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Explicit Plancherel formula for the p-adic group $\mathrm{GL}(n)$

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Abstract

We provide an explicit Plancherel formula for the p-adic group $\mathrm{GL}(n)$. We determine explicitly the Bernstein decomposition of Plancherel measure, including all numerical constants. We also prove a transfer-of-measure formula for $\mathrm{GL}(n)$. ***To cite this article: A.-M. Aubert, R. Plymen, C. R. Acad. Sci. Paris, Ser. I 338 (2004).***

Résumé

Formule de Plancherel explicite pour le groupe p-adique $\mathrm{GL}(n)$**.** Nous obtenons une formule de Plancherel explicite pour le groupe p-adique $\mathrm{GL}(n)$. Nous déterminons explicitement la décomposition de Bernstein de la mesure de Plancherel, y compris les diverses constantes numériques. Nous prouvons aussi une formule de transfert pour $\mathrm{GL}(n)$. ***Pour citer cet article : A.-M. Aubert, R. Plymen, C. R. Acad. Sci. Paris, Ser. I 338 (2004).***

Version française abrégée

Soit F un corps local non archimédien de corps résiduel de cardinal q et soit ϖ une uniformisante de F. Soit $G = \mathrm{GL}(n)$ et soient m un entier divisant n et σ une représentation irréductible supercuspidale unitaire de $\mathrm{GL}(m)$. Nous désignons par r l'ordre du groupe cyclique formé par les caractères non ramifiés η de $\mathrm{GL}(m)$ tels que $\sigma \otimes \eta \simeq \sigma$ et par $f(\sigma^\vee \times \sigma)$ le conducteur de paires de $\sigma^\vee \times \sigma$ dans la terminologie de [3]. Soit M le sous-groupe de Levi standard de G, isomorphe à $\mathrm{GL}(l_1 m) \times \cdots \times \mathrm{GL}(l_k m)$, où $(l_1, \ldots, l_k)$ est une partition donnée de $e = n/m$. Pour $i = 1, \ldots, k$, soit $g_i = (l_i - 1)/2$ et notons $\pi_i = \mathrm{St}(\sigma, l_i)$ l'unique quotient irréductible de la représentation induite de G définie par le segment de Zelevinsky $\{|\,|^{-g_i}\sigma, \ldots, |\,|^{g_i}\sigma\}$ (voir [13] ou [7]). La représentation π_i est une représentation de la série discrète de $\mathrm{GL}(l_i m)$, [13, 9.3]. Nous notons χ_i un caractère non ramifié de $F^\times$, et nous posons $\zeta_i = \chi_i(\varpi)$, $z_i = \zeta_i^r$.

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Nous notons $\Omega(G)$ la variété de Bernstein : un point de $\Omega(G)$ est une classe de conjugaison sous G d'un couple (M', σ'), où M' est un sous-groupe de Levi de G et σ' une représentation irréductible supercuspidale de M'. Soit Ω une composante de $\Omega(G)$. Nous notons $\mathrm{Irr}^{\mathrm{t}}(G)_\Omega$ le sous-ensemble du dual tempéré $\mathrm{Irr}^{\mathrm{t}}(G)$ de G formé des représentations tempérées dont le caractère infinitésimal appartient à Ω. La décomposition de Bernstein (voir [2]) induit la partition suivante de $\mathrm{Irr}^{\mathrm{t}}(G)$:

$$\mathrm{Irr}^{\mathrm{t}}(G) = \bigsqcup \mathrm{Irr}^{\mathrm{t}}(G)_\Omega,$$

et cette dernière définit une décomposition $\nu = \bigsqcup \nu_\Omega$ de la mesure de Plancherel, où

$$\mathrm{d}\nu(\omega) = c(G|M)^{-2} \cdot \gamma(G|M)^{-1} \cdot \mu_{G|M}(\omega) \cdot d(\omega) \cdot \mathrm{d}\omega = \gamma(G|M) \cdot j(\omega)^{-1} \cdot d(\omega) \cdot \mathrm{d}\omega,$$

$d(\omega)$ étant le degré formel de ω et $j(\omega)$ défini par [12, IV.3].

