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Numerical Analysis

Convergence of the Lagrange–Galerkin method for a fluid–rigid system ☆

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Abstract

In this Note, we consider a Lagrange–Galerkin scheme to approximate a two dimensional fluid–rigid body problem. The system is modelled by the incompressible Navier–Stokes equations in the fluid part, coupled with ordinary differential equations for the dynamics of the rigid body. In this problem, the equations of the fluid are written in a domain whose variation is one of the unknowns. We introduce a numerical method based on the use of characteristics and on finite elements with a fixed mesh. Our main result asserts the convergence of this scheme. ***To cite this article: J. San Martín et al., C. R. Acad. Sci. Paris, Ser. I 339 (2004).***

Résumé

Convergence de la méthode de Lagrange–Galerkin pour un système fluide–rigide. Dans cette Note, nous considérons un schéma de Lagrange–Galerkin pour approcher un problème fluide–rigide. Le système est modélisé par les équations de Navier–Stokes incompressible, pour la partie fluide, couplées avec des équations différentielles ordinaires pour la dynamique du corps rigide. Dans ce problème, les équations du fluide sont écrites sur un domaine dont la variation est une des inconnues. Nous introduisons une méthode numérique basée sur l'utilisation des caractéristiques et des éléments finis associés à un maillage fixe. Notre résultat principal est la convergence de ce schéma. ***Pour citer cet article : J. San Martín et al., C. R. Acad. Sci. Paris, Ser. I 339 (2004).***

Version française abrégée

Soit $\mathcal{O}$ un domaine borné de $\mathbb{R}^2$ de frontière régulière contenant un solide rigide qui occupe le domaine $B(t)$ et un fluide visqueux remplissant le domaine $\Omega(t) = \mathcal{O} \setminus B(t)$. Nous supposons que le mouvement du système est

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modélisé par les Éqs. (1)–(7) ci-dessous. Pour simplifier, nous supposons de plus que le solide est un disque de rayon 1.

Nous résolvons ce problème d'interaction fluide–structure par une méthode de type Lagrange–Galerkin : la dérivée particulaire est traitée en utilisant la méthode des caractéristiques et nous discrétisons la variable spatiale en utilisant les éléments finis. Plus précisément, nous utilisons une formulation mixte *globale* dans le sens où les formes bilinéaires a et b utilisées ((12) et (13)) sont définies sur l'ensemble $\mathcal{O}$ entier. Le champ des vitesses du fluide est prolongé par celui du solide dans le domaine du solide, et la contrainte de rigidité de ce champ de vitesses est prise en compte à travers les espaces utilisés ((9) et (10)). Plus précisément, le schéma proposé est donné par (18)–(21). Dans cet algorithme, les composantes du triplet $(\mathbf{u}_h^k, p_h^k, \boldsymbol{\zeta}_h^k)$ représentent les approximations du champ de vitesses global, de la pression et de la position du centre de masse du solide rigide à $t = t_k$. Une des particularités importantes de cette discrétisation, par rapport à d'autres méthodes existantes, est que les éléments finis utilisés correspondent à un maillage fixe. Les espaces des éléments finis utilisés pour la vitesse et la pression sont définis par (15) et (16) et correspondent à l'intersection d'espaces d'éléments finis classiques pour un fluide seul avec nos espaces fonctionnels où l'on tient compte de la contrainte de rigidité du solide.

Le résultat principal de ce travail est le suivant.

