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On the motion of a rigid body in a two-dimensional ideal flow with vortex sheet initial data

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Abstract

A famous result by Delort about the two-dimensional incompressible Euler equations is the existence of weak solutions when the initial vorticity is a bounded Radon measure with distinguished sign and lies in the Sobolev space H^{-1}. In this paper we are interested in the case where there is a rigid body immersed in the fluid moving under the action of the fluid pressure. We succeed to prove the existence of solutions *à la Delort* in a particular case with a mirror symmetry assumption already considered by Lopes Filho et al. (2006) [11], where it was assumed in addition that the rigid body is a fixed obstacle. The solutions built here satisfy the energy inequality and the body acceleration is bounded. When the mass of the body becomes infinite, the body does not move anymore and one recovers a solution in the sense of Lopes Filho et al. (2006) [11].

1. Introduction

1.1. Motion of a body in a two-dimensional ideal flow

We consider the motion of a body $\mathcal{S}(t)$ in a planar ideal fluid which therefore occupies at time t the set $\mathcal{F}(t) := \mathbb{R}^2 \setminus \mathcal{S}(t)$. We assume that the body is a closed disk of radius one and has a uniform density $\rho > 0$. The equations modeling the dynamics of the system read

$$\frac{\partial u}{\partial t} + (u \cdot \nabla)u + \nabla p = 0 \quad \text{for } x \in \mathcal{F}(t), \tag{1.1}$$

$$\operatorname{div} u = 0 \quad \text{for } x \in \mathcal{F}(t), \tag{1.2}$$

$$u \cdot \mathbf{n} = h'(t) \cdot \mathbf{n} \quad \text{for } x \in \partial\mathcal{S}(t), \tag{1.3}$$

$$mh''(t) = \int_{\partial\mathcal{S}(t)} p\mathbf{n}\, ds, \tag{1.4}$$

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$$u|_{t=0} = u_0, \tag{1.5}$$
$$\big(h(0), h'(0)\big) = (0, \ell_0). \tag{1.6}$$

Here $u = (u_1, u_2)$ and p denote the velocity and pressure fields, $m = \rho\pi$ denotes the mass of the body while the fluid is supposed to be homogeneous of density 1, to simplify the equations, $\mathbf{n}$ denotes the unit outward normal on $\mathcal{F}(t)$, ds denotes the integration element on the boundary $\partial\mathcal{S}(t)$ of the body. In Eq. (1.4), $h(t)$ is the position of the center of mass of the body.

Eqs. (1.1) and (1.2) are the incompressible Euler equations, the condition (1.3) means that the boundary is impermeable, Eq. (1.4) is Newton's balance law for linear momentum: the fluid acts on the body through pressure force.

In the system above we omit the equation for the rotation of the rigid ball, which yields that the angular velocity of the rigid body remains constant when time proceeds, since the angular velocity is not involved in Eqs. (1.1)–(1.6).

1.2. Equations in the body frame

We start by transferring the previous equations in the body frame. We define:

$$\begin{cases} v(t,x) = u\big(t, x + h(t)\big), \\ q(t,x) = p\big(t, x + h(t)\big), \\ \ell(t) = h'(t), \end{cases}$$

so that Eqs. (1.1)–(1.6) become

$$\frac{\partial v}{\partial t} + \big[(v - \ell)\cdot\nabla\big]v + \nabla q = 0, \quad x \in \mathcal{F}_0, \tag{1.7}$$
$$\operatorname{div} v = 0, \quad x \in \mathcal{F}_0, \tag{1.8}$$
$$v \cdot \mathbf{n} = \ell \cdot \mathbf{n}, \quad x \in \partial\mathcal{S}_0, \tag{1.9}$$
$$m\ell'(t) = \int_{\partial\mathcal{S}_0} q\mathbf{n}\, ds, \tag{1.10}$$
$$v(0,x) = v_0(x), \quad x \in \mathcal{F}_0, \tag{1.11}$$
$$\ell(0) = \ell_0, \tag{1.12}$$

where $\mathcal{S}_0$ denotes the closed unit disk, which is the set initially occupied by the solid and $\mathcal{F}_0 := \mathbb{R}^2 \setminus \mathcal{S}_0$ is the one occupied by the fluid.

1.3. Vortex sheets data

Such a problem has been tackled by [13] in the case of a smooth initial data with finite kinetic energy, by [5] in the case of Yudovich-like solutions (with bounded vorticities) and by [4] in the case where the initial vorticity of the fluid has a L^p_c vorticity with $p > 2$. The index c is used here and in the sequel for "compactly supported". These works provided the global existence of solutions. Actually the result of [13] was extended in [12] to the case of a solid of arbitrary form, for which rotation has to be taken into account, and the works [5] and [4] deal with an arbitrary form as well. Furthermore we address in [6] the case of an initial vorticity in L^p_c with $p > 1$, in order to achieve the investigation of solutions "à la DiPerna–Majda", referring here to the seminal work [2] in the case of a fluid alone.

It is therefore natural to try to extend these existence results to the case, more singular, of vortex sheet initial data. In the case of a fluid alone, without any moving body, vortex sheet motion is a classical topic in fluid dynamics. Several approaches have been tried. Here we will follow the approach initiated by J.-M. Delort in [1] who proved global-in-time existence of weak solutions for the incompressible Euler equations when the initial vorticity is a compactly supported, bounded Radon measure with distinguished sign in the Sobolev space H^{-1}. The pressure smoothness in Delort's result is very bad so that it could be a priori argued that the extension to the case of an immersed body should be challenging since the motion of the solid is determined by the pressure forces exerted by the fluid on the solid boundary. However the problem (1.7)–(1.12) admits a global weak formulation where the pressure disappears. The drawback is that test functions involved in this weak formulation do not vanish on the interface between the solid and the fluid, an unusual fact in Delort's approach, where the solution rather satisfies a weak formulation of the equations which involves some test functions compactly supported in the fluid domain (which is open) and the boundary condition is prescribed in a trace sense.

Yet in the paper [10], the authors deal with the case of an initial vorticity compactly supported, bounded Radon measure with distinguished sign in H^{-1} in the upper half-plane, superimposed on its odd reflection in the lower half-plane. The corresponding initial velocity is then mirror symmetric with respect to the horizontal axis. In the course of proving the existence of solutions to this problem, they are led to introduce another notion of weak solution that they called *boundary-coupled weak solution*, which relies on a weak vorticity formulation which involves some test functions that vanish on the boundary, but not their derivatives. They have extended their analysis to the case of a fluid occupying the exterior of a symmetric fixed body in [11].

1.4. Mirror symmetry

In this paper, we assume that the initial velocities ℓ_0 and v_0 are mirror symmetric with respect to the horizontal axis given by the equation $x_2 = 0$. Our setting here can therefore be seen as an extension of the one in [11] from the case of a fixed obstacle to the case of a moving body.

For the body velocity $\ell_0 \in \mathbb{R}^2$ the mirror symmetry entails that ℓ_0 is of the form $\ell_0 = (\ell_{0,1}, 0)$. Let us now turn our attention to the fluid velocity. Let

$$\mathcal{F}_{0,\pm} := \{x \in \mathcal{F}_0 \mid \pm x_2 > 0\} \quad \text{and} \quad \Gamma_\pm := \partial\mathcal{F}_{0,\pm}.$$

If $x = (x_1, x_2) \in \mathcal{F}_{0,\pm}$ then we denote $\tilde{x} := (x_1, -x_2) \in \mathcal{F}_{0,\mp}$. To avoid any confusion let us say here that for a smooth vector field $u = (u_1, u_2)$, the mirror symmetry assumption means that for any $x \in \mathcal{F}_{0,\pm}$, $(u_1, u_2)(\tilde{x}) = (u_1, -u_2)(x)$.

This assumption has two important consequences. First the vorticity $\omega := \operatorname{curl} v := \partial_1 v_2 - \partial_2 v_1$ is odd with respect to the variable x_2 and therefore its integral over the fluid domain $\mathcal{F}_0$ vanishes. The other one is that the circulation of the initial velocity around the body vanishes. In such a case, it is very natural to consider finite energy velocity.

We will explain why in the next section by considering the link between velocity and vorticity.

Let us also mention that the analysis performed here can be adapted to the case of a body occupying a smooth, bounded, simply connected closed set, which is symmetric with respect to the horizontal coordinate axis, and which is not allowed to rotate (for instance because of the action of an exterior torque on the body preventing any rotation, or because the angular mass is infinite).