Nous calculons explicitement la fonction $\mu_{G|M}$ de Harish-Chandra, pour le groupe G, à l'aide de la formule de Langlands–Shahidi [11] et de la formule du produit de Harish-Chandra [12, V.2.1].

Théorème 0.1. *Supposons que la classe de conjugaison de* $(\mathrm{GL}(m)^e, \sigma^{\otimes e})$ *définisse un point de la composante de Bernstein* Ω. *On a alors, pour* $\omega = \chi_1\pi_1 \otimes \cdots \otimes \chi_k\pi_k$:

$$\mathrm{d}\nu_\Omega(\omega) = \gamma(G|M) \cdot q^{\sum_{1 \leqslant i<j \leqslant k} l_i l_j f(\sigma^\vee \times \sigma)} \cdot \prod \left| \frac{1 - z_j z_i^{-1} q^{gr}}{1 - z_j z_i^{-1} q^{-(g+1)r}} \right|^2 \cdot d(\omega) \cdot \mathrm{d}\omega,$$

où le produit est pris sur tous les i, j, g *qui vérifient les inégalités suivantes* : $1 \leqslant i < j \leqslant k$, $|g_i - g_j| \leqslant g \leqslant g_i + g_j$.

Nous passons ensuite au cas d'une composante de Bernstein $\Omega \subset \Omega(\mathrm{GL}(n))$ arbitraire. D'après Zelevinsky, tout couple (M, π), formé d'un sous-groupe de Levi M de G et d'une représentation π de la série discrète de M, se décompose de manière unique en $(M, \pi) = (M_1 \times \cdots \times M_t, \pi_1 \otimes \cdots \otimes \pi_t)$, où, d'une part, les éléments du support cuspidal de π_i sont équivalents, au sens où ils se déduisent l'un de l'autre par torsion non ramifiée, d'autre part, pour $i \neq j$, aucun élément du support cuspidal de π_i n'est équivalent à un élément du support cuspidal de π_j. Nous notons $\Omega = \Omega_1 \times \cdots \times \Omega_t$ la factorisation correspondante de Ω. La mesure de Plancherel respecte (à multiplication par une constante près) cette factorisation :

$$\nu_\Omega = \mathit{const.} \cdot \nu_{\Omega_1} \cdots \nu_{\Omega_t}.$$

Les mesures de Plancherel $\nu_{\Omega_1}, \ldots, \nu_{\Omega_t}$ sont données par le Théorème 0.1.

1. Introduction

We shall follow very closely the notation and terminology in Waldspurger [12]. Let F be a nonarchimedean local field with ring of integers $\mathfrak{o}_F$ and residue field of order q. Let $G = \mathrm{GL}(n) = \mathrm{GL}(n, F)$ and let $\mathcal{K} = \mathrm{GL}(n, \mathfrak{o}_F)$. Let H be a closed subgroup of G. We use the *standard* normalization of Haar measures, following [12, I.1, p. 240]. Then Haar measure μ_H on H is chosen so that $\mu_H(H \cap \mathcal{K}) = 1$. If $Z = A_G$ is the centre of G then we have $\mu_Z(Z \cap \mathcal{K}) = 1$. If $H = G$ then Haar measure $\mu = \mu_G$ is normalized so that the volume of $\mathcal{K}$ is 1.

Denote by Θ the set of pairs $(\mathcal{O}, P = MU)$ where P is a semi-standard parabolic subgroup of G and $\mathcal{O} \subset \mathcal{E}_2(M)$ is an orbit under the action of $\mathrm{Im}\, X(M)$. (Here $\mathcal{E}_2(M)$ is the set of equivalence classes of the discrete series of the Levi subgroup M, and $\mathrm{Im}\, X(M)$ is the group of the unitary unramified characters of G.)