Théorème 0.1. *Soit $C_0 > 0$ une constante fixée. On suppose que $\mathcal{O}$ est l'intérieur d'un polygone convexe et que $(\mathbf{u}, p, \boldsymbol{\zeta}, \omega)$ satisfait* (1)–(7), (23) *et* (24)*. De plus, on suppose que $\mathbf{f}$, $\mathbf{u}_0$ satisfont* (22) *et que (8) est vérifiée. Soient $\boldsymbol{\zeta}_h^k$, $\mathbf{u}_h^k$ et p_h^k les approximations introduites dans la Section* 3 *ci-dessous. Alors il existe des constantes C et τ^* indépendantes de h et de Δt telles que pour tout $0 < \Delta t \leqslant \tau^*$ et $h \leqslant C_0(\Delta t)^2$ on a*

$$\sup_{1\leqslant k\leqslant N} \left(\left|\boldsymbol{\zeta}(t_k) - \boldsymbol{\zeta}_h^k\right| + \left\|\mathbf{u}(t_k) - \mathbf{u}_h^k\right\|_{\mathcal{L}^2(\mathcal{O})} \right) \leqslant C\,\Delta t.$$

A notre connaissance le seul autre résultat d'existence de convergence pour un schéma numérique pour un problème fluide–structure est donné dans [7] pour un problème 1D voisin de celui que nous traitons ici. Notre méthode est inspirée de l'approximation utilisée pour les équations de Navier–Stokes (cf. [1,14] et [17]).

1. Introduction.

The aim of this Note is to analyze a Lagrange–Galerkin approximation of the equations modelling the motion of a two-dimensional rigid body immersed in a fluid. We assume that the system fluid–rigid body occupies a bounded domain $\mathcal{O}$ in $\mathbb{R}^2$. The solid is supposed to occupy at each instant t a closed connected subset $B(t) \subset \mathcal{O}$ which is surrounded by a viscous homogeneous fluid filling the domain $\Omega(t) = \mathcal{O} \backslash B(t)$.

The motion of the fluid is described by the classical Navier–Stokes equations, whereas the motion of the rigid body is governed by the balance equations for linear and angular momentum (Newton's laws). More precisely, we consider the following system coupling partial differential and ordinary differential equations.

$$\rho_f \frac{\partial \mathbf{u}}{\partial t} - \nu \Delta \mathbf{u} + \rho_f (\mathbf{u} \cdot \nabla)\mathbf{u} + \nabla p = \rho_f \mathbf{f}, \quad \mathbf{x} \in \Omega(t), t \in [0, T], \tag{1}$$

$$\operatorname{div} \mathbf{u} = 0, \quad \mathbf{x} \in \Omega(t),\ t \in [0, T], \tag{2}$$

$$\mathbf{u} = 0, \quad \mathbf{x} \in \partial\mathcal{O},\ t \in [0, T], \tag{3}$$

$$\mathbf{u} = \boldsymbol{\zeta}'(t) + \omega(t)\big(\mathbf{x} - \boldsymbol{\zeta}(t)\big)^{\perp}, \quad \mathbf{x} \in \partial B(t),\ t \in [0, T], \tag{4}$$

$$M\boldsymbol{\zeta}''(t) = -\int_{\partial B(t)} \boldsymbol{\sigma}\mathbf{n}\,\mathrm{d}\Gamma + \rho_s \int_{B(t)} \mathbf{f}(\mathbf{x}, t)\,\mathrm{d}\mathbf{x}, \quad t \in [0, T], \tag{5}$$

$$J\omega'(t) = -\int\limits_{\partial B(t)} \big(\mathbf{x}-\boldsymbol{\zeta}(t)\big)^{\perp}\cdot\boldsymbol{\sigma}\mathbf{n}\,\mathrm{d}\Gamma + \rho_s\int\limits_{B(t)} \big(\mathbf{x}-\boldsymbol{\zeta}(t)\big)^{\perp}\cdot\mathbf{f}(\mathbf{x},t)\,\mathrm{d}\mathbf{x}, \quad t\in[0,T], \tag{6}$$

$$\mathbf{u}(\mathbf{x},0)=\mathbf{u}_0(\mathbf{x}), \quad \boldsymbol{\zeta}(0)=\boldsymbol{\zeta}_0, \quad \boldsymbol{\zeta}'(0)=\boldsymbol{\zeta}_1, \quad \omega(0)=\omega_0. \tag{7}$$

In the above equations the unknowns are $\mathbf{u}(\mathbf{x},t)$ (the Eulerian velocity field of the fluid), $p(\mathbf{x},t)$ (the pressure of the fluid), $\boldsymbol{\zeta}(t)$ (the position of the mass center of the rigid body) and $\omega(t)$ (the angular velocity of the rigid body). Moreover, we have denoted by $\partial B(t)$ the boundary of the rigid body at instant t and by $\mathbf{n}(\mathbf{x},t)$ the unit normal to $\partial B(t)$ at the point $\mathbf{x}$ directed to the interior of the rigid body.

