{q,H}(B, M) \to HC^{p+q}(A \rtimes_H B)$, à valeurs dans la cohomologie cyclique ordinaire de *l'algèbre produit tensoriel tordu* $A \rtimes_H B$. Pour notre *cup-produit de deuxième type*, on suppose que la cogèbre C *opère* sur l'algèbre A. On construit alors une transformation naturelle pour tous

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les nombres naturels $p, q \geqslant 0$ $HC^p_H(C, M) \otimes HC^q_H(A, M) \to HC^{p+q}(A)$. Notre méthode, dans les deux cas, est une généralisation de la technique de Connes dans [2] : on construit des cocycles cycliques de Hopf comme des *caractères des cycles tordus* (H, M) et on identifie les cycles qui représentent un cocycle cyclique trivial.

Soient (Ω, d) une H-module algèbre DG à gauche, et (Γ, d) une H-comodule algèbre DG à gauche. On définit une algèbre DG $\Omega \rtimes_H \Gamma$ comme suit. Comme espace vectoriel, c'est un produit tensoriel gradué $\Omega \otimes \Gamma$. La multiplication et la différentiation sont données par $(\omega_1 \otimes \gamma_1)(\omega_2 \otimes \gamma_2) := (-1)^{\deg(\omega_2)\deg(\gamma_1)}\omega_1\gamma_1^{(-1)}(\omega_2) \otimes \gamma_1^{(0)}\gamma_2$ et $d(\omega \otimes \gamma) := d\omega \otimes \gamma + (-1)^{\deg(\omega)}\omega \otimes d\gamma$. Soit M un module SAYD gauche-gauche et $\int : \Gamma \to M$, $\int' : M \otimes \Omega \to k$, des (M, H)-traces fermées et graduées de dimensions p et q sur Γ et Ω, respectivement. On définit une transformation linéaire

$$\int'' : \Omega \rtimes_H \Gamma \to k, \qquad \int''(\omega \otimes \gamma) := \int'\left(\int \gamma\right) \otimes \omega.$$

Lemme 0.1. *$\int''$ est une trace fermée graduée de degré $p + q$ sur $\Omega \rtimes_H \Gamma$. Si $\int$ (resp. $\int'$) définit un cycle nul sur B (resp. A), alors $\int''$ définit un cycle nul sur $A \rtimes_H B$.*

Soient $\varphi \in Z^{p,H}(B, M)$, $\psi \in Z^q_H(A, M)$ des cocycles cycliques representés par $\int$ sur $(\Omega B, d)$, et $\int'$ sur $(\Omega A, d)$, respectivement. On considère la suite d'applications $\Omega(A \rtimes_H B) \xrightarrow{i} \Omega(A) \rtimes_H \Omega(B) \xrightarrow{\int''} k$. La première application est induite par l'inclusion naturelle $A \rtimes_H B \to \Omega(A) \rtimes_H \Omega(B)$. On définit le cup-produit $\varphi \# \psi$ comme étant le cocycle cyclique representé par la trace fermée graduée $\int'' \circ i$. En utilisant le Lemme 0.1, on obtient le théorème suivant :

Théorème 0.2. *Avec A, B, H, et M comme ci-dessus, on a le pairing naturel $HC^p_H(A, M) \otimes HC^{q,H}(B, M) \to HC^{p+q}(A \rtimes_H B)$.*

Exemple 1. Soit G un groupe discret agissant sur l'algèbre A par des automorphismes unitaires et prenons $C = H = kG$. Dans [11], la cohomologie Hopf-cyclique de C est calculée à partir de la cohomologie de groupe avec des coefficients triviaux : $HC^q_{kG}(kG, k) = \bigoplus_{i \geqslant 0} H^{q-2i}(G)$. Les groupes $HC^p_{kG}(A, k)$ sont la cohomologie du souscomplexe de cochaines cycliques invariantes sur A : $\varphi(ga_0, ga_1, \ldots, ga_p) = \varphi(a_0, a_1, \ldots, a_p)$, pour tout $g \in G$ et $a_i \in A$. Par conséquent on obtient le pairing suivant, introduit pour la première fois par A. Connes dans [3] : $HC^p_{kG}(A, k) \otimes H^q(G) \to HC^{p+q}(A \rtimes G)$.