1.5. A velocity decomposition

We denote by

$$G(x, y) := \frac{1}{2\pi} \ln \frac{|x - y|}{|x - y^*||y|}, \quad \text{where } y^* := \frac{y}{|y|^2},$$

the Green's function of $\mathcal{F}_0$ with Dirichlet boundary condition. We also introduce the functions

$$H(x) := \frac{x^\perp}{2\pi|x|^2} \quad \text{and} \quad K(x, y) := H(x - y) - H\big(x - y^*\big), \tag{1.13}$$

with the notation $x^\perp := (-x_2, x_1)$ when $x = (x_1, x_2)$, which are the kernels of the Biot–Savart operators respectively in the full plane and in $\mathcal{F}_0$. More precisely we define the operator $K[\omega]$ as acting on $\omega \in C_c^\infty(\mathcal{F}_0)$ through the formula

$$K[\omega](x) = \int_{\mathcal{F}_0} K(x, y)\omega(y)\, dy.$$

We will extend this definition to bounded Radon measures in the sequel but let us consider here the smooth case first to clarify the presentation. We also define the hydrodynamic Biot–Savart operator $K_{\mathcal{H}}[\omega]$ by

$$K_{\mathcal{H}}[\omega](x) = \int_{\mathcal{F}_0} K_{\mathcal{H}}(x, y)\omega(y)\, dy \quad \text{with } K_{\mathcal{H}}(x, y) := K(x, y) + H(x).$$

One easily verifies that

$$\lim_{|x|+|y|\to+\infty} K_{\mathcal{H}}(x,y) - H(x-y) = 0 \tag{1.14}$$

and that H and $K_{\mathcal{H}}[\omega]$ satisfy

$$\operatorname{div} H = 0, \quad \operatorname{curl} H = 0 \quad \text{in } \mathcal{F}_0, \quad H\cdot\mathbf{n} = 0 \quad \text{in } \partial\mathcal{S}_0, \quad \int_{\partial\mathcal{S}_0} H\cdot\mathbf{n}^{\perp}\,ds = -1, \quad \lim_{|x|\to+\infty} H = 0, \tag{1.15}$$

$$\operatorname{div} K_{\mathcal{H}}[\omega] = 0, \quad \operatorname{curl} K_{\mathcal{H}}[\omega] = \omega \quad \text{in } \mathcal{F}_0, \quad K_{\mathcal{H}}[\omega]\cdot\mathbf{n} = 0 \quad \text{in } \partial\mathcal{S}_0,$$
$$\int_{\partial\mathcal{S}_0} K_{\mathcal{H}}[\omega]\cdot\mathbf{n}^{\perp}\,ds = 0, \quad \lim_{|x|\to+\infty} K_{\mathcal{H}}[\omega] = 0. \tag{1.16}$$

Let us also define the Kirchhoff potential

$$\Phi_i(x) := -\frac{x_i}{|x|^2},$$

which satisfies

$$-\Delta\Phi_i = 0 \quad \text{for } x\in\mathcal{F}_0, \qquad \Phi_i\to 0 \quad \text{for } |x|\to+\infty, \qquad \frac{\partial\Phi_i}{\partial\mathbf{n}} = \mathbf{n}_i \quad \text{for } x\in\partial\mathcal{S}_0, \tag{1.17}$$

for $i = 1, 2$, where $\mathbf{n}_1$ and $\mathbf{n}_2$ are the components of the normal vector $\mathbf{n}$. Let us also observe $\nabla\Phi_i$ is in $C^\infty(\overline{\mathcal{F}_0}) \cap L^2(\mathcal{F}_0)$, and that the derivatives of higher orders of $\nabla\Phi_i$ are also in $L^2(\mathcal{F}_0)$.

Then we have the following decomposition result:

Lemma 1. *Let $\omega\in C_c^\infty(\mathcal{F}_0)$, $\ell := (\ell_1,\ell_2)\in\mathbb{R}^2$ and $\gamma\in\mathbb{R}$. Then there exists one only smooth divergence free vector field u such that $u\cdot\mathbf{n} = \ell\cdot\mathbf{n}$ on $\partial\mathcal{S}_0$, $\int_{\partial\mathcal{S}_0} u\cdot\mathbf{n}^\perp\,ds = \gamma$, $\operatorname{curl} u = \omega$ in $\mathcal{F}_0$ and such that u vanishes at infinity. Moreover $u = K[\omega] + \ell_1\nabla\Phi_1 + \ell_2\nabla\Phi_2 + (\alpha-\gamma)H$, where $\alpha := \int_{\mathcal{F}_0}\omega\,dx$.*

Proof. Combining (1.15), (1.16) and (1.17) we get the existence part. Regarding the uniqueness, it is sufficient to apply [8, Lemma 2.14]. □

Now our point is that considering some mirror symmetric velocities u and ℓ, assuming again that u is smooth with $\omega := \operatorname{curl} u$ in $C_c^\infty(\mathcal{F}_0)$, one has $\int_{\partial\mathcal{S}_0} u\cdot\mathbf{n}^\perp\,ds = 0$ and $\int_{\mathcal{F}_0}\omega\,dx = 0$, so that, according to the previous lemma, $u = K[\omega] + \ell_1\nabla\Phi_1$. One then easily infers from the definitions above that $u\in L^2(\mathcal{F}_0)$. The kinetic energy $m\ell^2 + \int_{\mathcal{F}_0} u^2\,dx$ of the system "fluid + body" is therefore finite. Let us also stress that u can also be written as $u = K_{\mathcal{H}}[\omega] + \ell_1\nabla\Phi_1$. Here the advantage of using $K_{\mathcal{H}}[\omega]$ rather than $K[\omega]$ is that we will make use of (1.14), which is not satisfied with $K(x,y)$ instead of $K_{\mathcal{H}}(x,y)$.

1.6. Cauchy data

Let us now define properly the Cauchy data we are going to consider in this paper. For a subset X of $\mathbb{R}^2$ we will use the notation $\mathcal{BM}(X)$ for the set of the bounded measures over X, $\mathcal{BM}_+(X)$ for the set of the positive measures over X, $\mathcal{BM}_c(X)$ the subspace of the measures of $\mathcal{BM}(X)$ which are compactly supported in X and, following the terminology of [11], we will say that a $\omega\in\mathcal{BM}(\mathcal{F}_0)$ is nonnegative mirror symmetric (*NMS* for short) if it is odd with respect to the horizontal axis and if it is nonnegative in the upper half-plane. This means that for any $\phi\in C_c(\mathcal{F}_0;\mathbb{R})$,

$$\int_{\mathcal{F}_0}\phi(x)\,d\omega(x) = -\int_{\mathcal{F}_0}\phi(\tilde{x})\,d\omega(x), \tag{1.18}$$

with the notation of Section 1.4.

We now extend the operator $K[\cdot]$ to any $\omega \in \mathcal{BM}(\overline{\mathcal{F}_0})$ by defining $K[\omega] \in C_c^\infty(\overline{\mathcal{F}_0})'$ through the formula

$$\forall f \in C_c^\infty(\overline{\mathcal{F}_0}), \quad \langle K[\omega], f\rangle := -\int_{\overline{\mathcal{F}_0}}\int_{\overline{\mathcal{F}_0}} G(x,y)\,\mathrm{curl}\, f(x)\,dx\,d\omega(y). \tag{1.19}$$

Let $\ell_{0,1} \in \mathbb{R}$ and $\ell_0 = (\ell_{0,1}, 0)$. Let $\omega_{0,+} \in \mathcal{BM}_{c,+}(\mathcal{F}_{0,+})$ and $\omega_{0,-}$ the corresponding measure in $\mathcal{F}_{0,-}$ obtained by odd reflection. We then denote $\omega_0 := \omega_{0,+} + \omega_{0,-}$ which is in $\mathcal{BM}(\mathcal{F}_0)$ and is NMS. We define accordingly the initial fluid velocity by $v_0 := K[\omega_0] + \ell_{0,1}\nabla\Phi_1$.

1.7. Weak formulation

Let us now give a global weak formulation of the problem by considering—for solution and for test functions—a velocity field on the whole plane, with the constraint to be constant on $\mathcal{S}_0$. We introduce the following space

$$\mathcal{H} = \{\Psi \in L^2(\mathbb{R}^2);\ \mathrm{div}\,\Psi = 0 \text{ in } \mathbb{R}^2,\ \nabla\Psi = 0 \text{ in } \mathcal{S}_0\},$$

which is a Hilbert space endowed with the scalar product

$$(\bar{u},\bar{v})_\rho := \int_{\mathbb{R}^2} (\rho\chi_{\mathcal{S}_0} + \chi_{\mathcal{F}_0})\bar{u}\cdot\bar{v} = m\ell_u\cdot\ell_v + \int_{\mathcal{F}_0} u\cdot v\,dx, \tag{1.20}$$

where the notation χ_A stands for the characteristic function of the set A, $\ell_u \in \mathbb{R}^2$ and $u \in L^2(\mathcal{F}_0)$ denote respectively the restrictions of $\bar{u}$ to $\mathcal{S}_0$ and $\mathcal{F}_0$. Let us stress here that because, by definition of $\mathcal{H}$, $\bar{u}$ is assumed to satisfy the divergence free condition in the whole plane, the normal component of these restrictions have to match on the boundary $\partial\mathcal{S}_0$. We will denote $\|\cdot\|_\rho$ the norm associated to $(\cdot,\cdot)_\rho$. Let us also introduce $\mathcal{H}_T$ the set of the test functions Ψ in $C^1([0,T];\mathcal{H})$ with its restriction $\Psi|_{[0,T]\times\overline{\mathcal{F}_0}}$ to the closure of the fluid domain in $C_c^1([0,T]\times\overline{\mathcal{F}_0})$.