The constants ρ_f and ρ_s are respectively the density of the fluid and of the rigid body. In the following, we assume that the densities of the fluid and of the solid are equal, that is

$$\rho_f=\rho_s=1, \tag{8}$$

and that the rigid body is a ball in $\mathbb{R}^2$. Assumption (8) is clearly restrictive but it is important for the forthcoming analysis, so that it is not clear that it can be removed. On the contrary, the assumption that the rigid body is a ball is not essential but it avoids some technicalities.

The constants M and J are the mass and the moment of inertia of the rigid body and the positive constant ν is the viscosity of the fluid. Moreover, $\mathbf{f}(\mathbf{x},t)$ is the applied force (per unit mass).

Finally, the stress tensor is defined by

$$\boldsymbol{\sigma}(\mathbf{x},t) = -p(\mathbf{x},t)\,\mathbf{Id}+2\nu\mathbf{D}(\mathbf{u}), \quad \text{where } D(u)_{k,l}=\frac{1}{2}\left(\frac{\partial u_k}{\partial x_l}+\frac{\partial u_l}{\partial x_k}\right).$$

The main difficulties of this problem are:

– the equations of the structure are coupled with those of the fluid,
– the domain of the fluid is variable and it is one of the unknowns of the problem (we thus have a free boundary problem).

The wellposedness of this type of system has been recently studied in a large number of papers (see, for instance, [3,4,9,16,8,19,18] and the references therein).

The literature on the numerical approximation of the solution of (1)–(7) also contains a large number of recent papers. A part of these papers is based on an Arbitrary Lagrangian Eulerian (ALE) formulation: see, for example, [7,13,12,10,11]. In the ALE method, at each time step, the mesh is moved with an arbitrary velocity in the fluid in order to follow the motion of the rigid body.

Another approach, developed in [5,6] is based on a fictitious domain formulation: the rigid bodies are fictitiously filled by the surrounding fluid and the constraint of rigid body motion is relaxed by introducing a distributed Lagrange multiplier.

As far as we know, the only proof of the convergence of one of these methods is given in [7] for a simplified problem in one space dimension. The main novelty brought in by our paper consists in the fact that we construct a new approximation method using a fixed mesh and that we prove a convergence result. This method is inspired by the Galerkin–Lagrange approximation which is commonly used for Navier–Stokes equations (see [1,14] and [17]).

2. Notations and preliminaries

For $\Omega\subset\mathbb{R}^2$ and $m\in\mathbb{N}$ we define the spaces

$$\mathcal{L}^2(\Omega)=\big[L^2(\Omega)\big]^2, \quad \mathcal{H}^m(\Omega)=\big[H^m(\Omega)\big]^2, \quad L_0^2(\Omega)=\left\{\varphi\in L^2(\Omega)\;\Big|\;\int\limits_{\Omega}\varphi=0\right\},$$

$$\mathcal{K}(\boldsymbol{\zeta}) = \{\mathbf{u} \in \mathcal{H}_0^1(\mathcal{O}) \mid \mathbf{D}(\mathbf{u}) = 0 \text{ in } B(\boldsymbol{\zeta})\}, \tag{9}$$

$$\widehat{\mathcal{K}}(\boldsymbol{\zeta}) = \{\mathbf{u} \in \mathcal{H}_0^1(\mathcal{O}) \mid \operatorname{div}\mathbf{u} = 0 \text{ in } \mathcal{O},\ \mathbf{D}(\mathbf{u}) = 0 \text{ in } B(\boldsymbol{\zeta})\}, \tag{10}$$