On dit qu'une cogèbre C agit sur A s'il existe une application $C \otimes A \to A$ telle que $c(ab) = c^{(1)}(a)c^{(2)}(b)$, $c(1) = \epsilon(c)1$, et $(hc)(a) = h(c(a))$, pour tout $c \in C$ $a, b \in A$, $h \in H$. Il est clair que les deux premières conditions sont équivalentes au fait que l'application d'évaluation $e : A \to \mathrm{Hom}_H(C, A)$, $e(a)(c) = c(a)$, est une application d'algèbre. Donc on obtient une application $e^* : HC^n(\mathrm{Hom}_H(C, A)) \to HC^n(A)$. Combinant avec la Proposition 3.2 nous obtenons le cup-produit de deuxième type :

Théorème 0.3. *Supposons que C agit sur A. Nous obtenons alors le pairing naturel*

$$\# = e^* \circ \cup : HC^p_H(C, M) \otimes HC^q_H(A, M) \to HC^{p+q}(A).$$

Les cas $p = 0$ ou $q = 0$ ont déjà été considérés dans [9]. Il y a été montré que cette application (pour $C = H$, $q = 0$, and $M = {}^\sigma k_\delta$) coïncide avec l'application caractéristique de Connes–Moscovici [4]. D'autre part, pour $C = H$ et $M = {}^\sigma k_\delta$, ce pairing est construit dans [7] en utilisant une autre méthode. Pour donner un autre exemple, posons $x = m \otimes_H c_0 \otimes c_1 \in Z^1_H(C, M)$ et $\phi \in Z^1_H(A, M)$. Alors $x \# \phi$ agit sur $A^{\otimes 3}$ comme suit :

$$x \# \phi(a_0 \otimes a_1 \otimes a_2) = \phi\big(m \otimes c_0^{(1)}(a_0) \otimes c_0^{(2)}(a_1)c_1(a_2)\big) - \phi\big(m \otimes c_0(a_0)c_1^{(1)}(a_1) \otimes c_1^{(2)}(a_2)\big).$$

1. Preliminaries

We denote the comultiplication, counit, and antipode of a Hopf algebra by Δ, ε, and S respectively. The letter H will always denote a Hopf algebra over a ground field k of characteristic zero. We use the Sweedler notation, with summation sign suppressed, to denote the comultiplication by $\Delta(h) = h^{(1)} \otimes h^{(2)}$ and its higher iterations. If M is a left H-comodule we write $\rho(m) = m^{(-1)} \otimes m^{(0)}$, where summation is understood, to denote its coaction $\rho : M \to H \otimes M$. Similarly if M is a right H-comodule, we write $\rho(m) = m^{(0)} \otimes m^{(1)}$ to denote its coaction ρ. A left–left *stable anti-Yetter–Drinfeld (SAYD) module* is a left H-module and left H-comodule M such that $\rho(hm) = h^{(1)}m^{(-1)}S^{-1}(h^{(3)}) \otimes h^{(2)}m^{(0)}$ and $m^{(-1)}m^{(0)} = m$ for all $h \in H$ and $m \in M$. These modules were introduced in [8,9] as the most general coefficients systems that one can introduce into Hopf cyclic-cohomology. One-dimensional SAYD modules correspond to Connes–Moscovici's modular pair in involution (δ, σ) on H [4–6] and will be denoted by $M = {}^{\sigma}k_{\delta}$.

An algebra A is called a left *H-module algebra* if it is a left H-module and its multiplication and unit maps are morphisms of H-modules. An algebra B is called a left *H-comodule algebra*, if B is a left H-comodule and its multiplication and unit maps are H-comodule maps. A left *H-module coalgebra* is a coalgebra C which is a left H-module such that its comultiplication and counit maps are H-linear.

We call the above three types of symmetries, *symmetries of type A, B, and C*, respectively. For each type there is an associated Hopf-cyclic cohomology theory with coefficients in an SAYD H-module M [8,9]. We denote these theories by $HC_H^n(A, M)$, $HC^{n,H}(B, M)$, and $HC_H^n(C, M)$ respectively. Connes–Moscovici's theory for Hopf algebras [4,5] correspond to the case $C = H$ with multiplication action, and $M = {}^{\sigma}k_{\delta}$. The dual theory of [11] for Hopf algebras correspond to $B = H$ with comultiplication coaction and $M = {}^{\sigma}k_{\delta}$. The H-equivariant cyclic cohomology of [1] correspond to $HC_H^n(A, M)$ with $M = H$ and conjugation action, and the twisted cyclic cohomology with respect to an automorphism correspond to $HC_H^n(A, M)$ with $H = k[x, x^{-1}]$ and $M = {}^{1}k_{\varepsilon}$. $HC_H^0(A, {}^{\sigma}k_{\delta})$ is the space of δ-invariant σ-traces on A in the sense of [5,6].