Two elements $(\mathcal{O}, P = MU)$ and $(\mathcal{O}', P' = M'U')$ are *associated* if there exists $s \in W^G$ such that $s \cdot M = M'$, $s\mathcal{O} = \mathcal{O}'$. We fix a set Θ/assoc of representatives in Θ for the classes of association. For $(\mathcal{O}, P = MU) \in \Theta$, we set $W(G|M) = \{s \in W^G\colon\ s \cdot M = M\}/W^M$, and

$$\mathrm{Stab}(\mathcal{O}, M) = \{s \in W(G|M)\colon\ s\mathcal{O} = \mathcal{O}\}.$$

Let $\mathcal{C}(G)$ denote the Harish-Chandra Schwartz space of G and let $I_P^G\omega$ denote the normalized induced representation from ω. Let $f\in\mathcal{C}(G)$, $\omega\in\mathcal{E}_2(M)$. We will write

$$\pi = I_P^G\omega, \quad \pi(f)=\int f(g)\pi(g)\,\mathrm{d}g, \quad \theta_\omega^G(f)=\operatorname{trace}\pi(f).$$

Theorem 1.1 (The Plancherel Formula [12, VIII.1.1]). *For each $f\in\mathcal{C}(G)$ and each $g\in G$ we have*

$$f(g)=\sum c(G|M)^{-2}\cdot\gamma(G|M)^{-1}\cdot\left|\mathrm{Stab}(\mathcal{O},M)\right|^{-1}\cdot\int_{\mathcal{O}}\mu_{G|M}(\omega)\,d(\omega)\theta_\omega^G\left(\lambda(g)f^\vee\right)\mathrm{d}\omega,$$

where the sum is over all the pairs $(\mathcal{O},P=MU)\in\Theta/assoc$.

The map

$$(\mathcal{O},P=MU)\to\mathrm{Irr}^{\mathrm{t}}(G), \qquad \omega\mapsto I_P^G\omega$$

determines a *bijection*

$$\bigsqcup(\mathcal{O},P=MU)/\mathrm{Stab}(\mathcal{O},M)\to\mathrm{Irr}^{\mathrm{t}}(G).$$

The tempered dual $\mathrm{Irr}^{\mathrm{t}}(G)$ acquires, by transport of structure, the structure of *disjoint union of countably many compact orbifolds*.

According to [12, V.2.1], the function $\mu_{G|M}$ is a rational function on $\mathcal{O}$. We have $\mu_{G|M}(\omega)\geqslant 0$ and $\mu_{G|M}(s\omega)=\mu_{G|M}(\omega)$ for each $s\in W^G$, $\omega\in\mathcal{O}$. This invariance property implies that $\mu_{G|M}$ *descends* to a function on the orbifold $\mathcal{O}/\mathrm{Stab}(\mathcal{O},M)$. We can view $\mu_{G|M}$ either as an *invariant* function on the orbit $\mathcal{O}$ or as a function on the orbifold $\mathcal{O}/\mathrm{Stab}(\mathcal{O},M)$.

The normalized Haar measure $d\omega$ on the compact torus $\mathcal{O}$ descends to the *canonical measure* on the orbifold $\mathcal{O}/\mathrm{Stab}(\mathcal{O},M)$. With respect to this canonical measure, the Plancherel *density* is given by

$$c(G|M)^{-2}\cdot\gamma(G|M)^{-1}\cdot\mu_{G|M}(\omega)\cdot d(\omega)=\gamma(G|M)\cdot j(\omega)^{-1}\cdot d(\omega), \tag{1}$$

where $d(\omega)$ is the formal degree of ω and $j(\omega)$ is defined as in [12, IV.3]. It is precisely this expression which we will compute explicitly.