Definition 2. Let be given $\bar{v}_0 \in \mathcal{H}$ and $T > 0$. We say that $\bar{v} \in C([0,T];\mathcal{H} - w)$ is a weak solution of (1.7)–(1.12) in $[0,T]$ if for any test function $\Psi \in \mathcal{H}_T$,

$$\big(\Psi(T,\cdot),\bar{v}(T,\cdot)\big)_\rho - \big(\Psi(0,\cdot),\bar{v}_0\big)_\rho = \int_0^T \left(\frac{\partial\Psi}{\partial t},\bar{v}\right)_\rho dt + \int_0^T\int_{\mathcal{F}_0} v\cdot\big[\big((v-\ell_v)\cdot\nabla\big)\Psi\big]\,dx\,dt. \tag{1.21}$$

Definition 2 is legitimate since a classical solution of (1.7)–(1.12) in $[0,T]$ is also a weak solution. This follows easily from an integration by parts in space which provides

$$(\partial_t\bar{v},\Psi)_\rho = \int_{\mathcal{F}_0} v\cdot\big[\big((v-\ell_v)\cdot\nabla\big)\Psi\big]\,dx,$$

and then from an integration by parts in time.

1.8. Results

Our main result is the following.

Theorem 3. *Let be given a Cauchy data $\bar{v}_0 \in \mathcal{H}$ as described in Section 1.6. Let $T > 0$. Then there exists a weak solution of (1.7)–(1.12) in $[0,T]$. In addition this solution preserves the mirror symmetry and satisfies the energy inequality: for any $t \in [0,T]$, $\|\bar{v}(t,\cdot)\|_\rho \leqslant \|\bar{v}_0\|_\rho$. Moreover the acceleration ℓ' of the body is bounded in $[0,T]$.*

Let us slightly specify the last assertion. Actually the proof will provide a bound of $\|\ell'\|_{L^\infty(0,T)}$ which only depends on the body mass m and on the initial energy $\|\bar{v}_0\|_\rho$, but not on T.

Let us also stress that it is straightforward, by an energy estimate, to prove that the weak solution above enjoys a weak–strong uniqueness property. Then, applying Theorem 1 of [15], it follows that uniqueness holds for a G_δ dense subset of $\mathcal{H}$ endowed with its weak topology.

When the mass m of the body goes to infinity one should expect that the body cannot move anymore so that the system (1.7)–(1.12) degenerates into the Euler equations in $\mathcal{F}_0$, that is

$$\frac{\partial v}{\partial t} + [v \cdot \nabla]v + \nabla q = 0, \quad x \in \mathcal{F}_0, \tag{1.22}$$

$$\operatorname{div} v = 0, \quad x \in \mathcal{F}_0, \tag{1.23}$$

$$v \cdot \mathbf{n} = 0, \quad x \in \partial\mathcal{S}_0, \tag{1.24}$$

$$v(0, x) = v_0(x), \quad x \in \mathcal{F}_0. \tag{1.25}$$

The following result justifies this intuition for the weak solutions obtained in Theorem 3. In this case one will obtain at the limit some solutions of (1.22)–(1.25) in the sense of the following definition. We will denote $C^1_{c,\sigma}(\overline{\mathcal{F}_0})$ (respectively $C^1_{c,\sigma}([0,T] \times \overline{\mathcal{F}_0})$) the subspace of the vector fields in $C^1_c(\overline{\mathcal{F}_0}, \mathbb{R}^2)$ (respectively $C^1_c([0,T] \times \overline{\mathcal{F}_0}, \mathbb{R}^2)$) which are divergence free and tangent to the boundary $\partial\mathcal{F}_0$ (respectively tangent to $\partial\mathcal{F}_0$ for any $t \in [0,T]$). We denote by $L^2_\sigma(\mathcal{F}_0)$ the closure of $C^1_{c,\sigma}(\overline{\mathcal{F}_0})$ for the L^2 topology.

Definition 4. Let be given $v_0 \in L^2_\sigma(\mathcal{F}_0)$ and $T > 0$. We say that $v \in C([0,T]; L^2_\sigma(\mathcal{F}_0) - w)$ is a weak solution of (1.22)–(1.25) in $[0,T]$ if for any test function $\Psi \in C^1_{c,\sigma}([0,T] \times \overline{\mathcal{F}}_0; \mathbb{R}^2)$,

$$\int_{\mathcal{F}_0} \Psi(T,\cdot) \cdot v(T,\cdot)\,dx - \int_{\mathcal{F}_0} \Psi(0,\cdot) \cdot v_0\,dx = \int_0^T \int_{\mathcal{F}_0} \frac{\partial \Psi}{\partial t} \cdot v\,dx\,dt + \int_0^T \int_{\mathcal{F}_0} v \cdot \big[(v \cdot \nabla)\Psi\big]\,dx\,dt. \tag{1.26}$$

Observe that in Definition 4 the test functions are not required to vanish in the neighborhood of the boundary, they are only tangent, what makes this definition slightly more stringent than the definition used in Delort or Schochet's papers [1,16] where the boundary condition (1.24) is prescribed in a trace sense. This is the counterpart for the velocity formulation of the notion of boundary-coupled solution introduced in the vorticity formulation in [10] and [11].

Theorem 5. *Let be given a Cauchy data $\bar{v}_0 \in \mathcal{H}$ as in Theorem 3. Assume furthermore that $\ell_0 = 0$. Let $T > 0$. For any mass $m > 0$, consider a weak solution $\bar{v}^m$ as given by Theorem 3. Then there exists a sequence $(m_k)_{k\in\mathbb{N}}$ converging to $+\infty$ such that $(\bar{v}^{m_k})_k$ converges in $C([0,T]; \mathcal{H} - w)$ to $\bar{v}$ with $\ell = 0$ and v is a weak solution of (1.22)–(1.25) in $[0,T]$. Moreover $(\ell^{m_k})_k$ converges to 0 in $W^{1,\infty}([0,T])$.*

This result extends Theorem 4 of [14] to more irregular solutions, but for a more particular geometry.

Next section is devoted to the proof of Theorem 3 whereas the third and last one will be devoted to the proof of Theorem 5.

2. Proof of Theorem 3

A general strategy for obtaining a weak solution is to smooth out the initial data so that one gets a sequence of initial data which launch some classical solutions, and then to pass to the limit with respect to the regularization parameter in the weak formulation of the equations.

2.1. A regularized sequence

Let $(\eta_n)_n$ be a sequence of radial mollifiers. We therefore consider the sequence of regularized initial vorticities $(\omega^n_0)_n$ given by $\omega^n_0 := (\omega_0) * \eta_n$ and some corresponding initial velocities $(\bar{v}^n_0)_n$ in $\mathcal{H}$ with

$$\bar{v}^n_0 := \ell_0 \text{ in } \mathcal{S}_0 \text{ and } \bar{v}^n_0 := v^n_0 := K\big[\omega^n_0\big] + \ell_{0,1}\nabla\Phi_1 \text{ in } \mathcal{F}_0.$$

Then the $(\omega^n_0)_n$ are smooth, compactly supported in $\mathcal{F}_0$ (at least for n large enough), NMS and bounded in $L^1(\mathcal{F}_0)$, and $(\bar{v}^n_0)_n$ converges weakly in $\mathcal{H}$ to v_0.

Let $(\bar{v}^n)_n$ in $C([0,T]; \mathcal{H})$ be the classical solutions of (1.7)–(1.12) in $[0,T]$ respectively associated to the sequence $(\bar{v}^n_0)_n$ of initial data (cf. [13]). According to Lemma 1 the restriction v^n of $\bar{v}^n$ to $\mathcal{F}_0$ splits into

$$v^n = u^n + \nabla \Phi^n \quad \text{where } u^n := K\big[\omega^n\big] \quad \text{and} \quad \Phi^n := \ell_1^n \Phi_1. \tag{2.1}$$

Observe in particular that from now on we denote ℓ^n for ℓ_{v^n}.