2. Covariant differential calculi

By a differential graded (DG) left H-module algebra we mean a graded left H-module algebra $\Omega = \bigoplus_{i \geqslant 0} \Omega^i$ endowed with a graded derivation d of degree 1 such that $d^2 = 0$ and d is H-linear. Let A be a left H-module algebra. Its universal differential calculus [2] $(\Omega A, d)$ is a DG H-module algebra with the H-action

$$h \cdot (a_0 da_1 \cdots da_n) := h^{(1)}(a_0) dh^{(2)}(a_1) \cdots dh^{(n+1)}(a_n).$$

By a DG H-module coalgebra we mean a graded H-module coalgebra $\Theta = \bigoplus_{i \geqslant 0} \Theta_i$ endowed with a graded coderivation d of degree -1 such that $d^2 = 0$ and d is H-colinear. Let C be a left H-module coalgebra. Its universal differential calculus $(\Omega^c C, d)$, defined in [10], is a left DG H-module coalgebra under the H-action

$$h \cdot (c_0 \otimes c_1 \cdots \otimes c_n) = h^{(1)}(c_0) \otimes h^{(2)}(c_1) \otimes \cdots \otimes h^{(n+1)}(c_n).$$

By a DG left H-comodule algebra we mean a DG algebra (Γ, d) such that Γ is a graded left H-comodule algebra and the derivation d is an H-comodule map. If B is a left H-comodule algebra, its universal calculus $(\Omega B, d)$ is a DG left H-comodule algebra with the left H-coaction defined by

$$b_0 db_1 \cdots db_n \mapsto (b_0^{(-1)} \cdots b_n^{(-1)}) \otimes b_0^{(0)} db_1^{(0)} \cdots db_n^{(0)}.$$

Definition 2.1. Let Ω be a DG left H-module algebra and M be a left–left SAYD module. By a closed graded (H, M)-trace of degree n on Ω we mean a linear map $\int : M \otimes \Omega^n \to k$ such that for all $h \in H$, $m \in M$, and $\omega, \omega_1, \omega_2$ in Ω of appropriate degrees, we have:

$$\int h^{(1)}m \otimes h^{(2)}\omega = \varepsilon(h)\int m \otimes \omega, \qquad \int m \otimes \mathrm{d}\omega = 0,$$
$$\int m \otimes \omega_1\omega_2 = (-1)^{\deg(\omega_1)\deg(\omega_2)}\int m^{(0)} \otimes S^{-1}(m^{(-1)})(\omega_2)\omega_1.$$

Lemma 2.1. *Let A be a left H-module algebra and $\rho: A \to \Omega^0$ be an H-linear algebra homomorphism. Then the cochain $\varphi: M \otimes A^{\otimes(n+1)} \to k$,*

$$\varphi(m, a_0, \ldots, a_n) = \int m \otimes \rho(a_0)\,\mathrm{d}\rho(a_1)\cdots\mathrm{d}\rho(a_n),$$

is a cyclic cocycle in $Z^n_H(A, M)$. The map $\int \mapsto \varphi$ (for $\rho = \mathrm{id}$) defines a 1–1 correspondence between closed graded (H, M)-traces on ΩA and $Z^n_H(A, M)$.

Definition 2.2. Let Γ be a DG left H-comodule algebra and M be a left–left SAYD module. By a closed graded (H, M)-trace of degree n on Γ we mean a linear map $\int: \Gamma^n \to M$ such that $\int$ is H-colinear, and for all m, $\gamma, \gamma_1, \gamma_2$ of appropriate degrees $\int m \otimes \mathrm{d}\gamma = 0$, and

$$\int \gamma_1\gamma_2 = (-1)^{\deg(\gamma_1)\deg(\gamma_2)}\gamma_2^{(-1)} \cdot \int \gamma_2^{(0)}\gamma_1.$$

Lemma 2.2. *Let B be a left H-comodule algebra and $\rho: B \to \Gamma^0$ an H-colinear algebra homomorphism. Then the cochain $\varphi: B^{\otimes(n+1)} \to M$,*

$$\varphi(b_0, \ldots, b_n) = \int \rho(b_0)\,\mathrm{d}\rho(b_1)\cdots\mathrm{d}\rho(b_n),$$

is a cyclic cocycle in $Z^{n,H}(B, M)$. The map $\int \mapsto \varphi$ (for $\rho = \mathrm{id}$) defines a 1–1 correspondence between closed graded (H, M)-traces on ΩA as in Definition 3.3 and $Z^{n,H}(B, M)$.