The Bernstein variety $\Omega(G)$ is defined as follows: a point in $\Omega(G)$ is the G-conjugacy class of a pair (M',σ') formed by a Levi subgroup M' of G and an irreducible supercuspidal representation σ' of M'. Let $\mathrm{Irr}^{\mathrm{t}}(G)_\Omega$ denote the set of those tempered representations whose infinitesimal characters belong to Ω. We have, as in [9], the following partition of $\mathrm{Irr}^{\mathrm{t}}(G)$:

$$\mathrm{Irr}^{\mathrm{t}}(G)=\bigsqcup\mathrm{Irr}^{\mathrm{t}}(G)_\Omega.$$

Theorem 1.2 (The Bernstein Decomposition [9]). *The Plancherel measure ν admits a canonical Bernstein decomposition*

$$\nu=\bigsqcup\nu_\Omega,$$

where Ω is a component in the Bernstein variety $\Omega(G)$. The domain of each ν_Ω is a finite union of orbifolds of the form $\mathcal{O}/\mathrm{Stab}(\mathcal{O},M)$.

2. Calculation of the Harish-Chandra μ-function

Let m be an integer dividing n. We set $e = n/m$ and take for M the Levi subgroup $\mathrm{GL}(l_1 m) \times \cdots \times \mathrm{GL}(l_k m)$, where $(l_1, \ldots, l_k)$ is a partition of e. Let σ be an irreducible unitary supercuspidal representation of $\mathrm{GL}(m)$. Let $f(\sigma^\vee \times \sigma)$ denote the conductor for the pair $\sigma^\vee \times \sigma$ in the sense of [3].

Definition 2.1. The *torsion number* r of the representation σ is the order of the cyclic group of all those unramified characters η for which $\sigma \otimes \eta \cong \sigma$.

Let $\mathrm{St}(\sigma, e)$ denote the unique irreducible quotient of the induced representation defined by the Zelevinsky segment $\{|\det|^{-g}\sigma, \ldots, |\det|^{g}\sigma\}$, where $g = (e-1)/2$.

Theorem 2.2. *Let* $\pi = \mathrm{St}(\sigma, e)$. *Then*

$$\frac{d(\pi)}{d(\sigma)^e} = \frac{m^{e-1}}{r^{e-1}e} \cdot q^{(e^2-e)(f(\sigma^\vee \times \sigma)+r-2m^2)/2} \cdot \frac{(q^r-1)^e}{q^{er}-1} \cdot \frac{|\mathrm{GL}(em, q)|}{|\mathrm{GL}(m, q)|^e}.$$

For $i = 1, \ldots, k$, let $\pi_i = \mathrm{St}(\sigma, l_i)$. Then π_i is in the discrete series of $\mathrm{GL}(l_i m)$. Let χ_i be an unramified character of $F^\times$. We will write $\zeta_i = \chi_i(\varpi)$, $z_i = \zeta_i^r$. Let $\mathbb{T}^k$ denote the standard compact torus of dimension k, and let $P_{S_m}(X)$ denote the Poincaré polynomial of the Coxeter group S_m, so that

$$P_{S_m}\left(q^{-1}\right) = \frac{|\mathrm{GL}(m, q)|}{q^{m^2-m}(q-1)^m}.$$

Using the Harish-Chandra product formula (see [12, V.2.1]) and the Langlands–Shahidi formula in [11] (see also [10]), we prove the following result, in which the calculation of $\mu_{G|M}$ extends a classical result of Macdonald [8].

Theorem 2.3. *Let* $\omega = \chi_1\pi_1 \otimes \cdots \otimes \chi_k\pi_k$ *so that* ω *is in the discrete series of* M. *As a function on the compact torus* $\mathbb{T}^k$ *with coordinates* $(z_1, \ldots, z_k)$ *the Plancherel density is given by the formula* (1), *with*

$$c(G|M) = \frac{\prod_{1\leqslant i<j\leqslant k} P_{S_{n_i+n_j}}(q^{-1})}{P_{S_n}(q^{-1}) \cdot \prod_{i=1}^k (P_{S_{n_i}}(q^{-1}))^{k-2}}, \qquad \gamma(G|M) = \frac{P_{S_n}(q^{-1})}{P_{S_{n_1}}(q^{-1}) \times \cdots \times P_{S_{n_k}}(q^{-1})},$$