where $\boldsymbol{\zeta} \in \mathbb{R}^2$ and $B(\boldsymbol{\zeta}) = \{\mathbf{x} \in \mathbb{R}^2, |\mathbf{x} - \boldsymbol{\zeta}| \leqslant 1\}$. If the solution $\mathbf{u}$ of (1)–(7) is extended by

$$\mathbf{u}(\mathbf{x}, t) = \boldsymbol{\zeta}'(t) + \omega(t)\big(\mathbf{x} - \boldsymbol{\zeta}(t)\big)^{\perp} \quad \forall \mathbf{x} \in B\big(\boldsymbol{\zeta}(t)\big),$$

then, we easily see that $\mathbf{u}(t) \in \widehat{\mathcal{K}}(\boldsymbol{\zeta}(t))$. *In the sequel, the solution* $\mathbf{u}$ *will be extended as above.*

We also define:

$$M(\boldsymbol{\zeta}) = \{p \in L_0^2(\mathcal{O}) \mid p = 0 \text{ in } B(\boldsymbol{\zeta})\}, \tag{11}$$

$$a(\mathbf{u}, \mathbf{v}) = 2\nu \int_{\mathcal{O}} \mathbf{D}(\mathbf{u}) : \mathbf{D}(\mathbf{v})\, \mathrm{d}\mathbf{x} \quad \forall \mathbf{u}, y\mathbf{v} \in \mathcal{H}^1(\mathcal{O}), \tag{12}$$

$$b(\mathbf{u}, p) = -\int_{\mathcal{O}} \operatorname{div}(\mathbf{u})\, p\, \mathrm{d}\mathbf{x} \quad \forall \mathbf{u} \in \mathcal{H}^1(\mathcal{O})\ \forall p \in L_0^2(\mathcal{O}). \tag{13}$$

3. Full discretization and statement of the main result

In order to discretize the problem (1)–(7) with respect to the space variable we introduce two families of finite element spaces. We first define a family of finite element spaces which approximate the space $\mathcal{K}(\boldsymbol{\zeta})$ defined in (9). Let h denote a discretization parameter, $0 < h < 1$ and let P_1 be the space of all affine functions in $\mathbb{R}^2$.

Consider a quasi-uniform triangulation $\mathcal{T}_h$ of $\mathcal{O}$, as defined, for instance, in [2, p. 106] (this assumption will be accepted in the remaining part of this Note). We associate to this triangulation two classical approximation spaces used in the mixed finite element methods for the Stokes system. The first space, classically used for the approximation of the velocity field in the mixed statement of the Stokes system, is denoted by $\mathcal{W}_h$ and it is defined as the subspace of $\mathcal{H}_0^1(\mathcal{O})$ formed by the P_1-bubble finite elements associated to $\mathcal{T}_h$. The second space, classically used for the approximation of the pressure in the mixed statement of the Stokes system, is denoted by E_h and it is defined by

$$E_h = \{q \in C(\overline{\mathcal{O}}) \mid q_{|T} \in P_1(T) \text{ for all } T \in \mathcal{T}_h\}. \tag{14}$$

For our problem we use two spaces which are related to the presence of the rigid body. The first one, which is used for the approximation of the velocity field is denoted by $\mathcal{K}_h(\boldsymbol{\zeta})$ and it is defined by

$$\mathcal{K}_h(\boldsymbol{\zeta}) = \mathcal{W}_h \cap \mathcal{K}(\boldsymbol{\zeta}) \quad \forall \boldsymbol{\zeta} \in \mathcal{O}. \tag{15}$$

The second one, which is used for the approximation of the pressure, is denoted by $M_h(\boldsymbol{\zeta})$ and it is defined by

$$M_h(\boldsymbol{\zeta}) = E_h \cap M(\boldsymbol{\zeta}) \quad \forall \boldsymbol{\zeta} \in \mathcal{O}. \tag{16}$$

We also define the finite element space (see [14])

$$\mathcal{R}_h = \{\mathbf{rot}\,\varphi_h \mid \varphi_h \in E_h,\ \varphi_h = 0 \text{ on } \partial\mathcal{O}\}.$$

We denote by $\mathbf{P}$ the orthogonal projection from $\mathcal{L}^2$ onto $\mathcal{R}_h$.