Definition 2.3. Let Θ be a DG left H-module coalgebra and M be a left–left SAYD H-module. By an n-dimensional closed graded (H, M)-cotrace on Θ we mean an element $x = \sum_i m_i \otimes \theta_i \in M \otimes_H \Theta_n$ such that $(1 \otimes_H d)x = 0$, and

$$\sum_i m_i^{(0)} \otimes \theta_i^{(2)} \otimes m_i^{(-1)}\theta_i^{(1)} = \sum_i (-1)^{\deg(\theta^{(1)})\deg(\theta^{(2)})} m_i \otimes \theta_i^{(1)} \otimes \theta_i^{(2)}.$$

Lemma 2.3. *Let C be a left H-module coalgebra and $\rho: \Theta_0 \to C$ be an H-linear coalgebra map. Let $\tilde{\rho}: M \otimes_H \Theta_n \to M \otimes_H C^{\otimes(n+1)}$ be the natural co-extension of ρ. Then $\int x := \tilde{\rho}(x)$ is a cyclic cocycle in $Z^n_H(C, M)$. The map $x \mapsto \int x$ (for $\rho = \mathrm{id}$) defines a 1–1 correspondence between closed graded (H, M)-cotraces of degree n on $\Omega^c C$ and $Z^n_H(C, M)$.*

Extending the terminology of [2], we call the data $(\Omega, d, \int, H, M, \rho)$ an *Ω-cycle* over the algebra A and the corresponding Hopf-cyclic cocycle its *character*. Similarly for Γ-cycles and Θ-cycles. They correspond to symmetries of type A, B and C, respectively.

2.1. Vanishing Ω, Γ and Θ-cycles

Let B be a left H-comodule algebra and $u \in B$ be an invertible H-coinvariant element in the sense that $\rho(u) = 1 \otimes u$. The inner automorphism $\mathrm{Ad}_u: B \to B$, $\mathrm{Ad}_u(b) = ubu^{-1}$ is H-colinear and hence induces a map $\mathrm{Ad}_u^*: HC^{n,H}(B, M) \to HC^{n,H}(B, M)$ for all n.

Proposition 2.4. *We have* $\mathrm{Ad}_u^* = \mathrm{id}$.

Proof. One checks that the homotopy operator κ defined by

$$\kappa f(b_0,\ldots,b_{n-1})=\sum_{i=0}^{n}(-1)^i f(b_0u^{-1},\ldots,ub_iu^{-1},u,b_{i+1},\ldots,b_{n-1})$$

is H-colinear and is a contracting homotopy for $\mathrm{Ad}_u^*-\mathrm{id}$ on the Hopf–Hochschild complex. The result now follows by applying Connes' long exact sequence. □

The algebra of $n\times n$ matrices over B, $M_n(B)$, is a left H-comodule algebra in a natural way and the map $i:B\to M_n(B)$, $b\mapsto b\otimes e_{11}$, is H-colinear and hence induces a map $i^*:HC^{p,H}(M_n(B),M)\to HC^{p,H}(B,M)$. We define a map $\mathrm{Tr}:C^{p,H}(B,M)\to C^{p,H}(M_n(B),M)$

$$(\mathrm{Tr}\,\varphi)(b_0\otimes m_0,\ldots,b_p\otimes m_p)=\mathrm{tr}(m_0\cdots m_p)\varphi(b_0,\ldots,b_p).$$

The relation $i^*\circ\mathrm{Tr}=\mathrm{id}$ is easily verified. Although we won't need it for the construction of cup products in this paper, we pause to mention that we have now all the tools to prove a Morita invariance theorem for Hopf-cyclic cohomology theory of any type. For example we have:

Proposition 2.5 (Morita invariance). *Let B be a unital left H-comodule algebra and M be an* SAYD *module. Then i^* induces an isomorphism on Hopf–Hochschild and Hopf-cyclic cohomology of B with coefficients in M.*

The following lemma is an adaptation of a lemma of Connes [2] to our context:

Lemma 2.4. *Let $f:B\to B$ be an H-colinear algebra homomorphism and X an invertible H-coinvariant element of $M_2(B)$ such that*

$$X\begin{bmatrix}b&0\\0&f(b)\end{bmatrix}X^{-1}=\begin{bmatrix}0&0\\0&f(b)\end{bmatrix}$$

for all $b\in B$. Then for any SAYD *module M, $HC^{n,H}(B,M)=0$ for all n.*

Definition 2.6. We say that a Γ-cycle is vanishing if Γ^0 satisfies the condition of the above lemma.