$$j(\omega)^{-1} = c(G|M)^{-2} \cdot \gamma(G|M)^{-2} \cdot \mu_{G|M}(\omega) = q^{\sum_{1\leqslant i<j\leqslant k} l_i l_j f(\sigma^\vee \times \sigma)} \cdot \prod \left| \frac{1 - z_j z_i^{-1} q^{gr}}{1 - z_j z_i^{-1} q^{-(g+1)r}} \right|^2,$$

where the product is taken over those i, j, g *for which the following inequalities hold*: $1 \leqslant i < j \leqslant k$, $|g_i - g_j| \leqslant g \leqslant g_i + g_j$, *and* $d(\omega) = d(\pi_1) \cdots d(\pi_k)$, *where the* $d(\pi_i)$ *are given by Theorem* 2.2.

3. The Bernstein decomposition of Plancherel measure

We now pass to the general case of a component $\Omega \subset \Omega(\mathrm{GL}(n))$. We can think of a component Ω in the Bernstein variety $\Omega(\mathrm{GL}(n))$ as a vector $(\sigma_1, \ldots, \sigma_t)$ of irreducible supercuspidal representations of smaller general linear groups $\mathrm{GL}(m_1), \ldots, \mathrm{GL}(m_t)$: the entries of this vector are determined up to tensoring with unramified quasicharacters and permutation. If the vector is $(\sigma_1, \ldots, \sigma_1, \ldots, \sigma_t, \ldots, \sigma_t)$ with σ_j repeated e_j times, $1 \leqslant j \leqslant t$, and $\sigma_1, \ldots, \sigma_t$ pairwise distinct (after unramified twist) then we will make the following definition.

Definition 3.1. The natural numbers $e_1, \ldots, e_t$ are the *exponents* of Ω.

Each representation σ_i of $\mathrm{GL}(m_i)$ has a torsion number r_i.

We may choose each representation σ_i of $\mathrm{GL}(m_i)$ to be unitary: in which case σ_i has a formal degree $d_i = d(\sigma_i)$.

We will denote by $f_{ij} = f(\sigma_i^\vee \times \sigma_j)$ the conductor of the pair $\sigma_i^\vee \times \sigma_j$.

In this way, the Bernstein component $\Omega \subset \Omega(\mathrm{GL}(n))$ yields up the following *fundamental invariants*:

- the cardinality q of the residue field of F,
- the sizes $m_1, m_2, \ldots, m_t$ of the small general linear groups,
- the exponents $e_1, e_2, \ldots, e_t$,
- the torsion numbers $r_1, r_2, \ldots, r_t$,
- the formal degrees $d_1, d_2, \ldots, d_t$,
- the conductors for pairs $f_{11}, f_{12}, \ldots, f_{tt}$.

Our Plancherel formulas are built from precisely these numerical invariants.

Consider, once again, the component $\Omega \subset \Omega(\mathrm{GL}(n))$ with exponents $e_1, \ldots, e_t$. It determines t components $\Omega_1, \ldots, \Omega_t$ with separate exponents $e_1, \ldots, e_t$. The component $\Omega_j \subset \Omega(\mathrm{GL}(m_j e_j))$ contains the conjugacy class of the cuspidal pair $(\mathrm{GL}(m_j)^{e_j}, \sigma_j^{\otimes e_j})$. With Ω_j so defined, we will write

$$\Omega = \Omega_1 \times \cdots \times \Omega_t.$$

Each component Ω_j admits a single exponent, and so places us in the context of Theorem 2.3.

Theorem 3.2. *If $\Omega = \Omega_1 \times \cdots \times \Omega_t$ then we have*

$$\nu_\Omega = const. \cdot \nu_{\Omega_1} \cdots \nu_{\Omega_t}$$

where $\nu_{\Omega_1}, \ldots, \nu_{\Omega_t}$ are given by Theorem 2.3.