Moreover these solutions preserve, for any t in $[0,T]$, the mirror symmetry (this follows from the uniqueness of the Cauchy problem for classical solutions), the kinetic energy:

$$\big\|\bar{v}^n(t,\cdot)\big\|_\rho = \big\|\bar{v}_0^n\big\|_\rho, \tag{2.2}$$

and the L^1 norm of the vorticity on the upper and lower half-planes:

$$\big\|\omega^n(t,\cdot)\big\|_{L^1(\mathcal{F}_{0,\pm})} = \big\|\omega_0^n\big\|_{L^1(\mathcal{F}_{0,\pm})}. \tag{2.3}$$

This last property can be obtained from the vorticity equation:

$$\partial_t \omega^n + \big(v^n - \ell^n\big)\cdot \nabla \omega^n = 0. \tag{2.4}$$

As already said before a classical solution is a fortiori a weak solution, thus for any test function Ψ in $\mathcal{H}_T$,

$$\big(\Psi(T,\cdot), \bar{v}^n(T,\cdot)\big)_\rho - \big(\Psi(0,\cdot), \bar{v}_0^n\big)_\rho = \int_0^T \left(\frac{\partial \Psi}{\partial t}, \bar{v}^n\right)_\rho dt + \int_0^T \int_{\mathcal{F}_0} v^n \cdot \big[\big(\big(v^n - \ell^n\big)\cdot\nabla\big)\Psi\big]\, dx\, dt. \tag{2.5}$$

Using the bounds (2.2) and (2.3), we obtain that there exists a subsequence $(\bar{v}^{n_k})_k$ of $(\bar{v}^n)$ which converges to $\bar{v}$ in $L^\infty(0,T;\mathcal{H})$ weak* and such that $(\omega^{n_k})_k$ converges to ω weak* in $L^\infty(0,T;\mathcal{BM}(\overline{\mathcal{F}_0}))$. In particular we have that $(\ell^{n_k})_k$ converges to ℓ in $L^\infty(0,T)$ weak* and that $(v^{n_k})_k$ converges to v weak* in $L^\infty(0,T;L^2(\mathcal{F}_0))$ weak*, where ℓ and v denote respectively the restrictions to $\mathcal{S}_0$ and $\mathcal{F}_0$ of $\bar{v}$, and are mirror symmetric, so that the vector ℓ is of the form $(\ell_1, 0)$. We deduce that ω is NMS for almost every t in $(0,T)$, in the sense that for any $\phi \in C_c(\overline{\mathcal{F}_0};\mathbb{R})$,

$$\int_{\overline{\mathcal{F}_0}} \phi(x)\, d\omega(x) = -\int_{\overline{\mathcal{F}_0}} \phi(\tilde{x})\, d\omega(x), \tag{2.6}$$

with the notation of Section 1.4.

Thanks to the Lebesgue dominated convergence theorem we deduce that the total mass of ω vanishes for almost every t in $(0,T)$, that is

$$\omega(t,\overline{\mathcal{F}_0}) = 0. \tag{2.7}$$

Also, passing to the limit in the formula (1.19) yields $u(t,\cdot) = K[\omega(t,\cdot)]$ for almost every t in $(0,T)$.

Our goal now is to prove that the limit obtained satisfies the weak formulation (1.21). Unfortunately, the weak convergences above are far from being sufficient to pass to the limit. We will first improve these convergences with respect to the time variable. More precisely in the next section we will give an estimate of the body acceleration which will allow to obtain convergence in $C([0,T])$ of a subsequence of the solid velocities.

2.2. *Estimate of the body acceleration*

The goal of this section is to prove the following.

Lemma 6. *The sequence $((\ell^n)')_n$ is bounded in $L^\infty(0,T)$.*

Proof. Let ℓ be in $\mathbb{R}^2$. Then we define Ψ in $\mathcal{H}$ by setting $\Psi = \ell$ in $\mathcal{S}_0$ and $\Psi = \nabla(\ell_1\Phi_1 + \ell_2\Phi_2)$ in $\mathcal{F}_0$. Therefore, $\bar{v}^n$ being a classical solution of the system (1.7)–(1.12), one has

$$\big(\partial_t \bar{v}^n, \Psi\big)_\rho = \int_{\mathcal{F}_0} v^n \cdot \big[\big(\big(v^n - \ell^n\big)\cdot\nabla\big)\Psi\big]\, dx. \tag{2.8}$$

By using the definition of the scalar product in (1.20), (1.17) and the boundary condition (1.9) we obtain

$$\big(\partial_t \bar{v}^n, \Psi\big)_\rho = \ell^T \mathcal{M}\big(\ell^n\big)', \quad \text{with } \mathcal{M} := mId_2 + \bigg(\int_{\mathcal{F}_0} \nabla\Phi_i \cdot \nabla\Phi_j \, dx\bigg)_{i,j},$$

which is a 2×2 positive definite symmetric matrix that stands for the added mass of the body which, loosely speaking, measures how much the surrounding fluid resists the acceleration as the body moves through it.

Now we use that $\nabla\Psi$ is in $L^2(\mathcal{F}_0) \cap L^\infty(\mathcal{F}_0)$ and (2.2) to get that the right hand side of (2.8) is bounded uniformly in n. Therefore $(\ell^n)'$ is bounded in $L^\infty(0,T)$. $\square$

In particular we deduce from this, (2.2) and Ascoli's theorem that there exists a subsequence, that we still denote $(\ell^{n_k})_k$, which converges strongly to ℓ in $C([0,T])$. Moreover by weak compactness, we also have that $((\ell^{n_k})')_k$ converges to ℓ' in $L^\infty(0,T)$ weak*.

2.3. A decomposition of the nonlinearity

The main difficult term to pass to the limit into (2.5) is the third one because of its nonlinear feature. We first use (2.1) to obtain for any test function Ψ in $\mathcal{H}_T$,

$$\int_0^T \int_{\mathcal{F}_0} v^n \cdot \big[\big(\big(v^n - \ell^n\big) \cdot \nabla\big)\Psi\big] \, dx \, dt = T_1^n + T_2^n + T_3^n$$

where

$$T_1^n := \int_0^T \int_{\mathcal{F}_0} u^n \cdot \big[\big(u^n \cdot \nabla\big)\Psi\big] \, dx \, dt,$$

$$T_2^n := \int_0^T \int_{\mathcal{F}_0} u^n \cdot \big[\big(\big(\nabla\Phi^n - \ell^n\big) \cdot \nabla\big)\Psi\big] \, dx \, dt,$$

$$T_3^n := \int_0^T \int_{\mathcal{F}_0} \nabla\Phi^n \cdot \big[\big(\big(u^n + \nabla\Phi^n - \ell^n\big) \cdot \nabla\big)\Psi\big] \, dx \, dt.$$

From what precedes we infer that $(T_2^{n_k})_k$ and $(T_3^{n_k})_k$ converge respectively to T_2 and T_3, where

$$T_2 := \int_0^T \int_{\mathcal{F}_0} u \cdot \big[\big((\nabla\Phi - \ell) \cdot \nabla\big)\Psi\big] \, dx \, dt, \quad T_3 := \int_0^T \int_{\mathcal{F}_0} \nabla\Phi \cdot \big[\big((u + \nabla\Phi - \ell) \cdot \nabla\big)\Psi\big] \, dx \, dt,$$

where $\Phi := \ell_1 \Phi_1$.

The term T_1^n is more complicated. We would like to use vorticity to deal with this term, as in Delort's method where ruling out vorticity concentrations (formation of Dirac masses) allows to deal with the nonlinearity. However there is a difference here: the test function Ψ involved in the term T_1^n is not vanishing in general in the neighborhood of the boundary $\partial\mathcal{F}_0$, and not even on this boundary. We will use several arguments to fill this gap. In the next section we point out the role played by the normal trace of test functions.

2.4. Introduction of the vorticity in the nonlinearity

Let us start with the following lemma.

Lemma 7. *Let ω be smooth compactly supported in $\mathcal{F}_0$ such that $u := K[\omega]$ is in $L^2(\mathcal{F}_0)$. Then, for $\Psi \in C_c^1(\overline{\mathcal{F}_0})$ divergence free,*

$$\int_{\mathcal{F}_0} u \cdot \big[(u \cdot \nabla)\Psi\big] \, dx = -\frac{1}{2} \int_{\partial\mathcal{S}_0} \big|u \cdot \mathbf{n}^\perp\big|^2 \Psi \cdot \mathbf{n} \, ds + \int_{\mathcal{F}_0} \omega u \cdot \Psi^\perp \, dx. \tag{2.9}$$

Assume in addition that ω is NMS, then, also for $\Psi \in C_c^1(\overline{\mathcal{F}_0})$ divergence free,

$$\int_{\mathcal{F}_0^\pm} u \cdot \big[(u \cdot \nabla)\Psi\big]\, dx = -\frac{1}{2} \int_{\Gamma_\pm} \big|u \cdot \mathbf{n}^\perp\big|^2 \Psi \cdot \mathbf{n}\, ds + \int_{\mathcal{F}_0^\pm} \omega u \cdot \Psi^\perp\, dx. \tag{2.10}$$

Proof. Let us focus on the proof of (2.10); the proof of (2.9) being similar. First we observe that u is smooth, divergence free, in $L^2(\mathcal{F}_0)$ and is tangent to $\Gamma_\pm$ (since ω is NMS). Now, using that u and Ψ are divergence free, we obtain (see Appendix A.1 for a proof)

$$u \cdot \big[(u \cdot \nabla)\Psi\big] = u^\perp \cdot \nabla\big(\Psi^\perp \cdot u\big) + \Psi \cdot \nabla\left(\frac{1}{2}|u|^2\right). \tag{2.11}$$

Therefore integrating by parts, using that u is tangent to $\Gamma_\pm$, that $\operatorname{div} u^\perp = -\omega$ and that Ψ is divergence free, we get the desired result. □

Let us first recall what happens when Ψ is in $C_c^1(\mathcal{F}_0)$. This will already provide some useful informations in the next section.