Lemma 2.5. *Let $\varphi:B^{\otimes(n+1)}\to M$ be an H-colinear map. Then φ is a coboundary if and only if φ is the character of a vanishing Γ-cycle.*

Let C be a left H-module coalgebra and $\chi\in\mathrm{Hom}_H(C,k)$ be an H-linear convolution invertible functional on C. The *co-inner* automorphism $\mathrm{Ad}_\chi^c:C\to C$ defined by $\mathrm{Ad}_\chi(c)=\chi(c^{(1)})c^{(2)}\chi^{-1}(c^{(3)})$ is H-linear and hence for any $n\geqslant 0$ induces a map

$$\mathrm{Ad}_\chi^*:HC_H^n(C,M)\to HC_H^n(C,M).$$

Proposition 2.7. *We have* $\mathrm{Ad}_\chi^*=\mathrm{id}$.

Lemma 2.6. *Let $f:C\to C$ be an H-linear coalgebra homomorphism and χ a convolution invertible H-linear functional on the coalgebra $M_2(C)$ such that*

$$\chi\left(\begin{bmatrix}c^{(1)}&0\\0&f(c^{(1)})\end{bmatrix}\right)\begin{bmatrix}c^{(2)}&0\\0&f(c^{(2)})\end{bmatrix}\chi^{-1}\left(\begin{bmatrix}c^{(3)}&0\\0&f(c^{(3)})\end{bmatrix}\right)=\begin{bmatrix}0&0\\0&f(c)\end{bmatrix},$$

for all $c\in C$. Then for any SAYD *module M and for any $n\geqslant 0$, $HC_H^n(C,M)=0$.*

Definition 2.8. We say that a Θ-cycle (Θ,d,x,H,M) is vanishing if Θ_0 satisfies the condition of Lemma 2.6.

Lemma 2.7. *Let $y\in Z_H^n(C,M)$. Then y is a coboundary if and only if y is the character of a vanishing Θ-cycle.*

The notion of vanishing Γ-cycle is defined along parallel lines with analogue of Lemma 3.11 and 3.15 proved in a similar way.

3. Cup products of the second kind

In this section C is a left H-module coalgebra and A is a left H-module algebra. Let (Ω, d) be a DG left H-module algebra and (Θ, d) be a DG left H-module coalgebra. We define the *convolution DG algebra* $\mathrm{Hom}_H(\Theta, \Omega)$ as follows. As a graded vector space in degree n it has $\bigoplus_{i+j=n} \mathrm{Hom}_H(\Theta_i, \Omega^j)$. One checks that with convolution product $f * g(\theta) := (-1)^{\deg(g)\deg(\theta^{(1)})} f(\theta^{(1)}) g(\theta^{(2)})$, and differential $\mathrm{d}f := [\mathrm{d}, f]$ (graded commutator), $\mathrm{Hom}_H(\Theta, \Omega)$ is a DG algebra.

Let $\int$ be a closed graded (M, H)-trace of degree p on Ω and x a closed graded (M, H)-cotrace of degree q on Θ. We define a functional $\int'$ on $\mathrm{Hom}_H(\Theta, \Omega)$ by $\int' f := \int (\mathrm{id}_M \otimes f)(x)$.

Proposition 3.1. *$\int'$ is a closed graded trace of degree $p + q$ on $\mathrm{Hom}_H(\Theta, \Omega)$. If $\int$ (resp. x) defines a vanishing cycle on A (resp. C), then $\int'$ defines a vanishing cycle on $\mathrm{Hom}_H(C, A)$.*

Now let $\varphi \in Z_H^p(C, M)$ be represented by x and $\psi \in Z_H^q(A, M)$ by $\int$. Consider the sequence of maps

$$\Omega\big(\mathrm{Hom}_H(C, A)\big) \xrightarrow{i} \mathrm{Hom}_H(\Omega^c C, \Omega A) \xrightarrow{\int'} k,$$

where the first map is defined using the universal property of Ω. We let $\varphi \cup \psi$ to be the character of the cycle $\int' \circ i$. Using Proposition 5.1 we obtain

Proposition 3.2. *We have a well-defined pairing*:

$$\cup : HC_H^p(C, M) \otimes HC_H^q(A, M) \to HC^{p+q}\big(\mathrm{Hom}_H(C, A)\big).$$

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