4. Transfer-of-measure formula

We recall the situation at the beginning of Section 2. We have an integer m dividing n, $e = n/m$, σ is an irreducible unitary supercuspidal representation of $\mathrm{GL}(m)$ with torsion number r, and K is a local field such that the cardinality of its residue field is $q_K = q^r$.

Let $G = \mathrm{GL}(n, F)$, $G_0 = \mathrm{GL}(e, K)$. Let $M \simeq \mathrm{GL}(l_1 m) \times \cdots \times \mathrm{GL}(l_k m)$, $M_0 \simeq \mathrm{GL}(l_1) \times \cdots \times \mathrm{GL}(l_k)$, where $(l_1, \ldots, l_k)$ is a partition of e. Let $\Omega \subset \Omega(G)$ be defined as follows: Ω is the Bernstein component in $\Omega(G)$ which contains the conjugacy class of the cuspidal pair $(\mathrm{GL}(m)^e, \sigma^{\otimes e})$. Then Ω has the single exponent e.

Let $\Omega_0 \subset \Omega(G_0)$ be defined as follows: Ω_0 is the Bernstein component in $\Omega(G_0)$ which contains the conjugacy class of the cuspidal pair $(T, 1)$, where T is the diagonal subgroup of G_0. The component Ω_0 parametrizes those irreducible smooth representations of $\mathrm{GL}(e, K)$ which admit nonzero Iwahori fixed vectors. Then Ω_0 has the single exponent e, and we have $\mathrm{Irr}^{\mathrm{t}}\mathrm{GL}(n, F)_\Omega \cong \mathrm{Irr}^{\mathrm{t}}\mathrm{GL}(e, K)_{\Omega_0}$, see [9].

The theory of types of [5] produces a canonical extension K of F such that r is equal to the residue index of K with respect to F. Indeed, let (J, λ) be a maximal simple type in $\mathrm{GL}(m)$ contained in σ, and let $\mathfrak{A}$ and $E = F[\beta]$ respectively denote the corresponding hereditary order in $A = M(m, F)$ and the corresponding field extension of F (see [5, (5.5.10(iii))]). We have $r = m/e(E|F)$, where $e(E|F)$ denotes the ramification index of E with respect to F. Let B denote the centraliser of E in A. We set $\mathfrak{B} := \mathfrak{A} \cap B$. Then $\mathfrak{B}$ is a maximal hereditary order in B, and let K be an unramified extension of E which normalises it and is maximal with respect to that property, as in [5, (5.5.14)]. Then r is equal to the residue index of K with respect to F. Thus q^r is equal to the order q_K of the residue field of K. Also the number q^r is the one which occurs for the Hecke algebra $\mathcal{H}(\mathrm{GL}(m), \lambda)$ associated to (J, λ) in [5, (5.6.6)].

Let ν (resp. ν_0) denote Plancherel measure on the tempered dual of G (resp. G_0). Let ν_Ω, ν_{Ω_0} denote the corresponding Bernstein components. The support of ν_Ω (resp. ν_{Ω_0}) is a compact Hausdorff space. This compact space is an *extended quotient*, see [9].

A transfer-measure-formula appears in [4]. Their proof uses the techniques of Hecke algebras. Our method is different. We use our explicit Plancherel formulas, and also an extension of some of the results in [3].

Let (J^G, λ^G) denote a simple type attached to the Bernstein component Ω, see [5,6]. Let I denote an Iwahori subgroup of G_0.

Theorem 4.1. *The support of ν_Ω is homeomorphic to the support of ν_{Ω_0} and we have*

$$\frac{\mu_G(J^G)}{\dim\lambda^G}\cdot \mathrm{d}\nu_\Omega(\omega)=\mu_{G_0}(I)\cdot\frac{r^k}{m^k}\cdot d\nu_{\Omega_0}(\omega_0),$$

where $\omega=\chi_1\pi_1\otimes\cdots\otimes\chi_k\pi_k$ and ω_0 denotes the corresponding representation of M_0.

Detailed proofs of the results announced in this Note may be found in [1].

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