Lemma 8. *Let ω in $\mathcal{BM}(\mathcal{F}_0)$, diffuse (that is $\omega(\{x\}) = 0$ for any $x \in \mathcal{F}_0$), with vanishing total mass, such that $u := K[\omega] \in L^2(\mathcal{F}_0)$. Let $\Psi \in C_c^1(\mathcal{F}_0)$ divergence free. Then*

$$\int_{\mathcal{F}_0} u \cdot (u \cdot \nabla\Psi)\, dx = -\frac{1}{2} \iint_{\mathcal{F}_0 \times \mathcal{F}_0} H_{\Psi^\perp}(x, y)\, d\omega(x)\, d\omega(y), \tag{2.12}$$

where

$$H_f(x, y) := f(x) \cdot K_{\mathcal{H}}(x, y) + f(y) \cdot K_{\mathcal{H}}(y, x).$$

When ω is smooth, the previous lemma follows from Lemma 7: it suffices to plug the definition of the Biot–Savart operator in the second term of the right hand side of (2.9) and to symmetrize. The gain of this symmetrization is that the auxiliary function $H_f(x, y)$ is bounded, whereas the Biot–Savart kernels $K(x, y)$ and $K_{\mathcal{H}}(x, y)$ are not. More precisely it also follows from the analysis in [16] that:

Proposition 9. *There exists a constant M_2 depending only on $\mathcal{F}_0$ such that for any $f \in C_c^1(\mathcal{F}_0; \mathbb{R}^2)$,*

$$\big|H_f(x, y)\big| \leqslant M_2 \|f\|_{W^{1,\infty}(\mathcal{F}_0)} \quad \forall x, y \in \mathcal{F}_0,\ x \neq y. \tag{2.13}$$

Proposition 9 is also true if one substitutes $K(x, y)$ to $K_{\mathcal{H}}(x, y)$ in the definition of H_f above. However the choice of $K_{\mathcal{H}}(x, y)$ seems better since it implies the extra property that for any $f \in C_c^1(\mathcal{F}_0; \mathbb{R}^2)$, H_f is tending to 0 at infinity, thanks to (1.14).

Using this, one infers that Lemma 8 also holds true for any diffuse measure by a regularization process. Let us refer again here to [16] for more details, or to the sequel of this paper where we will slightly extend this.

2.5. Temporal estimate of the fluid

We have the following.

Lemma 10. *There exists a subsequence $(\bar{v}^{n_k})_k$ of $(\bar{v}^n)_n$ which converges to $\bar{v}$ in $C([0, T]; \mathcal{H} - w)$, and such that $(\omega^{n_k})_k$ of $(\omega^n)_n$ converges to $\omega := \operatorname{curl} v$ in $C([0, T]; \mathcal{BM}(\mathcal{F}_0) - w^*)$.*

Proof. Let us consider a divergence free vector field Ψ in $C_c^\infty(\mathcal{F}_0)$, so that

$$\int_{\mathcal{F}_0} \Psi \cdot \partial_t v^n\, dx = \big(\Psi, \partial_t \bar{v}^n\big)_\rho = T_1^n + T_2^n + T_3^n,$$

where, thanks to Lemma 8,

$$T_1^n := -\frac{1}{2} \iint\limits_{\mathcal{F}_0\times\mathcal{F}_0} H_{\Psi^\perp}(x,y)\omega^n(x)\omega^n(y)\,dx\,dy.$$

We can infer from Proposition 9 and (2.2) that

$$\left|\int\limits_{\mathcal{F}_0} \Psi \cdot \partial_t v^n\,dx\right| \leqslant C\|\Psi\|_{H^1\cap W^{1,\infty}(\mathcal{F}_0)}.$$

Moreover using that for any $\phi \in C_c^\infty(\mathcal{F}_0)$ then $\Psi = \nabla^\perp\phi$ is a divergence free vector field in $C_c^\infty(\mathcal{F}_0)$, we get

$$\left|\int\limits_{\mathcal{F}_0} \phi \cdot \partial_t \omega^n\,dx\right| = \left|\int\limits_{\mathcal{F}_0} \Psi \cdot \partial_t v^n\,dx\right| \leqslant C\|\Psi\|_{H^1\cap W^{1,\infty}(\mathcal{F}_0)} \leqslant C\|\phi\|_{H^2\cap W^{2,\infty}(\mathcal{F}_0)}.$$

It is therefore sufficient to use the Sobolev embedding theorem and the following version of the Aubin–Lions lemma with $M > 2$ and with

1. $X = L^2(\mathcal{F}_0)$, $Y = H_0^M(\mathcal{F}_0)$, the completion of $C_c^\infty(\mathcal{F}_0)$ in the Sobolev space $H^M(\mathcal{F}_0)$, and $f_n = v^n$; and with
2. $X = C_0(\mathcal{F}_0)$ and $Y = H_0^{M+1}(\mathcal{F}_0)$ and $f_n = \omega^n$.

□

Lemma 11. *Let X and Y be two Banach spaces such that Y is dense in X and X is separable. Assume that $(f_n)_n$ is a bounded sequence in $L^\infty(0,T;X')$ such that $(\partial_t f_n)_n$ is bounded in $L^1(0,T;Y')$. Then $(f_n)_n$ is relatively compact in $C([0,T];X'-w^*)$.*

The proof of Lemma 11 is given in Appendix A.2 for sake of completeness.

A first consequence of the previous result is that we can pass to the limit the left hand side of (2.5): for any test function Ψ in $\mathcal{H}_T$, as $k \to +\infty$,

$$\big(\Psi(T,\cdot), \bar{v}^{n_k}(T,\cdot)\big)_\rho - \big(\Psi(0,\cdot), \bar{v}_0^{n_k}\big)_\rho \to \big(\Psi(T,\cdot), \bar{v}(T,\cdot)\big)_\rho - \big(\Psi(0,\cdot), \bar{v}_0\big)_\rho.$$

2.6. *Extension of the symmetrization process to tangent test functions*

Let us go back to the issue of passing to the limit Eq. (2.5) for a general test function Ψ in $\mathcal{H}_T$. The only remaining issue is to pass to the limit into the term involving T_1^n. We are going to use the following generalizations of Lemma 8 and Proposition 9.

Proposition 12. *There exists a constant M_2 depending only on $\mathcal{F}_0$ such that (2.13) holds true for any $f \in C_c^1(\overline{\mathcal{F}_0};\mathbb{R}^2)$ normal to the boundary.*

Proposition 12 can be proved thanks to the formula (1.13). Actually it can also be seen as a particular case of [11], Theorem 1. An extension to the case of several obstacles is given in [7].

Using Proposition 12, we can obtain the following.

Lemma 13. *Let ω in $\mathcal{BM}(\overline{\mathcal{F}_0})$, diffuse (that is $\omega(\{x\}) = 0$ for any $x \in \overline{\mathcal{F}_0}$), with vanishing total mass, and such that $u := K[\omega] \in L^2(\mathcal{F}_0)$. Then for any $\Psi \in C_{c,\sigma}^1(\overline{\mathcal{F}_0})$,*

$$\int\limits_{\mathcal{F}_0} u\cdot(u\cdot\nabla\Psi)\,dx = -\frac{1}{2}\iint\limits_{\overline{\mathcal{F}_0}\times\overline{\mathcal{F}_0}} H_{\Psi^\perp}(x,y)\,d\omega(x)\,d\omega(y). \tag{2.14}$$

Proof. Let $\Psi \in C^1_{c,\sigma}(\overline{\mathcal{F}_0})$. By mollification there exists a sequence of smooth functions ω^ε, with vanishing total mass, converging to ω weakly* in $\mathcal{BM}(\overline{\mathcal{F}_0})$ and such that $u^\varepsilon := K[\omega^\varepsilon]$ converges strongly to u in $L^2(\mathcal{F}_0)$. Moreover, for any ε, it follows from Lemma 7 that

$$\int_{\mathcal{F}_0} u^\varepsilon \cdot \big(u^\varepsilon \cdot \nabla\Psi\big)\,dx = -\frac{1}{2}\iint_{\mathcal{F}_0\times\mathcal{F}_0} H_{\Psi^\perp}(x,y)\omega^\varepsilon(x)\omega^\varepsilon(y)\,dx\,dy. \tag{2.15}$$

As $\varepsilon \to 0$, the left hand side of (2.15) converges to the one of (2.14). On the other hand, we use the following lemma, borrowed from [3], with $X = \overline{\mathcal{F}_0} \times \overline{\mathcal{F}_0}$, $\mu_\varepsilon = \omega^\varepsilon \otimes \omega^\varepsilon$, $f = H_{\Psi^\perp}$, $F = \{(x,x)/x \in \overline{\mathcal{F}_0}\}$, to pass to the limit the right hand side. $\square$

Lemma 14. *Let X be a locally compact metric space. Let $(\mu_\varepsilon)_{\varepsilon\in(0,1)}$ be a sequence in $\mathcal{BM}(X)$ converging, as $\varepsilon \to 0$ to μ weakly* in $\mathcal{BM}(X)$ and $(\nu_\varepsilon)_\varepsilon$ be a sequence in $\mathcal{BM}_+(X)$ converging to ν weakly* in $\mathcal{BM}(X)$, with, for any ε, $|\mu_\varepsilon| \leqslant \nu_\varepsilon$. Let F be a closed subset of X with $\nu(F) = 0$. Let f be a Borel bounded function on X, continuous on $X \setminus F$ and such that*

$$\forall\eta > 0, \quad \exists K \subset X / \, |f| \leqslant \eta \quad \text{on } X \setminus K. \tag{2.16}$$

Then $\int_X f\,d\mu_\varepsilon \to \int_X f\,d\mu$ as $\varepsilon \to 0$.