Let N be a positive integer. We denote $\Delta t = T/N$ and $t_k = k\Delta t$. Assume that the approximate solution $(\mathbf{u}_h^k, p_h^k, \boldsymbol{\zeta}_h^k)$ of (1)–(7) at $t = t_k$ is known. We describe below the numerical scheme allowing to determinate the approximate solution $(\mathbf{u}_h^{k+1}, p_h^{k+1}, \boldsymbol{\zeta}_h^{k+1})$ at $t = t_{k+1}$. First, we compute $\boldsymbol{\zeta}_h^{k+1} \in \mathbb{R}^2$ by

$$\boldsymbol{\zeta}_h^{k+1} = \boldsymbol{\zeta}_h^k + \mathbf{u}_h^k\big(\boldsymbol{\zeta}_h^k\big)\Delta t. \tag{17}$$

We denote by $\mathbf{Pu}_h^k$ the projection of $\mathbf{u}_h^k$ onto $\mathcal{R}_h$. Then, we define the characteristic function $\bar{\boldsymbol{\psi}}_{\boldsymbol{h}}^{\boldsymbol{k}}$ associated to the fully discretized velocity field as the solution of

$$\begin{cases} \dfrac{\mathrm{d}}{\mathrm{d}t}\bar{\boldsymbol{\psi}}_{\boldsymbol{h}}^{\boldsymbol{k}}(t;t_{k+1},\mathbf{x}) = \mathbf{Pu}_h^k\big(\bar{\boldsymbol{\psi}}_{\boldsymbol{h}}^{\boldsymbol{k}}(t;t_{k+1},\mathbf{x})\big), \\ \bar{\boldsymbol{\psi}}_{\boldsymbol{h}}^{\boldsymbol{k}}(t_{k+1};t_{k+1},\mathbf{x}) = \mathbf{x}. \end{cases} \tag{18}$$

We also define

$$\overline{\boldsymbol{X}}_{\boldsymbol{h}}^{\boldsymbol{k}}(\mathbf{x}) = \bar{\boldsymbol{\psi}}_{\boldsymbol{h}}^{\boldsymbol{k}}(t_k;t_{k+1},\mathbf{x}) \quad \forall \mathbf{x} \in \mathcal{O} \tag{19}$$

and one can check that $\overline{\boldsymbol{X}}_{\boldsymbol{h}}^{\boldsymbol{k}}(\mathcal{O}) = \mathcal{O}$.

Then, we define $(\mathbf{u}_h^{k+1}, p_h^{k+1}) \in \mathcal{K}_h(\boldsymbol{\zeta}_h^{k+1}) \times M_h(\boldsymbol{\zeta}_h^{k+1})$ as the solution of the problem:

$$\left(\frac{\mathbf{u}_h^{k+1} - \mathbf{u}_h^k \circ \overline{\boldsymbol{X}}_{\boldsymbol{h}}^{\boldsymbol{k}}}{\Delta t}, \boldsymbol{\varphi}\right) + a\big(\mathbf{u}_h^{k+1}, \boldsymbol{\varphi}\big) + b\big(\boldsymbol{\varphi}, p_h^{k+1}\big) + \big(\mathbf{f}_h^{k+1}, \boldsymbol{\varphi}\big) \quad \forall \boldsymbol{\varphi} \in \mathcal{K}_h\big(\boldsymbol{\zeta}_h^{k+1}\big), \tag{20}$$