A proof of Lemma 14 is provided as Appendix A.3 for sake of completeness.

2.7. A slowly varying lift

Yet the test function Ψ in T_1^n is not normal to the boundary so that we still cannot apply Lemma 13. The following lemma, which is somehow reminiscent of the fake layer constructed in [17], allows to correct this with an arbitrarily small collateral damage.

Lemma 15. *Let $\Psi \in \mathcal{H}$. Then there exist $(\tilde{\Psi}^\varepsilon)_{0<\varepsilon\leqslant 1}$ some smooth compactly supported divergence free vector fields on $\overline{\mathcal{F}_0}$ such that $\tilde{\Psi}^\varepsilon = \ell_\Psi$ on $\partial\mathcal{S}_0$ and such that $\|\nabla\tilde{\Psi}^\varepsilon\|_{L^\infty(\mathcal{F}_0)} \to 0$ when $\varepsilon \to 0^+$.*

Proof. Let ξ be a smooth cut-off function from $[0,+\infty)$ to $[0,1]$ with $\xi(0) = 1$, $\xi'(0) = 0$ and $\xi(r) = 0$ for $r \geqslant 1$. Then define for $x \in \overline{\mathcal{F}_0}$ and $0 < \varepsilon \leqslant 1$,

$$-\tilde{\Psi}^\varepsilon(t,x) := \nabla^\perp\big(\xi\big(\varepsilon\big(|x|-1\big)\big)\ell_\Psi^\perp \cdot x\big) = \xi\big(\varepsilon\big(|x|-1\big)\big)\ell_\Psi + \Sigma^\varepsilon(\varepsilon x), \tag{2.17}$$

where we have denoted, for $X \in \mathbb{R}^2$ with $|X| \geqslant \varepsilon$,

$$\Sigma^\varepsilon(X) := \ell_\Psi^\perp \cdot X\xi'\big(|X|-\varepsilon\big)\frac{1}{|X|}X^\perp.$$

It is not difficult to see that Σ^ε and $\tilde{\Psi}^\varepsilon$ are smooth and compactly supported, and that $(\|\Sigma^\varepsilon(\cdot)\|_{Lip(\mathcal{F}_0)})_{0<\varepsilon\leqslant 1}$ is bounded. Now that $\tilde{\Psi}^\varepsilon$ is divergence free follows from the first identity in (2.17). Let us now use the second one. First it shows that for x in $\partial\mathcal{F}_0$, that is for $|x| = 1$, $\tilde{\Psi}^\varepsilon(x) = \ell_\Psi(x)$. Finally, we infer from the chain rule that $\|\nabla\tilde{\Psi}^\varepsilon\|_{L^\infty(\mathcal{F}_0)} \to 0$ when $\varepsilon \to 0^+$. $\square$

Let Ψ be in $\mathcal{H}_T$. Lemma 15 provides a family $(\tilde{\Psi}^\varepsilon)_{0<\varepsilon\leqslant 1}$, the time t being here a harmless parameter. Let us also introduce

$$\check{\Psi}^\varepsilon := \Psi - \tilde{\Psi}^\varepsilon,$$

which is in $C^1([0,T];\mathcal{H})$ and satisfies $\check{\Psi}^\varepsilon \cdot \mathbf{n} = 0$ on $\partial\mathcal{S}_0$. We split T_1^n into $T_1^n = \check{T}_1^{n,\varepsilon} + \tilde{T}_1^{n,\varepsilon}$ with

$$\check{T}_1^{n,\varepsilon} := \int_0^T\int_{\mathcal{F}_0} u^n \cdot \big[\big(u^n\cdot\nabla\big)\check{\Psi}^\varepsilon\big]\,dx\,dt \quad \text{and} \quad \tilde{T}_1^{n,\varepsilon} := \int_0^T\int_{\mathcal{F}_0} u^n \cdot \big[\big(u^n\cdot\nabla\big)\tilde{\Psi}^\varepsilon\big]\,dx\,dt.$$

We are going to prove that T_1^n converges to $T_1 = \check{T}_1^\varepsilon + \tilde{T}_1^\varepsilon$ with

$$\check{T}_1^\varepsilon := \int_0^T \int_{\mathcal{F}_0} u \cdot \left[(u \cdot \nabla)\check{\Psi}^\varepsilon \right] dx\, dt, \qquad \tilde{T}_1^\varepsilon := \int_0^T \int_{\mathcal{F}_0} u \cdot \left[(u \cdot \nabla)\tilde{\Psi}^\varepsilon \right] dx\, dt.$$

Thanks to (2.2) and Lemma 15, $\limsup_n |\tilde{T}_1^{n,\varepsilon}| + |\tilde{T}_1^\varepsilon| \to 0$ when $\varepsilon \to 0^+$, so that in order to achieve the proof of Theorem 3 it is sufficient to prove that for $\varepsilon > 0$,

$$\check{T}_1^{n,\varepsilon} \to \check{T}_1^\varepsilon \quad \text{when } n \to \infty. \tag{2.18}$$

Actually we are going to first prove that for $\varepsilon > 0$, when $n \to \infty$,

$$\int_0^T \iint_{\overline{\mathcal{F}_0} \times \overline{\mathcal{F}_0}} H_{(\check{\Psi}^\varepsilon)^\perp}(x, y)\, \omega^n(t, x)\, \omega^n(t, y)\, dx\, dy\, dt \to \int_0^T \iint_{\overline{\mathcal{F}_0} \times \overline{\mathcal{F}_0}} H_{(\check{\Psi}^\varepsilon)^\perp}(x, y)\, d\omega_t(x)\, d\omega_t(y)\, dt, \tag{2.19}$$

and that the measure $\omega_t := \omega(t, \cdot)$ is diffuse for almost every time $t \in (0, T)$. Then we will apply Lemma 13 to $f = (\check{\Psi}^\varepsilon)^\perp$ to get

$$\check{T}_1^{n,\varepsilon} = -\frac{1}{2} \int_0^T \iint_{\overline{\mathcal{F}_0} \times \overline{\mathcal{F}_0}} H_{(\check{\Psi}^\varepsilon)^\perp}(x, y)\, \omega^n(t, x) \omega^n(t, y)\, dx\, dy\, dt$$

and

$$\check{T}_1^\varepsilon = -\frac{1}{2} \int_0^T \iint_{\overline{\mathcal{F}_0} \times \overline{\mathcal{F}_0}} H_{(\check{\Psi}^\varepsilon)^\perp}(x, y)\, d\omega_t(x)\, d\omega_t(y)\, dt,$$

and this will prove (2.18).

Now, in order to prove (2.19) we would like to proceed as in the convergence part of the proof of Lemma 13. However there are some extra difficulties in particular because of the time dependence. We will proceed in three steps.

2.8. *Non-concentration of the vorticity up to the boundary*

Here we are going to prove a result of non-concentration of the vorticity up to the boundary. We will here also follow closely [11]. We first isolate the following identity which will be useful twice in the sequel.

Lemma 16. *Let ϕ be a smooth function on $\overline{\mathcal{F}_{0,+}}$ with bounded derivatives up to second order. Then*

$$\int_{\mathcal{F}_{0,+}} \phi \partial_t \omega^n\, dx = \frac{1}{2} \int_{\Gamma_+} \left| u \cdot \mathbf{n}^\perp \right|^2 \nabla\phi \cdot \mathbf{n}^\perp\, ds - \int_{\mathcal{F}_{0,+}} u^n \cdot \left[\left(u^n \cdot \nabla\right)\Psi \right] dx + \int_{\mathcal{F}_{0,+}} \nabla\phi \cdot \left(\ell_1^n \nabla\Phi_1 - \ell^n\right)\omega^n\, dx, \tag{2.20}$$

where $\Psi := \nabla^\perp \phi$.

Proof. Using Eq. (2.4) and an integration by parts, we have

$$\partial_t \int_{\mathcal{F}_{0,+}} \phi \omega^n = \int_{\mathcal{F}_{0,+}} \phi \partial_t \omega^n\, dx = -\int_{\mathcal{F}_{0,+}} \phi\left(v^n - \ell^n\right) \cdot \nabla\omega^n\, dx = \int_{\mathcal{F}_{0,+}} \nabla\phi \cdot \left(v^n - \ell^n\right)\omega^n\, dx.$$

Using now the decomposition (2.1) we get

$$\partial_t \int_{\mathcal{F}_{0,+}} \phi \omega^n\, dx = I_1^n + I_2^n, \tag{2.21}$$

where

$$I_1^n := \int_{\mathcal{F}_{0,+}} \nabla\phi \cdot u^n \omega^n \, dx \quad \text{and} \quad I_2^n := \int_{\mathcal{F}_{0,+}} \nabla\phi \cdot \big(\ell_1^n \nabla\Phi_1 - \ell^n\big)\omega^n \, dx.$$

Using Lemma 7 with $\Psi := \nabla^\perp \phi$, we get

$$I_1^n = \frac{1}{2}\int_{\Gamma_+} \big|u \cdot \mathbf{n}^\perp\big|^2 \nabla\phi \cdot \mathbf{n}^\perp \, ds - \int_{\mathcal{F}_{0,+}} u^n \cdot \big[\big(u^n \cdot \nabla\big)\Psi\big] \, dx. \qquad \square$$

We first use the previous identity to obtain the following estimate.

Lemma 17. *Let ϕ be a smooth function on $\overline{\mathcal{F}_{0,+}}$ with bounded derivatives up to second order. Then there exists $C > 0$ which depends only on ϕ, on $\|\bar{v}_0^n\|_\rho$ and on $\|\omega_0^n\|_{L^1(\mathcal{F}_0)}$ such that*

$$\frac{1}{2}\int_0^T \int_{\Gamma_+} \big|u^n \cdot \mathbf{n}^\perp\big|^2 \nabla\phi \cdot \mathbf{n}^\perp \, ds \, dt \leqslant C.$$

Proof. We integrate in time (2.21) to get

$$\frac{1}{2}\int_0^T \int_{\Gamma_+} \big|u^n \cdot \mathbf{n}^\perp\big|^2 \nabla\phi \cdot \mathbf{n}^\perp \, ds \, dt \leqslant -\int_0^T I_2^n \, dt + \int_{\mathcal{F}_{0,+}} \phi\omega^n(T,\cdot) \, dx - \int_{\mathcal{F}_{0,+}} \phi\omega_0^n \, dx + \int_0^T \int_{\mathcal{F}_{0,+}} u^n \cdot \big[\big(u^n \cdot \nabla\big)\Psi\big] \, dx \, dt.$$

It remains to use trivial bounds and (2.2)–(2.3) to conclude. $\square$

Using a smooth perturbation of $\arctan(x_1)$ instead of ϕ yields that there exists $C > 0$ such that for any n,

$$\int_0^T \int_{\Gamma_+} \frac{1}{1+x_1^2}\big|u^n \cdot \mathbf{n}^\perp\big|^2 \, ds \leqslant C. \tag{2.22}$$

Then one infers from (2.2) and (2.22), following exactly the proof of Lemma 2 in [11], that for any compact $K \subset \overline{\mathcal{F}_{0,+}}$ there exists $C > 0$ such that for any $0 < \delta < 1$, for any n,

$$\int_0^T \sup_{x \in K} \int_{B(x,\delta) \cap \mathcal{F}_{0,+}} \big|\omega^n(t,y)\big| \, dy \, dt \leqslant C|\log\delta|^{-1/2}. \tag{2.23}$$

Of course we can proceed similarly on $\mathcal{F}_{0,-}$.

2.9. Temporal estimate of the fluid up to the boundary

Using again the identity (2.20), together with (2.22) and some trivial bounds we obtain that $(\partial_t \omega^n)_n$ is bounded in $L^1(0,T;Y')$ with

$$Y := \big\{\phi \in C^2(\overline{\mathcal{F}}_0)/\phi,\ \big(1+x_1^2\big)\nabla\phi \text{ and } \nabla^2\phi \text{ are bounded}\big\},$$

endowed with the norm

$$\|\phi\|_Y := \sup_{x\in\overline{\mathcal{F}}_0} \big(\big|\phi(x)\big| + \big(1+x_1^2\big)\big|\nabla\phi(x)\big| + \big|\nabla^2\phi(x)\big|\big).$$

Therefore thanks to Lemma 11 we obtain that $(\omega^n)_n$ is relatively compact in $C([0,T];\mathcal{BM}(\overline{\mathcal{F}_0}) - w^*)$.

As a first consequence (2.6), (2.7) and $u = K[\omega]$ hold true for any t in $[0,T]$. Moreover since the vorticities ω^n are NMS, we also have that the sequence $(|\omega^n|)_n$ is relatively compact in $C([0,T];\mathcal{BM}_+(\overline{\mathcal{F}_0}) - w^*)$, there exists a subsequence which converges weakly* to, say, σ.

Thus there exists a subsequence $(\omega^{n_k}\otimes\omega^{n_k})_k$ which converges to $\omega\otimes\omega$ in $C([0,T];\mathcal{BM}(\overline{\mathcal{F}_0}\times\overline{\mathcal{F}_0}) - w^*)$ and such that $(|\omega^{n_k}|\otimes|\omega^{n_k}|)_k$ converges to $\sigma\otimes\sigma$ in $C([0,T];\mathcal{BM}_+(\overline{\mathcal{F}_0}\times\overline{\mathcal{F}_0}) - w^*)$.

2.10. End of the proof of Theorem 3

In order to achieve the proof of Theorem 3 let us now prove that (2.19) holds true. Using Fatou's lemma we deduce from (2.23) that $\sup_{x\in\overline{\mathcal{F}_0}}\sigma(t,\{x\}) = 0$ for any t in $[0,T]$. Thus if we denote by $F := \{(x,x)/\ x\in\overline{\mathcal{F}_0}\}$ then $(\sigma\otimes\sigma)(t,F) = 0$ for any t in $[0,T]$. Using Proposition 12 and the following time-dependent variant of Lemma 14 with $X = \overline{\mathcal{F}_0}\times\overline{\mathcal{F}_0}$, $\mu^{n_k} = \omega^{n_k}\otimes\omega^{n_k}$, $\nu^{n_k} = |\omega^{n_k}|\otimes|\omega^{n_k}|$, $f = H_{(\check{\Psi}^\varepsilon)^\perp}$, we obtain (2.19).

Lemma 18. *Let X be a locally compact metric space and $T > 0$. Let*

- *$(\mu^n)_{n\in(0,1)}$ be a sequence in $C([0,T];\mathcal{BM}(X))$ converging, as $n\to+\infty$ to μ in $C([0,T];\mathcal{BM}(X) - w^*)$ and $(\nu^n)_n$ be a sequence in $C([0,T];\mathcal{BM}_+(X))$ converging to ν in $C([0,T];\mathcal{BM}_+(X) - w^*)$, with, for any $n\in\mathbb{N}$, for any $t\in[0,T]$, $|\mu^n(t,\cdot)| \leqslant \nu^n(t,\cdot)$,*
- *F be a closed subset of X with $\nu(t,F) = 0$ for any t in $[0,T]$,*
- *f be a Borel bounded function on $[0,T]\times X$, continuous on $[0,T]\times(X\setminus F)$ and such that*

$$\forall\varepsilon>0,\quad \exists K\subset X/|f|\leqslant\varepsilon\quad \text{on } [0,T]\times(X\setminus K).$$

Then $\int_0^T\int_X f(x)\,d\mu_t^n(x)\,dt \to \int_0^T\int_X f(x)\,d\mu_t(x)\,dt$ as $n\to+\infty$.

The proof of Lemma 18 follows the one of Lemma 14 and we will therefore skip it.

3. Proof of Theorem 5

As the proof of Theorem 5 will follow quite closely the one of Theorem 3, we will only outline the differences. The first observation is that, since $\ell_0 = 0$, the energy inequality guaranteed by Theorem 3 reads

$$m\big(\ell^m\big)^2(t,\cdot) + \int_{\mathcal{F}_0} v^m(t,\cdot)^2\,dx \leqslant \int_{\mathcal{F}_0} v_0^2\,dx. \tag{3.1}$$

Therefore $(\ell^m)_m$ converges to 0 in $L^\infty(0,T)$ as m goes to $+\infty$.

Then proceeding as in Section 2.2 we obtain that $(\ell^m)_m$ converges to 0 in $W^{1,\infty}([0,T])$ as m goes to $+\infty$.

The inequality (3.1) also provides that $(v^m)_m$ is bounded in $L^\infty(0,T;L^2_\sigma(\mathcal{F}_0))$ so that there exists a subsequence $(v^{m_k})_k$ converges in $L^\infty(0,T;L^2_\sigma(\mathcal{F}_0))$ weak* to v.