$$b\big(\mathbf{u}_h^{k+1}, q\big) = 0 \quad \forall q \in M_h\big(\boldsymbol{\zeta}_h^{k+1}\big), \tag{21}$$

where $\mathbf{f}_h^{k+1}$ is the $\mathcal{L}^2$-projection of $\mathbf{f}^{k+1} = \mathbf{f}(t_{k+1})$ on $(E_h)^2$. We take $\boldsymbol{\zeta}_h^0 = \boldsymbol{\zeta}^0$ and the initial approximate velocity $\mathbf{u}_h^0$ is the $\mathcal{H}_0^1$-projection of $\mathbf{u}_0$ onto $\mathcal{K}_h(\boldsymbol{\zeta}_h^0)$.

In the sequel, we suppose that

$$\begin{aligned} &\mathbf{f} \in C\big([0,T];\mathcal{H}^1(\mathcal{O})\big), \quad \mathbf{u}_0 \in \mathcal{H}^2(\Omega), \quad \operatorname{div}(\mathbf{u}_0) = 0 \quad \text{in } \Omega, \\ &\mathbf{u}_0 = 0 \quad \text{on } \partial\mathcal{O}, \quad \mathbf{u}_0(\mathbf{y}) = \boldsymbol{\zeta}_1 + \omega_0(\mathbf{y} - \boldsymbol{\zeta}_0)^\perp \quad \text{on } \partial B. \end{aligned} \tag{22}$$

The corresponding solution $(\mathbf{u}, p, \boldsymbol{\zeta}, \omega)$ of problem (1)–(7) will be assumed to satisfy the following regularity hypotheses

$$\begin{cases} \mathbf{u} \in C\big([0,T];\mathcal{H}^2\big(\Omega(t)\big)\big) \cap H^1\big(0,T;\mathcal{L}^2\big(\Omega(t)\big)\big), \\ D_t^2\mathbf{u} \in L^2\big(0,T;\mathcal{L}^2\big(\Omega(t)\big)\big), \quad \mathbf{u} \in C\big([0,T];\mathcal{C}^{0,1}(\overline{\mathcal{O}})\big), \\ p \in C\big([0,T];\mathcal{H}^1\big(\Omega(t)\big)\big), \quad \boldsymbol{\zeta} \in \mathcal{H}^3(0,T), \quad \omega \in H^2(0,T). \end{cases} \tag{23}$$

Moreover, we assume that

$$\operatorname{dist}\big(B(t), \partial\mathcal{O}\big) > 0 \quad \forall t \in [0,T]. \tag{24}$$

Our main result is the following.

Theorem 3.1. *Let $C_0 > 0$ be a fixed constant. Suppose that $\mathcal{O}$ is the interior of a convex polygon and that $(\mathbf{u}, p, \boldsymbol{\zeta}, \omega)$ is a solution of* (1)–(7) *satisfying* (23) *and* (24). *Moreover, assume that* $\mathbf{f}$, $\mathbf{u}_0$ *satisfy* (22) *and that* (8) *holds. Consider the functions $\boldsymbol{\zeta}_h^k$, $\mathbf{u}_h^k$ and p_h^k defined in this section. Then there exist two positive constants C and τ^* not depending on h and on Δt such that for all $0 < \Delta t \leqslant \tau^*$ and for all $h \leqslant C_0\,(\Delta t)^2$ we have*

$$\sup_{1 \leqslant k \leqslant N}\big(\big|\boldsymbol{\zeta}(t_k) - \boldsymbol{\zeta}_h^k\big| + \big\|\mathbf{u}(t_k) - \mathbf{u}_h^k\big\|_{\mathcal{L}^2(\mathcal{O})}\big) \leqslant C\,\Delta t.$$

Remark 1. For the Navier–Stokes system, the same type of result is obtained in [14] for $h \leqslant C_0\Delta t$ and in [17] for $h^2 \leqslant C_0\Delta t \leqslant C_1 h^\sigma$ and $\sigma > 1/2$ (for h and Δt small enough).

The proof of the above result is given in [15].

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