Let us now prove that v is a weak solution of (1.22)–(1.25) in $[0,T]$. Let us consider $\Psi\in C^1_{c,\sigma}([0,T]\times\overline{\mathcal{F}}_0)$. Then extending Ψ by 0 in $\mathcal{S}_0$, we observe that $\Psi\in\mathcal{H}_T$ so that, for any $k\in\mathbb{N}$,

$$\begin{aligned}&\int_{\mathcal{F}_0}\Psi(T,\cdot)\cdot\bar v^{m_k}(T,\cdot)\,dx - \int_{\mathcal{F}_0}\Psi(0,\cdot)\cdot\bar v_0^{m_k}\,dx\\ &\quad= \int_0^T\frac{\partial\Psi}{\partial t}\cdot\bar v^{m_k}\,dx\,dt + \int_0^T\int_{\mathcal{F}_0} v^{m_k}\cdot\big[\big(\big(v^{m_k}-\ell_{v^{m_k}}\big)\cdot\nabla\big)\Psi\big]\,dx\,dt.\end{aligned} \tag{3.2}$$

The last term can be split as in Section 2.3 in

$$\int_0^T\int_{\mathcal{F}_0} v^m\cdot\big(\big(v^m-\ell^m\big)\cdot\nabla\big)\Psi\,dx\,dt = T_1^m+T_2^m+T_3^m$$

where

$$T_1^m := \int_0^T\int_{\mathcal{F}_0} u^m\cdot\big[\big(u^m\cdot\nabla\big)\Psi\big]\,dx\,dt,$$

$$T_2^m := \int_0^T\int_{\mathcal{F}_0} u^m\cdot\big[\big(\big(\nabla\Phi^m-\ell^m\big)\cdot\nabla\big)\Psi\big]\,dx\,dt,$$

$$T_3^m := \int_0^T\int_{\mathcal{F}_0} \nabla\Phi^m\cdot\big[\big(\big(u^m+\nabla\Phi^m-\ell^m\big)\cdot\nabla\big)\Psi\big]\,dx\,dt.$$

We can already deduce from what precedes that $(T_2^{m_k})_k$ and $(T_3^{m_k})_k$ converge to 0.

Next we observe that the vorticities ω^m given by Theorem 3 are diffuse and satisfy for any $t\in[0,T]$, $\|\omega^m(t,\cdot)\|_{\mathcal{BM}(\overline{\mathcal{F}_0})}\leqslant\|\omega_0\|_{\mathcal{BM}(\mathcal{F}_0)}$. Moreover the temporal estimates of Sections 2.2 and 2.9 and the non-concentration estimate of Section 2.8 are uniform in the limit $m\to+\infty$, so that we can proceed as in the proof of Theorem 3 in order to prove that, up to a subsequence, $(v^{m_k})_k$ converges in $C([0,T];L^2_\sigma(\mathcal{F}_0)-w)$ to v, which entails that the first term of the left hand side of (3.2) converges to the first term of the left hand side of (1.26), and that $T_1^{m_k}$ converges to the last term of (1.26) when k goes to $+\infty$. Actually the proof is even a little bit simplified since the test function Ψ is tangent to the boundary so that the lift of Section 2.7 is not necessary here.

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Appendix A

A.1. Proof of (2.11)

Let us denote by $L:=-\Psi\cdot\nabla(\frac{1}{2}|u|^2)+u\cdot[(u\cdot\nabla)\Psi]$ and by $R:=u^\perp\cdot\nabla(\Psi^\perp\cdot u)$. We extend L and R into

$$\begin{aligned}L=\sum_{i=1}^8 L_i &= -\Psi_1u_1\partial_1u_1-\Psi_1u_2\partial_1u_2-\Psi_2(\partial_2u_2)u_2-\Psi_2(\partial_2u_1)u_1\\&\quad+u_1^2\partial_1\Psi_1+u_1u_2\partial_1\Psi_2+u_1u_2\partial_2\Psi_1+u_2^2\partial_2\Psi_2,\\ R=\sum_{i=1}^8 R_i &= u_2(\partial_1\Psi_2)u_1+u_2\Psi_2\partial_1u_1-u_2^2\partial_1\Psi_1-u_2\Psi_1\partial_1u_2\\&\quad-u_1^2\partial_2\Psi_2-u_1\Psi_2\partial_2u_1+\Psi_1u_2\partial_2u_2+u_1u_2\partial_2\Psi_1,\end{aligned}$$

and observe that $L_1=R_7$, $L_2=R_4$, $L_3=R_2$, $L_4=R_6$, $L_5=R_5$, $L_6=R_1$, $L_7=R_8$, $L_8=R_3$, where we use that u is divergence free for the first and third equalities, and that Ψ is divergence free for the fifth and last equalities.

A.2. Proof of Lemma 11

We follow the strategy of Appendix C of [9]. Let $\overline{B}(0,R)$ be a closed ball of X' containing all the values $f_n(t)$ for all $t\in[0,T]$, for all $n\in\mathbb{N}$. Since X is separable, this ball is a compact metric space for the w^* topology. Indeed since

Y is dense in X, one distance for this topology is given as follows: let $(\phi_j)_{j\geqslant 1}$ be a sequence of Y dense in X, and define, for f, g in $\overline{B}(0,R)$,

$$d(f,g) := \sum_{j\geqslant 1} \frac{1}{2^j} \frac{|\langle f-g, \phi_j\rangle_{X',X}|}{1+|\langle f-g,\phi_j\rangle_{X',X}|}.$$

Let $\varepsilon > 0$ and k such that $\frac{1}{2^k} < \varepsilon$. For any $t, s \in [0,T]$, for all $n \in \mathbb{N}$,

$$d\big(f_n(t), f_n(s)\big) \leqslant \sup_{1\leqslant j\leqslant k} \big|\big\langle f_n(t) - f_n(s), \phi_j\big\rangle_{X',X}\big| + \varepsilon.$$

But using now that $(\partial_t f_n)_n$ is bounded in $L^1(0,T;Y')$ we get that

$$\sup_{1\leqslant j\leqslant k} \big|\big\langle f_n(t) - f_n(s), \phi_j\big\rangle_{X',X}\big| \to 0 \quad \text{when } t-s \to 0.$$

Therefore the sequence $(f_n)_n$ is equicontinuous in $C([0,T]; \overline{B}(0,R) - w^*)$.

Thanks to the Arzela–Ascoli theorem we deduce the desired result.

A.3. *Proof of Lemma 14*

Let us denote by $I_\varepsilon := \int_X f\,d\mu_\varepsilon - \int_X f\,d\mu$. Let $\eta > 0$. We are going to prove that for ε small enough, $|I_\varepsilon| \leqslant 4\eta$.

Let $M := \nu(X) + \sup_\varepsilon \nu_\varepsilon(X)$ which is finite by the Banach–Steinhaus theorem. Using (2.16) we obtain that there exists a compact subset K of X such that $|f| \leqslant \eta/M$ on $X \setminus K$. Let us decompose I_ε into $I_\varepsilon = I_\varepsilon^1 + I_\varepsilon^2$ with

$$I_\varepsilon^1 := \int_{X\setminus K} f\,d\mu_\varepsilon - \int_{X\setminus K} f\,d\mu \quad \text{and} \quad I_\varepsilon^2 := \int_K f\,d\mu_\varepsilon - \int_K f\,d\mu.$$

First we have $|I_\varepsilon^1| \leqslant \eta$ thanks to the previous choice of K. It therefore remains to prove that for ε small enough, $|I_\varepsilon^2| \leqslant 3\eta$.

Now, let us introduce a smooth cut-off function ξ on $\mathbb{R}$ such that $\xi(x) = 1$ for $|x| \leqslant 1$ and $\xi(x) = 0$ for $|x| \geqslant 2$. We denote, for $\delta > 0$ and $x \in X$, $\beta_\delta(x) := \xi(\frac{\operatorname{dist}(x,F)}{\delta})$. We decompose I_ε^2 into $I_\varepsilon^2 = I_{\varepsilon,\delta}^3 + I_{\varepsilon,\delta}^4$, where

$$I_{\varepsilon,\delta}^3 := \int_K \beta_\delta f\,d\mu_\varepsilon - \int_K \beta_\delta f\,d\mu \quad \text{and} \quad I_{\varepsilon,\delta}^4 := \int_K (1-\beta_\delta) f\,d\mu_\varepsilon - \int_K (1-\beta_\delta) f\,d\mu.$$

We have

$$\big|I_{\varepsilon,\delta}^3\big| \leqslant \|f\|_\infty \bigg(\int_K \beta_\delta\,d\nu_\varepsilon + \int_K \beta_\delta\,d\nu \bigg) \leqslant 2\|f\|_\infty \int_K \beta_\delta\,d\nu + \eta$$

for ε small enough, by weak-* convergence. Since $\nu(F) = 0$ there exists $\delta > 0$ such that $2\|f\|_\infty \int_K \beta_\delta\,d\nu \leqslant \eta$.

Now using for this δ that $(1-\beta_\delta)f$ is continuous on X and that $(\mu_\varepsilon)_\varepsilon$ is converging to μ weakly-* in $\mathcal{BM}(X)$, we get $|I_{\varepsilon,\delta}^4| \leqslant \eta$ for ε small enough.

Gathering all the estimates yields the result.

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